The idea of the cylindrical matched feed (tri-mode feed) was proposed by Rudge and Adatia in 1975 [31]. Thereafter very little efforts have been made to practically implement the tri-mode horn and to present the experimental verifications. Most of the work carried out in this direction is based on the computer simulations. Through numerical results, Prasad and Shafai [45] have proved that the tri-mode feed can significantly improve the cross-polar performance of the offset parabolic reflector antenna. Bahadori and Rahmat-Samii [46, 47] have also used a tri-mode matched feed to simulate the performance of a gravitationally balanced back-to-back offset reflector antenna system. They have achieved an additional 20 dB improvement in the cross-polarization over a conventional Potter horn [48]. Shee and Smith [49] have proposed an algorithm to reduce the cross-polarization of an offset reflector illuminated by a cluster of tri-mode horns. However, they have not reported the measured results.

To the best of the author’s knowledge, the measured performance of a tri-mode matched feed has not been reported in the open literature. Also, no measured data on a circularly polarized offset parabolic reflector antenna, with a tri-mode matched feed, is reported in the published literature.

The primary objective of this chapter is to present the design of a tri-mode matched feed using the conjugate field matching technique and to present the measured performance of the said feed in conjunction with an offset reflector antenna. In a smooth-walled cylindrical waveguide, $\text{TE}_{11}$, $\text{TM}_{11}$ and $\text{TE}_{21}$ modes are combined in proper amplitude and phase to configure a tri-mode conjugate matched feed. The return-loss characteristic and primary radiation patterns of the proposed tri-mode horn
are presented in this chapter. The chapter also includes the comparison of matched feed performance with that of the conventional Potter horn. The results of several parametric studies are also summarized in the present chapter. Further, an additional feed design is also proposed to improve the cross-polar band-width of the offset reflector antenna. Finally, three possible applications of the proposed tri-mode feed are discussed.

4.1 Field Expressions For the Cylindrical Matched Feed

As mentioned in Table 2.1, matched feed can be implemented in a smooth-walled cylindrical structure by adding two additional higher order modes, i.e., TE_{21} and TM_{11} mode in proper amplitude and phase proportion with the fundamental TE_{11} mode. In this, a small component of TE_{21} mode compensates the asymmetric cross-polarization introduced by the offset geometry. The TM_{11} mode is also added to improve the axial symmetry of the feed co-polar components and to suppress the cross-polar components which otherwise radiate into the diagonal planes of the feed far-field pattern [50]. Accordingly, matched feed in a cylindrical structure can be considered as the extension of the well-known Potter horn [48] in which the TM_{11} mode is added with the fundamental TE_{11} mode.

For a tri-mode matched feed with an aperture radius of ‘a’, the \( \theta(\text{Theta}) \) and \( \phi(\text{Phi}) \) components of the far-field radiation pattern are [51],

\[
E_\theta = E_{0}^{\text{TE}_{11}} + c_1 E_{0}^{\text{TM}_{11}} + c_2 E_{0}^{\text{TE}_{21}} \quad (4.1)
\]

\[
E_\phi = E_{\phi}^{\text{TE}_{21}} + c_1 E_{\phi}^{\text{TM}_{11}} + c_2 E_{\phi}^{\text{TE}_{21}} \quad (4.2)
\]

where,
\[ c_1 = \text{arbitrary constant defining the relative power in TM}_{11} \text{ mode with respect to the fundamental TE}_{11} \text{ mode} \]

\[ c_2 = \text{arbitrary constant defining the relative power in TE}_{21} \text{ mode with respect to the fundamental TE}_{11} \text{ mode} \]

Using the general expressions of the polar and the azimuthal radiation pattern components of the TE and the TM waves, as given by Silver [41];

\[ E_{\theta}^{TE_{11}}, E_{\theta}^{TM_{11}}, E_{\phi}^{TE_{21}}, E_{\phi}^{TM_{11}}, E_{\phi}^{TE_{21}}, \] can be written as,

\[ E_{\theta}^{TE_{11}} = j^2 \frac{\omega a}{2R} \left( 1 + \frac{\rho_{11H}}{k} \cos \theta \right) \cdot J_1 \left( k a \sin \theta \right) \cdot \sin \phi \cdot e^{-jK} \] (4.3)

\[ E_{\theta}^{TM_{11}} = j^2 \frac{\omega a}{2R} \left( 1 + \frac{\rho_{11H}}{k} \cos \theta \right) \cdot \left( \frac{J_1 \left( k a \sin \theta \right)}{k a \sin \theta} \right) \cdot \left( \frac{J_1 \left( k a \sin \theta \right)}{1 - \left( \frac{k a \sin \theta}{k a \sin \theta} \right)^2} \right) \cdot \sin \phi \cdot e^{-jK} \] (4.4)

