RECTANGULAR MATCHED FEED

As discussed in chapter 1, the high cross-polarization of the offset parabolic reflector antenna results in boresight-jitter when the offset reflector antenna is used in an application of monopulse tracking radar. The boresight-jitter affects the tracking accuracies and puts a major limitation on the use of offset reflector configuration for precision tracking radars. This limitation can be overcome by illuminating the offset reflector by a dual-mode rectangular matched feed in place of a conventional pyramidal horn.

Rudge and Adatia [39] presented a very brief idea of a rectangular matched feed in a conference paper. However, this paper does not include the results of a rectangular matched feed as a single element. Also, the necessary design details and the detailed parametric analysis are not covered in the paper. Therefore, it was felt that the detailed investigations on the rectangular matched feed will be of significant importance. The further motivation behind this study is the advantages of a rectangular structure, like ease of fabrication, ease of excitation, large gain and feasible applications in monopulse tracking radars.

This chapter presents the design and the experimental results of a dual-mode rectangular matched feed. In order to develop this feed, higher order TE\textsubscript{11} mode has been added in correct amplitude and phase with the fundamental TE\textsubscript{01} mode in a smooth-walled rectangular feed structure. The proposed feed was then used as a primary feed to illuminate a linearly polarized offset parabolic reflector antenna. Through experimental results, it has been verified that such a feed cancels out the undesired high cross-polarization introduced by the offset geometry. In addition, the
radiation patterns have been also obtained for a circularly polarized matched feed and the performance improvement in terms of ‘beam squinting’ has been discussed.

### 3.1 FIELD EXPRESSIONS FOR THE RECTANGULAR MATCHED FEED

For a rectangular matched feed with an aperture dimension of a x b, with TE<sub>01</sub> and TE<sub>11</sub> mode at the feed aperture satisfying a specified amplitude and phase ratio, the \( \theta \) (Theta) and \( \phi \) (Phi) components of the far-field radiation pattern are [40],

\[
E_\theta = E_\theta^{TE_{01}} + \alpha E_\theta^{TE_{11}} \tag{3.1}
\]

\[
E_\phi = E_\phi^{TE_{01}} + \alpha E_\phi^{TE_{11}} \tag{3.2}
\]

where, \( \alpha \) is the arbitrary constant defining the relative power in TE<sub>11</sub> mode with respect to the fundamental TE<sub>01</sub> mode. Using the general expressions for \( E_\theta \) and \( E_\phi \) from [41], the expressions for \( E_\theta^{TE_{01}}, E_\theta^{TE_{11}}, E_\phi^{TE_{01}}, E_\phi^{TE_{11}} \) and can be obtained as,

\[
E_\theta^{TE_{01}} = \frac{\sin \theta}{k_{01}} \left( 1 + \frac{\beta_{01}}{k} \cos \theta \right) \left[ \left( \frac{\pi}{b} \cos \phi \right)^2 - \left( \frac{\pi}{b} \cos \phi \right)^2 \right] \Psi_{01}(\theta, \phi) \tag{3.3}
\]

\[
E_\theta^{TE_{11}} = \frac{\sin \theta}{k_{11}} \left( 1 + \frac{\beta_{11}}{k} \cos \theta \right) \left[ \left( \frac{\pi}{a} \sin \phi \right)^2 - \left( \frac{\pi}{b} \cos \phi \right)^2 \right] \Psi_{11}(\theta, \phi) \tag{3.4}
\]

\[
E_\phi^{TE_{01}} = \sin \theta \sin \phi \cos \phi \left( \cos \theta + \frac{\beta_{01}}{k} \right) \Psi_{01}(\theta, \phi) \tag{3.5}
\]

\[
E_\phi^{TE_{11}} = \sin \theta \sin \phi \cos \phi \left( \cos \theta + \frac{\beta_{11}}{k} \right) \Psi_{11}(\theta, \phi) \tag{3.6}
\]

where, \( k = \frac{2\pi}{\lambda} \) = free-space propagation constant, \( \beta_{01} \) and \( \beta_{11} \) can be obtained from,

\[
\beta_{mn} = \sqrt{k^2 - k_{mn}^2} = \text{phase constant of the (mn)\textsuperscript{th} mode} \tag{3.8}
\]

\[
k_{mn} = \sqrt{\left( \frac{mn\pi}{a} \right)^2 + \left( \frac{mn\pi}{b} \right)^2} = \text{cutoff wave number of the (mn)\textsuperscript{th} mode} \tag{3.9}
\]

similarly, \( \Psi_{01}(\theta, \phi) \) and \( \Psi_{11}(\theta, \phi) \) can be obtained from,
\[ \Psi_{mn}(\theta, \phi) = \frac{\sin\left(\frac{\pi}{\lambda} \sin \theta \cos \phi + \frac{mn}{\lambda}\right)}{\left(\frac{\pi}{\lambda} \sin \theta \cos \phi\right)^2 - \left(\frac{mn}{\lambda}\right)^2} \]

