Chapter 3

THREE DIMENSIONAL GEOMETRIC MODEL FOR WOVEN FIBER COMPOSITES

The mechanical behavior of woven fiber composites depends on reinforcing fiber geometry. So a three dimensional geometric model is developed which gives a clear idea of spatial orientations of the yarn in the reinforcing fiber network. A repeat unit cell of dimensions $u$, $v$ and $w$ as shown in Fig: 3.1 is chosen for this geometric model.

The fiber architecture of three dimensional Carbon – Silicon carbide (C-SiC) composite shown in Fig: 3.2 is of 8H satin weave pattern. Each yarn is crimped along its path due to interlacing of yarns. Warp, fill and Z – direction yarns are mutually perpendicular to each other with no crimp in Z – direction yarns.

To increase the fiber volume fraction, the crimp angle $\gamma$ is increased, and then fill direction yarns will come closer. The spatial orientation of the yarn is influenced by planar angle ($\Theta$) and the crimp angle ($\gamma$).
Fig: 3.1 Unit Cell

Fig: 3.2. The fiber architecture of 3-D C-SiC
The central path of the yarn \( i \) can be expressed as crimp function:

\[
z_1 = f(x_1) \tag{3.1}
\]

and the crimp angle along its central path can be given by:

\[
\gamma = \tan^{-1}\left(\frac{dz_1}{dx_1}\right) = \tan^{-1}\left[\frac{df(x_1)}{dx_1}\right] \tag{3.2}
\]

The spatial orientation of any yarn \( i \) can be fully identified by its planar orientation angle \( \Theta \) and the crimp function \( f(x_1) \) or crimp angle \( \gamma \). Now the three dimensional fabric geometry is completely described. We assume circular cross-section of the yarn in warp, fill and \( Z \) (through thickness) directions. Any two yarns interlaced with each other, one will be tangent with the other’s circular cross-section on the two departing points as shown in Fig: 3.3.

Fig: 3.3 The spatial orientation of central path of the wrapping yarn \( i \)
The central path or the crimp function \( f(x_1) \) of a yarn in the unit cell of a woven fiber composite can be identified by the two mathematical functions:

1) The circular function of the wrapping segments AB and DE.
2) Rectilinear function of the rectilinear segment BD between two circular segments as shown in Fig: 3.3.

The central path of the wrapping segment AB or DE of the yarn \( i \) can be described by circular equations as:

\[
(x_1-a_i)^2+(z_1-b_j)^2=4r^2
\]  

(3.3)

where \((a_j, b_j)\) represent central coordinates of the wrapped yarn \( j \) on the plane \( x_1 - z_1 \); \( r \) is the radius of the yarns \( i, j \) and \( k \).

The central path of the rectilinear segment BD of the yarn \( i \) between the two circular segments is given by a rectilinear equation:

\[
z_1=mx_1 + p
\]  

(3.4)

where \( m \) is the slope of the rectilinear segment BD; \( p \) is a constant.

The equation of the circular segment AB from (3.3) can be written as:

\[
(z_1 + b_j)^2 = 4r^2 - (x_1-a_i)^2
\]

\[
(z_1 + b_j) = \sqrt{4r^2 - (x_1-a_i)^2}
\]

\[
z_1 = \sqrt{4r^2 - (x_1-a_i)^2} - b_j
\]

Coordinates of B are:

\[
B(x_1, - z_1) = B [ (a_j + G), - z_1 ]
\]

\[
x_1 \text{ coordinate of } C = \frac{(a_k - a_i) + a_j}{2} = \frac{a_k + a_j}{2}
\]

\[
z_1 \text{ coordinate of } C = \frac{(b_k - b_j)}{2} + b_j = \frac{(b_j + b_k)}{2}
\]

where \((a_k, b_k)\) are the central coordinates of the circular cross-section of the yarn \( k \) on \( x_1 - z_1 \) plane.
Slope of the rectilinear segment BC:

\[ z_1 = mx_1 + p \]

\[ m = \frac{[(bj + bk) / 2 + z_1]}{[(ak + aj) / 2 - (aj + G)]} \]

\[ m = \frac{[(bj + bk) / 2 + \sqrt{4r^2 - (x_1-aj)^2} - bj]}{[(ak + aj) / 2 - (aj + G)]} \]

(3.5)

From the circular equation, we can also obtain the slope of the tangent line at the point B which is BD as:

\[ z_1 = \sqrt{4r^2 - (x_1-aj)^2} - bj \]

Slope, \[ m = \frac{dz_1}{dx_1} \] at \[ x_1 = (aj + G) \]

\[ m = \frac{1}{2} \left( 4r^2 - (x_1-aj)^2 \right)^{-1/2} \left\{ -2 (x_1-aj) \right\} \]

\[ m = -\frac{(x_1-aj)}{\sqrt{4r^2 - (x_1-aj)^2}} \]

\[ m = -\frac{G}{\sqrt{4r^2 - G^2}} \]   \hspace{1cm} (3.6)

Equating (3.5) and (3.6)

\[ \frac{[(bj + bk) / 2 + \sqrt{4r^2 - G^2} - bj]}{[(ak + aj) / 2 - (aj + G)]} = - \frac{G}{\sqrt{4r^2 - G^2}} \]

\[ \sqrt{4r^2 - G^2} (bk - bj) = 8r^2 - 4G^2 - G(aj - ak) \]

Squaring on both sides and simplifying:

\[ 16G^4 + 8G^3(aj-ak) + G^2[(aj-ak)^2 - 64r^2 + (bk-bj)^2] - 16Gr^2 + 64r^4 - 4r^2(bk-bj)^2 = 0 \]

(3.7)
In equation (3.7), \( (a_k-a_j); (b_j-b_k) \) represent the space between the two yarns \( j \) and \( k \) in horizontal and vertical directions respectively. The solution of this equation, by trial and error method, gives the value \( G=0.0565 \) which can be substituted back into equation (3.6) to find the slope, \( m = - \frac{G}{\sqrt{(4r^2 - G^2)}} = -0.232 = \tan(\gamma) \). Now the crimp angle \( \gamma \) can be given as, \( \gamma = \tan^{-1}(-0.232) = -13.06^0 \)

Hence, in the composite material, for a fiber volume fraction of 40%, the values of \( G \), slope and crimp angle are determined. By using the above three dimensional geometric analysis, the orientation of any yarn \( i \), in space in a unit cell; and the three dimensional geometry of the reinforcing fiber for a woven composite have been completely determined.

From Fig: 3.3 it is quite evident that, when crimp angle \( \gamma \) is increased, the fill direction yarns come closer, thereby increasing the fiber volume fraction. The increase in fiber volume fraction changes failure behavior of the composite material from catastrophic to near ductile, thereby increasing the fracture toughness. The fiber geometry and the mechanical properties of a woven composite depend on the following important geometric parameters.

1) The number of fibers \( N_f \), the diameter of the fiber \( d_f \) and the fiber volume fraction \( v_f \) in a yarn.

2) The cross sectional area of the composite \( (A_c) \) which can be given as: \( A_c = \pi d_f^2 N_f / 4v_f \)

3) Length of the yarn \( l \) and diameter \( d \) with aspect ratio, \( (l/d) \) nearing infinity.
4) Length $u$, width $v$, and height $w$ of the unit cell.

5) Planar orientation angle $\Theta$ ($0^0$ in this case), the yarn space $(a_k - a_j)$ and $(b_j - b_k)$ of the two yarns $j$ and $k$. 