\[ E_{\phi}^{TE_{21}} = j^3 \frac{\omega a}{2R} \left( 1 + \frac{\rho_{21H}}{k} \cos \theta \right) \cdot J_2 \left( k a \sin \theta \right) \cdot \sin (2\phi) \cdot e^{-jK} \] (4.5)

\[ E_{\phi}^{TE_{11}} = j^2 \frac{\omega a}{2R} \cdot \left( \frac{\rho_{11H}}{k} \cos \theta \right) \cdot J_1 \left( k a \sin \theta \right) \cdot \sin \phi \cdot e^{-jK} \] (4.6)

\[ E_{\phi}^{TM_{11}} = 0 \] (4.7)

\[ E_{\phi}^{TE_{21}} = j^3 \frac{\omega a}{2R} \cdot \left( \frac{\rho_{21H}}{k} \cos \theta \right) \cdot J_2 \left( k a \sin \theta \right) \cdot \sin (2\phi) \cdot e^{-jK} \] (4.8)

In (4.3) to (4.8),

\[ a = \text{aperture radius of the feed (in meter)}, \]

\[ R = \text{distance from the aperture centre to the observation point}, \]

\[ \omega = \text{angular frequency (in rad/sec)}, \]

\[ \mu = \text{permeability (in henry/meter)}, \]

\[ k = \frac{2\pi}{\lambda} = \text{free-space propagation constant}, \] (4.9)

\[ J_1 = \text{first-order Bessel function of the first kind}, \]
\( J_2 \) = second-order Bessel function of the first kind,
\( J'_1 \) = first derivative of \( J_1 \) with respect to its argument,
\( J'_2 \) = first derivative of \( J_2 \) with respect to its argument,

\[
\beta_{11H} = \sqrt{k^2 - k_{11H}^2} = \text{propagation constant of the TE}_{11} \text{ mode,}
\]
\( \beta_{21H} = \sqrt{k^2 - k_{21H}^2} = \text{propagation constant of the TE}_{21} \text{ mode,} \)  
\( \beta_{11E} = \sqrt{k^2 - k_{11E}^2} = \text{propagation constant of the TM}_{11} \text{ mode,} \)

\[
k_{11H} = \frac{x_{11}}{a} = \text{cutoff wave number of the TE}_{11} \text{ mode,}
\]
\( k_{21H} = \frac{x_{21}}{a} = \text{cutoff wave number of the TE}_{21} \text{ mode,} \)
\( k_{11E} = \frac{x_{11}}{a} = \text{cutoff wave number of the TM}_{11} \text{ mode,} \)

\[
x'_{11} = k_{11H} \cdot a = \text{first root of } J'_1 = 1.841,
\]
\( x'_{21} = k_{21H} \cdot a = \text{first root of } J'_2 = 3.054, \)
\( x_{11} = k_{11E} \cdot a = \text{first root of } J_1 = 3.832 \)

After eliminating constants and combining (4.3) to (4.5),

\[
E_0 = \left[ \left( 1 + \frac{\beta_{11H}}{k} \cos \theta \right) \left( \frac{J_1(ka \sin \theta)}{ka \sin \theta} \right) \sin \phi \right] + \\
\quad c_1 \left[ \left( \frac{\beta_{11E}}{k} + \cos \theta \right) \left( \frac{J_1(ka \sin \theta)}{ka \sin \theta} \right) \left( \frac{1}{1 - \left( \frac{k_{11E}}{ka \sin \theta} \right)^2} \right) \sin \phi \right] + \\
\quad j c_2 \left[ 2 \left( 1 + \frac{\beta_{21H}}{k} \cos \theta \right) \left( \frac{J_2(ka \sin \theta)}{ka \sin \theta} \right) \sin 2\phi \right] 
\]

(4.19)

Similarly, after eliminating the constants and combining (4.6) to (4.8),

\[
E_\phi = \left[ \left( \frac{\beta_{11H}}{k} + \cos \theta \right) \left( \frac{J'_1(ka \sin \theta)}{1 - \left( \frac{k_{11H}}{ka \sin \theta} \right)^2} \right) \cos \phi \right] + j c_2 \left[ \left( \frac{\beta_{21H}}{k} + \cos \theta \right) \left( \frac{J'_2(ka \sin \theta)}{1 - \left( \frac{k_{21H}}{ka \sin \theta} \right)^2} \right) \cos 2\phi \right] 
\]

(4.20)
4.2 **Numerical Results**

This section presents the various numerical results including the radiation patterns of the cylindrical matched feed and that of the offset parabolic reflector antenna. In addition, the effects of various matched feed (tri-mode horn) parameters on the overall reflector performance are also discussed. All simulations were carried out at the operating frequency of 6.6 GHz for the offset reflector geometry described in section 3.2 (Fig. 3.1).