(3.10)

### 3.2 Numerical Results

The offset parabolic reflector geometry shown in Fig. 3.1 is taken into consideration for all numerical results presented in this section. The basic parameters of the reflector are shown as the focal length (F) of the parent parabola, the aperture diameter (D) of the offset reflector, the offset angle (\(\theta_0\)) and the half subtended angle (\(\theta^*\)). In Fig. 3.1, \(x', y', z'\) represent the symmetrical cartesian coordinates and \(x, y, z\) represent associated offset coordinates. The relation between the primed \((x', y', z')\) and the unprimed \((x, y, z)\) coordinates can be found from [21]. The numerical values of all the reflector parameters are listed in Table 3.1. All simulations were carried out at the operating frequency of 6.6 GHz.

![Offset reflector geometry under consideration](image)

**Fig. 3.1** Offset reflector geometry under consideration

\(D = 1.242\text{ m}, \ F/D = 0.82, \ \theta_0 = 34.81^\circ\)
Table 3.1 Offset reflector antenna parameters for the geometry under consideration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture diameter (D)</td>
<td>1.2420 m</td>
</tr>
<tr>
<td>Focal length (F)</td>
<td>1.0164 m</td>
</tr>
<tr>
<td>F/D ratio</td>
<td>0.82</td>
</tr>
<tr>
<td>Offset angle ($\theta_0$)</td>
<td>34.81°</td>
</tr>
</tbody>
</table>

3.2.1 Simulated Far-field (Primary) Radiation Patterns of the Rectangular Matched Feed

First, it was decided to carry out the simulations to predict the far-field radiation patterns of the rectangular matched feed. Using the expressions derived in section 3.1, a MATLAB based computer program was developed to compute such patterns. The results obtained through the computer program have been validated by comparison with the GRASP-8W results. As shown in Fig. 3.2, the results are in close agreement with those of the GRASP-8W. This in turn proves the accuracy of the field expressions as well as the computer codes.
3.2.2 Variation in Peak Cross-Polarization as a Function of Relative Power in TE$_{11}$ Mode ($\alpha$)

Using a MATLAB based computer program, the peak cross-polarization in the secondary radiation pattern (for $\Phi = 90^\circ$) was computed for different values of $\alpha$. Ludwig’s third definition [42] for cross-polarization was used while computing the cross-polar radiation. It is evident from the results plotted in Fig. 3.3, that the least peak cross-polarization is achieved only for a specific value of $\alpha$. Thus, proper care should be taken while deciding the contribution of TE$_{11}$ mode.

3.2.3 Offset Angles ($\theta_0$) Versus Relative Power in TE$_{11}$ Mode ($\alpha$)

After studying the variation in cross-polarization as a function of ‘$\alpha$’, further simulations were carried out to obtain the numerically optimized values of ‘$\alpha$’, which gives minimum cross-polarization in the secondary radiation pattern, for different offset angles ($\theta_0$). The outcome of this study is plotted in Fig. 3.4. Inspection of Fig.
3.4 reveals that the modal amplitude of TE$_{11}$ mode is a function of offset angle (θ$_0$), which increases with the offset angle.

**Fig. 3.3** Variation in peak cross-polarization as a function of relative power in TE$_{11}$ mode (α) - for F/D = 0.82 and θ$_0$ = 34.81°

**Fig. 3.4** Offset angles (θ$_0$) versus relative power in TE$_{11}$ mode (α)
3.2.4 Simulated Far-field (Secondary) Radiation Patterns of the Offset Reflector Antenna Fed by a Linearly Polarized Rectangular Matched Feed

In order to estimate the improvement in cross-polarization, it was necessary to obtain the secondary radiation pattern of an offset parabolic reflector. Using the physical optics (PO) based mathematical model [11], a computer program was developed to compute the radiation patterns of the linearly polarized offset reflector antenna. Using the said program, the far-field patterns were estimated for the offset reflector illuminated by two different feeds: (i) conventional pyramidal horn, and (ii) rectangular matched feed.

