This thesis entitled "Studies in Graph Theory - On (G,D)-number of a Graph" embodies the work done by K. Palani under the guidance of Dr. A. Nagarajan. The concept of domination in graphs was introduced by Ore[13]. Let $G = (V, E)$ be a finite undirected graph with neither loops nor multiple edges. A subset $D$ of $V(G)$ is a dominating set of $G$ if every vertex in $V - D$ is adjacent to at least one vertex of $D$. The minimum cardinality of a dominating set of $G$ is called the domination number of $G$ and is denoted by $\gamma(G)$. The concept of geodominating(or geodetic) set was introduced by Buckley and Harary in [1] and Chartrand, Harary and Zhang in [4]. A $u-v$ geodesic is a $u-v$ path of length $d(u,v)$. A set $S$ of vertices of $G$ is a geodominating(or geodetic) set of $G$ if every vertex of $G$ lies in an $x-y$ geodesic for some $x, y \in S$. Equivalently, a set $S \subseteq V(G)$ is said to be a geodetic set of $G$ if every vertex in $V - S$ lies in a geodesic joining two vertices of $S$. The minimum cardinality of a geodominating(or geodetic) set of $G$ is called the geodomination(or geodetic) number of $G$ and is denoted by $g(G)$.

For a graph $G = (V, E)$, some subsets of $V(G)$ are dominating sets of $G$ but not geodetic sets and some others are geodetic sets of $G$ but not dominating sets. Also, some subsets of $V(G)$ are both dominating and geodetic. We call such sets as $(G,D)$-sets of $G$.

Let $G = (V, E)$ be a connected graph with at least two vertices. A set $S$ of vertices of $G$ is a $(G,D)$-set of $G$ if $S$ is both dominating and geodetic set of $G$. The minimum cardinality of a $(G,D)$-set of $G$ is called the $(G,D)$-number of $G$ and is denoted by $\gamma_G(G)$.

In Chapter 1, we collect some basic definitions and theorems which are needed for the subsequent chapters. For graph theoretic terminology, we refer Harary [8]. For terms related to domination, we refer Haynes et. al. [10] and for geodetic results, we refer [1, 3, 4, 5, 6, 7, 11].

The terminology of changing and unchanging was first suggested by Harary [9]. Further, Carrington et.al.[2] surveyed the problems involved in changing and unchanging domination number of a graph. A detailed study of the effects on
domination number, when a vertex or edge is deleted from $G$ or an edge is added to $G$, is done by Haynes et al.\cite{10}.

In Chapter 2, the concepts of $(G, D)$-sets and $(G, D)$-number of a graph $G$ are introduced. Further, the $(G, D)$-number for various graphs are obtained. Its bounds are found and its relationship with various other parameters are established. Graphs with $(G, D)$-number equal to $p$ and equal to $p - 1$ are characterized. The following theorem is also established. "For three positive integers $a, b$ and $c$ with $b \geq 2$ and $\max\{a, b\} \leq c \leq a + b$, there exists a graph $G$ with $\gamma(G) = a$, $g(G) = b$ and $\gamma_G(G) = c$." The relation connecting diameter of a graph $G$ and $(G, D)$-number of $G$ is studied. Graphs with $(G, D)$-number equal to $2$ are characterized. Also, it is found that how the $(G, D)$-number of a non-complete connected graph is affected by addition of a single vertex. A detailed study of its effect on a path is carried out. The change in $(G, D)$-number due to removal of vertices or edges is also studied. Further, the change in $(G, D)$-number of a complete graph due to removal of the edges of standard subgraphs in $K_n$, such as path, cycle, clique and star are discussed. Several sections of this chapter has been published in \cite{14, 18}.

In Chapter 3, we study different kinds of $(G, D)$-numbers, viz, Connected $(G, D)$-number, Independent $(G, D)$-number, Split $(G, D)$-number, Strong split $(G, D)$-number, Upper $(G, D)$-number and Forcing $(G, D)$-number. We obtain bounds for the above parameters and also characterize graphs which attain such bounds. Further, the value of the above parameters for graphs like $K_p, P_n, C_n, W_n, K_{m,n}$ and complete $r$-partite graphs for $r \geq 3$. Some sections of this chapter has been published in \cite{15, 19, 20}.

Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and a graph theoretic parameter. Further they characterized the corresponding extremal graphs. In \cite{21}, Paulraj Joseph J and Arumugam S proved that $\gamma + \kappa \leq p$. In \cite{22}, they also proved that $\gamma + \chi \leq p + 1$ and characterized the classes of graphs for which the upper bound is obtained. In \cite{12, 23}, Paulraj Joseph J and Mahadevan G characterized the classes of graphs for which the sum of complementary connected domination number and chromatic number is of order up to $2p - 6$. 
Motivated by these results, in chapter 4, the sum of the parameter $\gamma_G(G)$ ($(G, D)$-number of $G$) with the existing graph theoretic parameter $\chi(G)$ (chromatic number of $G$) is found. First it is proved that for a graph $G$ on $p$ vertices, the sum of $(G, D)$-number of $G$ and chromatic number of $G$ is equal to $2p$ if and only if $G$ is complete. It is also observed that, for a non complete connected graph, this sum is less than or equal to $2p - 2$. Further, the graphs with this sum equal to $2p - 2$, $2p - 3$, $2p - 4$ and $2p - 5$ are also characterized. This chapter has been published in [16, 17].