4.2.1 *Simulated Far-field (Primary) Radiation Patterns of the Cylindrical Matched Feed*

Before simulating the performance of the reflector, it was necessary to examine the radiation properties of the individual matched feed element. Using the closed form field expressions derived for the cylindrical matched feed in the previous section, a MATLAB based computer program was developed to predict the far-field patterns of the feed in both E and H-plane. The numerical results obtained from the computer program were verified with those of the GRASP-8W results and are shown in Fig. 4.1. The close agreement of the computed results and the GRASP-8W results validates the field expressions and the accuracy of the MATLAB codes. Further, high cross-polarization in the $\Phi = 90^\circ$ plane was expected due to the presence of asymmetrical $TE_{21}$ mode. However, this feed cross-polarization will counter balance the cross-polarization added by the offset reflector.
4.2.2 Variation in Peak Cross-Polarization as a Function of Relative Power in $TE_{21}$ Mode ($c_2$)

In a tri-mode matched feed, the values of constants, $c_1$ and $c_2$ play a very crucial role in deciding the overall performance of the feed with the offset reflector. Therefore,
as a first step of the exhaustive study, both the constants were numerically optimized. The values which gave the least peak cross-polarization in the secondary radiation pattern for $\Phi l = 90^\circ$ plane are summarized in Table 4.1.

**Table 4.1** Numerically optimized values of the constants for the tri-mode horn

<table>
<thead>
<tr>
<th>$(c_1)$</th>
<th>$(c_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.14</td>
</tr>
<tr>
<td>$0^\circ$ w.r.t. TE$_{11}$ mode</td>
<td>-90$^\circ$ w.r.t. TE$_{11}$ mode</td>
</tr>
</tbody>
</table>

In order to examine the effect on the reflector cross-polarization for variation in $c_2$, an interesting study was carried out. The constant $c_1$ was set to its optimum value and the constant $c_2$ was varied from 0 to 1 in a step of 0.01. For each value of $c_2$, the reflector peak cross-polarization is shown in Fig. 4.2. From the results it is clear that, only for a specific value of $c_2$ the least peak cross-polarization can be achieved. It is important to note that an incorrect value of $c_2$ may degrade the cross-polar performance of the overall antenna system.

![Figure 4.2: Variation in peak cross-polarization as a function of relative power in TE$_{21}$ mode ($c_2$)](image-url)
4.2.3 Offset Angles ($\theta_0$) Versus Relative Power in TE$_{21}$ Mode ($c_2$)

This part of the thesis presents the outcome of an interesting study in which the numerically optimized values of constant $c_2$ were found out for different offset angles ($\theta_0$). In the beginning, for each value of the offset angle ($\theta_0$), the corresponding value of the half subtended angle ($\theta^*$) was calculated. Then, both these parameters were given as the input data to a MATLAB program. The program was developed to obtain the value of the constant $c_2$ which gives the lowest cross-polarization for the given offset reflector geometry. The results obtained through this program are plotted in Fig. 4.3. From the graph it can be concluded that, for an offset reflector with higher offset angle, the relative modal power in TE$_{21}$ mode should be high to compensate the reflector cross-polarization.

![Graph](image)

**Fig. 4.3** Offset angles ($\theta_0$) versus relative power in TE$_{21}$ mode ($c_2$)
4.2.4 Simulated Far-field (Secondary) Radiation Patterns of the Offset Reflector Antenna Fed by a Linearly Polarized Cylindrical Matched Feed

This sub-section summarizes the radiation performance of the offset parabolic reflector antenna illuminated by a linearly polarized feed. The reflector was illuminated by a conventional Potter horn (case-I) and by a cylindrical tri-mode feed (case-II). For both the cases, the co-polar and the cross-polar radiation patterns were estimated using the physical-optics approximation. It is important to note that the two-dimensional integral involved in the approximation must be solved with high accuracy. For the evaluation of such an integral, the Romberg integration method was found suitable.

Fig. 4.4 shows the estimated far-field radiation patterns for case-I. As visible in the figure, ideally there is no cross-polarization in the symmetric plane (Phî=0° plane). However, noticeable cross-polarization (approximately -25 dB) is observed in the asymmetric plane (Phî=90° plane). It is believed that, this high cross-polarization has resulted due to the asymmetry of the reflector (high offset angle and low F/D).

Fig. 4.5 shows the predicted co-polar and the cross-polar radiation patterns for case-II. As anticipated the cross-polarization is very low in the symmetric plane. Also, in the asymmetric plane, the cross-polarization is below -50 dB, which is quite low as compared to case-I (see Fig. 4.4). Based on these results, it can be concluded that the cylindrical tri-mode feed, in conjunction with offset reflector, significantly reduces the offset reflector cross-polarization.
Fig. 4.4 Simulated secondary radiation patterns of the offset reflector illuminated by a linearly polarized Potter horn

Fig. 4.5 Simulated secondary radiation patterns of the offset reflector illuminated by a linearly polarized tri-mode matched feed
4.2.5 Simulated Far-field (Secondary) Radiation Patterns of the Offset Reflector Antenna fed by a Circularly Polarized Cylindrical Matched Feed

After obtaining the satisfactory results with linear polarization, the tri-mode matched feed was tested for circular polarization. First, the reflector was illuminated by a circularly polarized Potter horn and the secondary radiation patterns were computed. As shown in Fig. 4.6, the radiation patterns are squinted from its boresight axis. The close examination of Fig. 4.6 reveals that the direction of the beam shifting is towards the right for the left circulation and towards the left for the right circulation.