In Fig. 3.5, the co-polar and cross-polar radiation patterns are shown for a linearly polarized offset reflector illuminated by a pyramidal horn. As expected, the cross-polarization level is very low for Phi = 0° plane, and approximately -25 dB in a Phi = 90° plane (asymmetric plane). Similarly, the radiation patterns for a rectangular matched feed illuminated offset reflector were obtained and the results are shown in Fig. 3.6. Comparison of results for a case of pyramidal horn (Fig. 3.5) and for a case of matched feed (Fig. 3.6) shows a substantial cross-polarization reduction in case of a matched feed fed reflector. This validates the concept of ‘conjugate field matching’ for a rectangular feed structure.
Fig. 3.5  Simulated secondary radiation patterns of the offset reflector illuminated by a linearly polarized pyramidal horn

Fig. 3.6  Simulated secondary radiation patterns of the offset reflector illuminated by a linearly polarized rectangular matched feed
3.2.5 Simulated Far-field (Secondary) Radiation Patterns of the Offset Reflector Antenna Fed by a Circularly Polarized Rectangular Matched Feed

As mentioned in the first chapter, the circularly polarized offset reflector antenna, when illuminated by a conventional feed (pyramidal horn in the present discussion), results into beam squinting effect. As shown in Fig. 3.7, the beam squinting effect shifts the main beam towards the right for left-handed circular polarization and towards the left for right-handed circular polarization. In order to solve this problem, the rectangular matched feed was used in place of a pyramidal horn to illuminate the reflector. The predicted radiation patterns were found ‘squint-free’ (see Fig. 3.8). Hence, it can be concluded that the rectangular matched feed removes beam squinting of a circularly polarized offset reflector antenna.

![Simulated seconday radiation pattern of the offset reflector illuminated by a circularly polarized pyramidal horn](image)

**Fig. 3.7** Simulated secondary radiation patterns of the offset reflector illuminated by a circularly polarized pyramidal horn
Simulated secondary radiation patterns of the offset reflector illuminated by a circularly polarized rectangular matched feed horn

### 3.3 Rectangular Matched Feed Design

The geometry of the proposed dual-mode rectangular matched feed [43] is shown in Fig. 3.9. For an operating frequency of 6.6 GHz, the horn dimensions were optimized using the commercially available antenna design software - high frequency structure simulator (HFSS) [44]. In order to excite the input section of the horn with a fundamental TE$_{01}$ mode, the physical dimensions of the input waveguide (W1 X W4) were selected as that of the standard WR-137 waveguide (15.80 X 34.85 mm$^2$). The higher order TE$_{11}$ mode was generated by a cylindrical metallic post. It is to be noted that the dimensions of the cylindrical post (diameter and the height) decide the amplitude of the TE$_{11}$ mode. Extensive computer simulations were carried out to decide the proportion of TE$_{11}$ mode. It has been observed that the height of the post affects the return-loss performance of the matched feed. Thus, the post height was
selected such that it generates the TE$_{11}$ mode in the required modal amplitude level while maintaining the satisfactory return-loss performance over a specified frequency band. The dimensions W2 X W5 ensures the smooth propagation of TE$_{11}$ mode. The required phase relationship (-90°) amongst the two modes (TE$_{01}$ and TE$_{11}$) was established by adjusting the horn phasing lengths L3 and L4. According to the requirement of the feed taper, the dimensions W3 X W6 were decided.

![Diagram of the proposed dual-mode rectangular matched feed](image_url)

**Fig. 3.9** The geometry of the proposed dual-mode rectangular matched feed (W1=15.80 mm, W2=34 mm, W3=60 mm, W4=34.85 mm, W5=36 mm, W6=80 mm, L1=30 mm, L2=40 mm, L3 =36 mm, L4=188 mm)

Before the actual fabrication, the permissible fabrication tolerance for the feed was determined. Fabrication tolerance corresponds to the permissible error during the fabrication process of the feed. It can be decided based on the ‘sensitivity analysis’, in which the critical dimensions (like post height, post diameter, etc. in the present horn design) of the feed were varied over a specified tolerance level and the RF performance was simulated. The highest tolerance level, which gave the acceptable RF performance, was finally decided. The feed was fabricated at the Space Applications
Centre (SAC), Indian Space Research Organization (ISRO), Ahmedabad, India. The photograph of the actual rectangular matched feed is shown in Fig. 3.10.