![Simulated secondary radiation patterns of the offset reflector illuminated by a circularly polarized Potter horn](image)

**Fig. 4.6** Simulated secondary radiation patterns of the offset reflector illuminated by a circularly polarized Potter horn

Later on, a circularly polarized tri-mode matched feed was used as a feed to simulate the performance of the offset reflector. The results for both left and right hand polarization are shown in Fig. 4.7. The radiation patterns are absolutely squint-free. This, in turn, justifies the use of tri-mode feed for the offset reflector antenna.
Fig. 4.7 Simulated secondary radiation patterns of the offset reflector illuminated by a circularly polarized tri-mode matched feed

4.3 CYLINDRICAL MATCHED FEED DESIGN

The geometry of the proposed tri-mode matched feed [52] is depicted in Fig. 4.8.

The feed has been modeled using the commercially available HFSS [44] software.

Fig. 4.8 HFSS simulated design of the proposed tri-mode matched feed (a) Front View (b) Side View

(D1=34 mm, D2=52 mm, D3=70 mm, L1=40 mm, L2=65 mm, L3=25 mm, L4=135 mm)
The performance of the feed with the offset parabolic reflector antenna was simulated using the TICRA’s reflector design software (GRASP-8W). The horn design was finalized after carrying out extensive computer simulations, both on HFSS and GRASP-8W. Based on the parametric study, the design, which gave the least peak cross-polarization in the secondary radiation pattern of an offset parabolic reflector antenna, was finally selected.

As discussed in [53], a specific mode will propagate through the waveguide, if its free-space propagation constant is greater than the cutoff wave number of that mode. Accordingly, to excite the input waveguide of the tri-mode horn by a pure TE$_{11}$ mode, it should satisfy the condition, $k > k_{11H}$ or $k > \frac{x'_{11}}{a_1}$. This condition was imposed to select the radius ($a_1$) of the input waveguide as, $a_1 > \frac{x'_{11}}{k}$ or $a_1 > \frac{x'_{11}}{k} \left(= \frac{1.841}{k}\right)$. Based on this fact, the diameter ($D_1 = 2a_1$) was chosen to be 34 mm.

**Excitation of the TE$_{21}$ Mode:**

As mentioned in [26], the non-unity azimuthal dependent modes like TE$_{21}$ mode can be generated by introducing some change in the coaxial symmetry of the waveguide. Practically, by inserting a metallic post (pin) into the waveguide or by offsetting the axis of the input waveguide with respect to the axis of the horn, such modes can be generated. In the proposed horn, three identical cylindrical posts (pins) were used to generate the TE$_{21}$ mode. The dimensions of the posts (diameter and the height) decide the amplitude of the TE$_{21}$ mode. Further, it has been observed that the height of the post affects the return-loss performance of the horn. Thus, the post dimensions were selected such that they generate the TE$_{21}$ mode in the required amplitude-level while maintaining the satisfactory return-loss performance over a specified frequency band. The diameter $D_2$ is selected such that it allows TE$_{21}$ mode to
propagate. The waveguide sections with diameter D1 and D2 are connected by means of a tapered section with effective length L2. The reason for selecting a tapered section instead of a sharp step junction is to ensure the desired return loss.

**Excitation of the TM\textsubscript{11} Mode:**

The TM\textsubscript{11} mode falls under the category of unity azimuthal modes. Such modes can be generated by abrupt or gradual changes in the diameter of the horn [26]. For example, in case of a well-known Potter horn [48], a step discontinuity is used to excite the TM\textsubscript{11} mode. In the proposed tri-mode horn, the TM\textsubscript{11} mode was introduced by a step (D2/D3). The amplitude of the TM\textsubscript{11} mode is controlled by the step size (D2/D3), while the phase is adjusted by the length L3 and L4. Further, for the smooth propagation of the TM\textsubscript{11} mode, the radius \( a_3 \) \((D3 = 2a_3)\) was chosen to satisfy the condition,

\[
a_3 > \frac{x_{11}}{k} \left( \frac{3.832}{k} \right)
\]

As reported in [50], for the optimum performance of a tri-mode horn, the TM\textsubscript{11} mode amplitude should be approximately -5 dB and that of the TE\textsubscript{21} mode should be -20 dB relative to the fundamental TE\textsubscript{11} mode. Also, there should be an in-phase relationship between the TE\textsubscript{11} and TM\textsubscript{11} mode and a quadrature-phase relationship between TE\textsubscript{11} and TE\textsubscript{21} mode. For the proposed tri-mode horn, the required modal amplitudes of all the three modes were obtained by carefully selecting the diameters D2, D3; and the post dimensions. The desired phase relationship amongst the three modes was established by adjusting the lengths L3 and L4. The optimized modal power and phases of the tri-mode horn for the 6 mm post height are listed in Table 4.2. The total power in all the three modes is equal to 1 watt.
Table 4.2 Optimized modal parameters for the tri-mode horn

<table>
<thead>
<tr>
<th>Mode</th>
<th>TE₁₁ mode</th>
<th>TM₁₁ mode</th>
<th>TE₂₁ mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power (dB)</td>
<td>-1.17</td>
<td>-6.54</td>
<td>-19.3</td>
</tr>
<tr>
<td>Phase (degree)</td>
<td>50.8</td>
<td>53</td>
<td>-33.6</td>
</tr>
</tbody>
</table>

After deciding the dimensions of the horn, the design was sent to the Fabrication Division. During the fabrication, it was very necessary to maintain the maximum accuracy of all the dimensions. A slight change in the dimensions may act as a discontinuity and may excite the higher order modes. This may adversely affect the performance of the horn. The photograph of the actual tri-mode feed is shown in Fig. 4.9.

![Photograph of the actual tri-mode feed](image)

**Fig. 4.9** The photograph of the proposed tri-mode matched feed (with transition)  
(a) Front View (b) Side View

### 4.4 Measured Results

#### 4.4.1 Feed Return-Loss Measurement

After receiving the feed from the fabrication department, all its physical dimensions were measured and it was confirmed that the deviations are under the specified tolerance limit. The feed was then tested for the return-loss. The feed return-loss was measured, (i) without cylindrical posts, and (ii) with cylindrical posts. As is evident from Fig. 4.10, the return-loss is slightly better in case of a feed without posts. However, the return-loss is under the acceptance limit for the feed with inserted posts.
Fig. 4.10 The measured return-loss characteristic of the tri-mode matched feed

4.4.2 Far-field (Primary) Radiation Patterns of the Cylindrical Matched Feed

After getting the satisfactory return-loss characteristic, the far-field radiation pattern (primary pattern) measurements of the individual matched feed were carried out. For measurements in two principle planes (E & H planes), the necessary setup was arranged in an anechoic chamber. For comparison purpose, the measured and the simulated results are superimposed and are shown in Fig. 4.11. For Phi = 90° plane, the measured radiation patterns were found to be in close agreement with the simulated radiation patterns. However, some variations between the simulated performance and the measured performance for Phi = 0° plane were observed. It is believed that, this deviations have been resulted may be because of the slight mis-alignment of the feed or mis-tuning of the post height. In some cases, the limitations of the simulation software are also responsible for such deviations.
Fig. 4.11 The normalized simulated and measured radiation patterns of the HFSS designed tri-mode matched feed (a) For $\Phi=0^\circ$ (b) For $\Phi=90^\circ$
This sub-section describes the results of secondary radiation patterns obtained for the linearly polarized offset reflector antenna fed by a tri-mode matched feed. Before the actual measurements, the mechanical alignment of the reflector and the feed was ensured. The photograph of the complete antenna system, captured at the CATF (CCR-75/60) is shown in Fig. 4.12.

For the two principal planes (\(\Phi = 0^\circ\) and \(\Phi = 90^\circ\)), different sets of radiation patterns measurements were obtained by changing the post height. After careful examination of all the results, it was found that the maximum cross-pol suppression occurs for the post height of 6mm. For this optimum post height, the results are shown in Fig. 4.13.
Fig. 4.13 Measured secondary radiation patterns of the offset reflector illuminated by a linearly polarized tri-mode matched feed (a) For \( \Phi = 0^\circ \) (b) For \( \Phi = 90^\circ \)
Generally in an offset reflector, the results of asymmetrical (Phi=90°) plane are of utmost importance as the asymmetric structure of the reflector highly affects the cross-polar performance in this plane. Considering this fact, Phi = 90° plane was chosen as the reference plane to estimate the actual improvement in the cross-polarization with tri-mode horn as the primary feed. As observed in Fig. 4.13 (b), the peak level of the relative cross-polarization has reduced to -38 dB for an asymmetric plane (Phi = 90°). This in turn proves that, the tri-mode matched feed can provide extra 13 dB suppression of the reflector cross-polarization in comparison to the conventional Potter horn (-25 dB).

4.4.4 Measurement of Cross-Polar Bandwidth

After ensuring the satisfactory performance of the tri-mode feed with the offset reflector, a detailed study was carried out to find out the cross-polar bandwidth of the proposed feed. For the proposed matched feed, the variation in cross-polarization with frequency is shown in Fig. 4.14.