Fig. 3.10  The photograph of the proposed dual-mode rectangular matched feed

After the fabrication of the feed, all its dimensions were measured using high precision digital vernier caliper to inspect the tolerance. The deviations were found under the specified tolerance limit and the feed was accepted for various measurements.

3.4 MEASURED RESULTS

This section, presents the measured results of the feed as well as that of the complete offset reflector antenna system. In the first sub-section, the measured results of the feed return-loss characteristic are presented. Following this, in the second sub-section, the simulated and the measured far-field patterns of the proposed dual-mode rectangular matched feed are presented. Finally, the same feed is used to measure the secondary radiation patterns of the linearly polarized offset parabolic reflector antenna. For secondary radiation pattern measurement, the offset geometry shown in Fig. 3.1 has been used.

3.4.1 Feed Return-Loss Measurement

The return-loss performance of the proposed feed was measured with the help of Agilent make Vector Network Analyzer (E8361A PNA series) and the results are
shown in Fig. 3.11. As evident from Fig. 3.11, the return-loss is better than 18 dB over a specified frequency band.

![Graph showing return-loss characteristic](image)

**Fig. 3.11** The measured return-loss characteristic of the rectangular matched feed

### 3.4.2 Far-field (Primary) Radiation Patterns of the Rectangular Matched Feed

The far-field (primary) radiation patterns of the proposed dual-mode rectangular matched feed were measured in an anechoic chamber. The simulated and the measured results are superimposed and are shown in Fig. 3.12. It is clear from Fig. 3.12 that the simulated and the measured radiation patterns are in close agreement. It is observed that the cross-polarization is below -40 dB in case of a Phi = 0° plane, while very high for Phi = 90° plane. This high cross-polarization of the feed (see Fig. 3.12(b)) will counter-balance the effect of high cross-polarization in the secondary radiation pattern of an offset parabolic reflector antenna.
Fig. 3.12  The normalized simulated and measured radiation patterns of the HFSS designed rectangular matched feed (a) For Phi=0° (b) For Phi=90°
3.4.3 Far-field (Secondary) Radiation Patterns of the Offset Reflector Antenna Fed by a Linearly Polarized Rectangular Matched Feed

The secondary radiation pattern measurements of the complete offset reflector antenna system were carried out at the compact antenna test facility - CATF (CCR-75/60) of Space Applications Centre (SAC), Indian Space Research Organization (ISRO), Ahmedabad, India. The offset reflector was illuminated by a dual-mode rectangular matched feed as shown in Fig. 3.13.

Fig. 3.13 The offset reflector and a rectangular matched feed under test at CATR (SAC, ISRO, Ahmedabad, India)
The measured secondary radiation patterns for the two principle planes, i.e. \( \Phi = 0^\circ \) and \( \Phi = 90^\circ \) are shown in Fig. 3.14. Since in an offset parabolic reflector antenna, the worst cross-polarization occurs in an asymmetrical \( (\Phi = 90^\circ) \) plane [1], the same was chosen to estimate the actual improvement in the cross-polarization. As shown in Fig. 3.14(b), the measured cross-polarization in case of a rectangular matched feed fed offset reflector is -34 dB, which is 9 dB less as compared to a pyramidal horn fed offset reflector (-25 dB). If the feed parameters (post dimensions) can be optimized, further reduction in the cross-polarization can be achieved. However, the experimental results confirm the concept of matched feed for an offset parabolic reflector antenna.
Fig. 3.14 Measured secondary radiation patterns of the offset reflector illuminated by a linearly polarized rectangular matched feed horn (a) For Phi=0° (b) For Phi=90°

3.5 CONCLUSION

In this chapter, the design of a rectangular matched feed and its performance with the offset parabolic reflector are presented. In a smooth-walled rectangular waveguide, two modes i.e., TE\textsubscript{01} and TE\textsubscript{11} were combined in proper amplitude and phase to configure a dual-mode rectangular matched feed. It has been experimentally verified that the said feed can suppress the cross-polar level of an offset reflector antenna up to 9 dB. Based on these experimental results, it can be concluded that the rectangular matched feed can become the most favorite option for the next generation offset radar (monopulse tracking radar) antennas.
PUBLICATIONS RELATED TO THE CHAPTER