![Measured cross-polar bandwidth of a tri-mode matched feed](image.png)

**Fig. 4.14** Measured cross-polar bandwidth of a tri-mode matched feed
Inspection of Fig. 4.14 reveals that the tri-mode feed offers additional 8 dB cross-pol reduction (compared to a conventional feed) over 230 MHz frequency range. This is approximately 3.5 % at the center frequency. This bandwidth is sufficient for remote sensing applications but may not be adequate for communication applications. Therefore, further investigations are required to find out a suitable technique to improve the cross-polar bandwidth of the tri-mode matched feed.

4.4.5 Far-field (Secondary) Radiation Patterns of the Offset Reflector Antenna Fed by a Circularly Polarized Cylindrical Matched Feed

This subsection presents the measured results of the offset reflector antenna illuminated by a circularly polarized tri-mode feed [54]. For generation of circular polarization, a separate polarizer was designed and fabricated. The photograph of the polarizer is shown in Fig. 4.15. The said polarizer was first tested in an anechoic chamber for its axial ratio measurement. Over a band of frequencies, the performance of the polarizer was found satisfactory. The polarizer was then assembled with a tri-mode matched feed as shown in Fig. 4.16.

![Fig. 4.15 The photograph of the polarizer (a) Front view (b) Side view](image)
While carrying out the measurements, the offset reflector was illuminated by a circularly polarized tri-mode feed. In the first phase the results were obtained for the matched feed without posts (i.e., without the TE$^{21}$ component). In this case a beam squint of 0.1° (from bore sight) was observed as shown in Fig. 4.17(a). This measured value of beam squint approximately matches with the theoretical value of 0.11° as derived by Rudge’s formula [55].

In the next phase, the same offset reflector was fed by the tri-mode matched feed with inserted posts. As shown in Fig. 4.18(a), for an optimum post height of 6 mm, a
squint-free radiation pattern was achieved. From these results, it can be concluded that, the circularly polarized tri-mode matched feed removes the beam squinting of the offset parabolic reflector antenna.

Fig. 4.18  (a) Measured secondary radiation patterns of the offset reflector fed by a circularly polarized tri-mode matched feed-with posts (b) Magnified section of a measured secondary radiation patterns

4.5 IMPROVEMENT IN CROSS-POLAR BANDWIDTH

This section presents an additional design of a tri-mode matched feed which offers wide cross-polar bandwidth as compared to the design discussed in section 4.3. Using the HFSS software, the design of the existing tri-mode matched feed (of Fig. 4.8) was modified such that a minimum of 10% cross-polar bandwidth can be achieved. The geometry of the modified tri-mode feed is shown in Fig. 4.19. In the modified feed design, in place of three cylindrical posts, total fifteen posts were employed (3 sets of five posts each placed 120° apart). The diameters of all the posts were kept equal while the heights were kept different. For proper understanding, a magnified view of the position of the posts is shown in Fig. 4.20. In all the three sets (placed 120° apart), the height of the central posts (post-1 in Fig. 4.20) were kept 6
mm. The heights of post-2 and post-3 were kept 5 mm, while those of post-4 and post-5 were set to be 4 mm in all the sets.

![Diagram](image)

Fig. 4.19 HFSS simulated design of the modified tri-mode matched feed (a) Front View (b) Side View

![Diagram](image)

Fig. 4.20 Magnified view of the positions of the posts in a modified tri-mode feed

With all the fifteen posts inserted into the feed, its return-loss was found satisfactory over the desired frequency band. Following this, the measurements were carried out to estimate the cross-pol suppression bandwidth of a modified tri-mode feed. From the results plotted in Fig. 4.21, it can be observed that the cross-pol suppression bandwidth has increased to 800 MHz (approximately 12%). With this improved cross-polar performance over a wide frequency band, the modified feed can be used for many practical applications.
4.6 APPLICATIONS

In the present chapter it has been shown that the use of tri-mode horn as the primary feed improves the cross-polar performance of the offset parabolic reflector antenna. This added benefit makes the offset reflector more suitable for many practical applications. Three of such applications are briefly discussed in this section.

4.6.1 Beam-Scanning with a Cluster of Cylindrical Matched Feeds

In beam-scanning applications, it is necessary to illuminate the reflector with a feed displaced away from the geometrical focus of the reflector. However, the displacement of the feed introduces the phase errors across the reflector surface, which ultimately reduces the directivity of the reflector [1]. This loss in the directivity can be compensated, if the beam scanning is controlled by a cluster of feeds in place of a single feed [56]. To form such a feed cluster, various feed options are available.

Fig. 4.21 Cross-polar bandwidth of the modified tri-mode matched feed
However, in the present thesis a cluster of cylindrical matched feeds (tri-mode feeds) has been considered, to illuminate the offset parabolic reflector antenna.

**Feed Locations in a Cluster:**

A cluster has been formed by arranging seven identical feed elements in a hexagonal structure[57] as shown in Fig. 4.22. The beam scanning angle of 4.8° (corresponding to approximately two beamwidths scan) was set by properly selecting the distance ‘L’ from the focal point.

**Fig. 4.22  Arrangement of the feed elements in a hexagonal cluster**

**Feed Excitation Coefficients:**

In order to improve the beam scanning performance of the reflector with the optimum directivity, it is necessary that the feed elements in a cluster should be properly excited [56]. One of the most popular computational techniques to find out the complex excitation coefficients for the feed elements is the conjugate field matching (CFM) scheme [58-59]. This technique has been also used for compensation of distorted reflectors [56, 60-62]. The CFM technique is further classified into the direct
conjugate field matching (DCFM) and the indirect conjugate field matching (ICFM).
For the present feed array (Fig. 4.22), the ICFM technique has been used to obtain the
excitation coefficients ($E_1, E_2, ... E_7$) of all the seven feeds in a cluster. In the ICFM
technique, first the reflector far-field (co-polar) values are calculated in the desired
direction for each individual feed element. Then the complex conjugate of these
computed far-fields are obtained. Finally, the resultant values are normalized and used
as the excitation coefficients for the respective feed element.

**Numerical Results:**

The numerical results for the beam scanning application are obtained for the
offset reflector geometry described in section 3.2 (Fig. 3.1). First, the directivity of the
offset reflector was calculated for a single matched feed element placed at the
geometrical focus of the reflector (un-scanned case). Next, the feed was moved from
the focal point (F) such that the main beam pointed towards 4.8° scan angle. Also, for
this scanned case, the reflector directivity was computed. The results are summarized
in Table 4.3. The results confirm the directivity loss of 0.34 (36.96 dB-36.62 dB) dB
for the beam scanning of approximately two beamwidths.

**Table 4.3 Directivity of the offset reflector illuminated by a single matched feed**

<table>
<thead>
<tr>
<th>Feed Position</th>
<th>Directivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Un-scanned case: Single matched feed at the focal point (F)</td>
<td>36.96 dB</td>
</tr>
<tr>
<td>Scanned case: Single matched feed at the distance ‘L’ from the focal point</td>
<td>36.62 dB</td>
</tr>
</tbody>
</table>

In order to compensate this loss in the directivity, the reflector was illuminated by
a cluster of seven-element matched feeds (Fig. 4.22). The predicted secondary
radiation patterns for this case are shown in Fig. 4.23. The computed results were also
compared with GRASP-8W results and were found in close agreement. In Fig. 4.23, it
is observed that the directivity is peaked at 37.30 dB, which is even better than that of
the un-scanned case (36.96 dB). This improvement in the directivity is attributed to the fact that the feed cluster reduces the aperture phase errors as well as the spill-over losses.

![Simulated secondary radiation patterns](image)

**Fig. 4.23** Simulated secondary radiation patterns of the offset reflector illuminated by a hexagonal cluster of cylindrical matched feeds

Following this, the overall cross-polar performance of the offset reflector was studied with two different feed clusters. First, a hexagonal cluster of cylindrical matched feeds was used to illuminate the offset reflector and then the cluster was replaced by a similar type of Potter horns’ cluster. The results have been obtained in the form of contours. For a case of matched feed cluster, the co-pol. and the cross-pol. contours are shown in Fig. 4.24(a) and Fig. 4.24(b), respectively. Similarly, the results for a Potter horn cluster are shown in Fig. 4.25(a) and Fig. 4.25(b). From the comparison of cross-polar contours (Fig. 4.24(b), and Fig. 4.25(b)), it reveals that the cross-polarization is lower (14 dB improvement) in case of an offset reflector with a cluster of matched feeds.
Fig. 4.24  Contour plots of offset reflector illuminated by a hexagonal cluster of cylindrical matched feeds (a) Co-polar plot (b) Cross-polar plot
Fig. 4.25 Contour plots of offset reflector illuminated by a hexagonal cluster of Potter horns (a) Co-polar plot (b) Cross-polar plot
4.6.2 Multiple-Beam Antenna using an Array of Cylindrical Matched Feeds

Multiple-beam antennas (MBA) find various applications in satellite based communications including direct-broadcast satellites (DBS), personal communication satellites (PCS), military communication satellites, etc. [63]. A detailed review on multibeam antennas can be found in [64]. Multiple-beam antennas use the concept of ‘frequency re-use’ to save the available bandwidth. It generates several beams by illuminating the reflector by a cluster of laterally displaced feeds. Usually an offset parabolic reflector configuration with high F/D ratio (F/D > 1) is preferred for the multiple-beam antenna system [11, 21, 63-65]. The high F/D ratio is chosen to ensure the minimum cross-polarization. However, the high F/D reflector configuration results in a bulky antenna structure and become a limitation for the satellite payload.

This problem can be overcome by illuminating the offset reflector antenna by the cylindrical matched feeds in place of the conventional feeds. As shown previously, the matched feed reduces the offset reflector cross-polarization, while maintaining the compact (F/D < 1) antenna structure. A practical application of multiple-beam antenna is discussed in the following paragraph.

In an application oriented multiple-beam antenna design, the primary goal was to generate five spot beams to cover the entire Indian landmass [66]. To keep the antenna system compact and of light weight, an offset parabolic reflector with F/D=0.6 was selected. In order to suppress the cross-polarization added by the offset geometry, it was decided to use cylindrical matched feed to illuminate the reflector. An array of five such matched feeds was designed to generate five spot beams and to cover the total Indian landmass. In a feed array, the locations of all the matched feeds were decided based on the beam positioning requirements and the physical accommodation
of the feeds. After designing the complete antenna system, its performance was simulated. The results are shown in Fig. 4.26, with five beams covering the entire Indian landmass. Close examination of Fig. 4.26 reveals that the edge of coverage gain (EOC) is approximately 33 dB. The cross-polarization was found below -32 dB. It is important to note that, similar performance with an array of conventional feeds can only be achieved by keeping the F/D ratio very large (F/D > 1). The higher F/D ratio will ultimately increase the size and the mass of the antenna structure.

![Indian landmass coverage using an offset reflector antenna illuminated by an array of five cylindrical matched feeds](image)

**Fig. 4.26** Indian landmass coverage using an offset reflector antenna illuminated by an array of five cylindrical matched feeds
4.6.3 Improving the Beam Efficiency by using a Tri-mode Horn as a Primary Feed

Beam efficiency is a fundamental antenna parameter used to judge the ability of an antenna system to distinguish between the signals received through its main lobe and those through the minor lobes [67]. For a specific value of half cone angle ($\theta_1$), the beam efficiency can be calculated as [68],

$$\text{Beam Efficiency (\%)} = \frac{P_{co}(\theta_1)}{P_{co}(\pi) + P_{xp}(\pi)} \times 100$$  \hspace{1cm} (4.21)

where,

$$P_{co}(\theta_1) = \int_0^\theta \int_0^{2\pi} |G_{co}(\theta, \phi)|^2 \cdot \sin \theta \cdot d\theta \cdot d\phi = \text{co-pol. power}$$  \hspace{1cm} (4.22)

$$P_{xp}(\theta_1) = \int_0^\theta \int_0^{2\pi} |G_{xp}(\theta, \phi)|^2 \cdot \sin \theta \cdot d\theta \cdot d\phi = \text{cross-pol. power}$$  \hspace{1cm} (4.23)

The total power integral is given as,

$$P = P_{co}(\pi) + P_{xp}(\pi)$$  \hspace{1cm} (4.24)

It is apparent from (4.21), that by reducing the cross-polarization, it is possible to improve the beam efficiency.

In radiometer applications, high beam efficiency is necessary to achieve the required contrast for the scene-brightness variation [69-70]. Also, very high beam efficiency of the order of 95-98% ensures minimum contributions from the sidelobes and effectively high spatial resolution.

As reported in [3], the offset parabolic reflector configuration is the most preferred antenna system for radiometric applications. However, the high cross-polarization, introduced by the offset geometry, reduces the beam efficiency and results in measurement errors [2]. Thus, it is necessary that the antenna system for the radiometric applications should be designed such that it radiates minimum energy in the cross-polarization. In this reference, the use of a tri-mode matched feed, in place of
a conventional feed, can suppress the unwanted high cross-polarization of an offset reflector and thereby improve the beam efficiency of the radiometer antenna.

A MATLAB program developed for computation of the secondary radiation pattern of an offset parabolic reflector has been extended to estimate the beam efficiency. Through computer simulations, the beam efficiency calculations were made, (i) for a Potter horn fed offset reflector antenna system and, (ii) for a tri-mode horn fed offset reflector antenna system. The results are plotted in Fig. 4.27, as a function of half-cone angle ($\theta_1$). As expected, the improvement in beam efficiency was achieved in case of a matched feed illuminated offset reflector, as compared to a dual-mode Potter horn fed offset reflector.

![Graph](image)

**Fig. 4.27** Variation in beam efficiency as a function of half cone beam angle

### 4.7 Conclusion

In this chapter, the design and development of a tri-mode conjugate matched feed is discussed. It is observed that, such a feed in conjunction with an offset
parabolic reflector can suppress the unwanted high cross-polarization in case of a linearly polarized offset reflector antenna, whereas in case of a circularly polarized antenna it removes the effect of beam squinting. This type of feed will be more suitable for a low F/D offset reflector configuration. However, proper care should be taken in selecting the horn dimensions, especially the height of the cylindrical posts. Finally, it is concluded that, the tri-mode matched is the best feed option for the offset reflector antenna with low F/D configuration.
PUBLICATIONS RELATED TO THE CHAPTER


