PREFACE

The thesis entitled “Studies in Graph Theory – Some Labeling Problems in Graphs and Related Topics” embodies the work done by the author under the guidance of Dr. A. Nagarajan.

Labeling of graphs is one of the most interesting problems in Graph Theory. A vertex labeling of a graph $G$ is an assignment of labels to the vertices of $G$ that induces for each edge $uv$ a label depending on the vertex labels of $u$ and $v$.


Several graph labelings have been introduced in Gallian survey [4].

The thesis contains the following 5 chapters:

(i) Preliminaries (Introduction)
(ii) Near Mean Graph
(iii) Near Super Mean Graph
(iv) Near Super Mean Number of a Graph
(v) Magic Graphoidal Graph

By a graph $G = (V,E)$, we mean a finite undirected graph without loops or multiple edges. For graph theoretic terminology we follow [2, 4, 6].
In Chapter 1, we collect some basic definitions and theorems on graphs which are needed for the subsequent chapters.

The Mean Labeling was introduced by Ponraj in [21] and further it was studied in [22, 24, 25, 26, 27, 29]. The concept of Near Graceful Labeling was introduced by Frucht [3] and Near $\alpha$-labeling was introduced by S.El – Zanati, M.Kenig and C.Vanden Eynden [33]. It motivates us to define the concept of Near Mean Labeling as follows.

Let $G = (V, E)$ be a simple graph of order $p$ and size $q$. $G$ is said to be Near Mean Graph (NMG) if $f : V(G) \rightarrow \{0, 1, 2, \ldots, q-1, q+1\}$ such that for each edge $uv$, the induced map $f^*$ defined by $f^*(uv) = \left\lfloor \frac{f(u) + f(v)}{2} \right\rfloor$ is injective, where $\left\lfloor x \right\rfloor$ denotes the least integer which is greater than or equal to $x$ and $f^*(E(G)) = \{1, 2, 3, \ldots, q\}$.

In Chapter 2, some preliminary results on Near Mean labeling are obtained and the total number of possible Near Mean labelings of a graph are found.

The Near Mean Labeling for the following graphs are also investigated:

i) Standard graphs : $P_n$, $C_m$, $P_n^2$, $P_n^+$, $K_n$ and $K_{1,n}$ [9].

ii) Product graphs : Book $K_{1,n} \times K_2$, Grid $P_n \times P_n$, Prism $P_m \times C_3$, $(m \geq 2)$, Corona of Ladder $L_n \odot K_1$ and Cuboid $C_4 \times P_m$ (m > 2) [8].
iii) Special type of graphs : $P_n + 2K_1$, Triangular snake $T_n$, Parachute $P_{2,n-2}$, $P_{a,b}$ (b-odd) and $P_{b}^{a}$ (b : odd) [12].

iv) Armed and Double armed crown of cycles : $C_4 \odot P_m$ and $C_4 \odot 2P_m$.

v) Families of Trees [10] : Bi-star, subdivided Bi-star, $P_m \odot 2K_1$, $P_m \odot 3K_1$, $P_m \odot K_{1,4}$ and $P_m \odot K_{1,3}$.

vi) Join of graphs [11] : $K_2 + mK_1$, $K_n^1 + 2K_2$, $S_m + K_1$ and $P_n + 2K_1$.

Super Mean Graph was introduced in [7] and studied in [30, 31]. The concept of Near Super Mean Graph is introduced as follows. Let $G = (V, E)$ be a graph with $p$ vertices and $q$ edges and let $f: V(G) \rightarrow \{0, 1, 2, \ldots, p+q-1, p+q+1\}$ be an injection. For every edge $e = uv$, $f^*$ on $E(G)$ is defined as

$$f^*(uv) = \begin{cases} 
\frac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\
\frac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd}
\end{cases}$$

Then $f$ is called Near Super Mean Labeling if $\{f(u) : u \in V(G)\} \cup \{f^*(e) : e \in E(G)\} \subseteq \{0, 1, 2, \ldots, p+q+1\}$ and the labels of $V(G)$ and $E(G)$ are distinct. A graph that admits a Near Super Mean Labeling is called a Near Super Mean Graph (NSMG).

In Chapter 3, we investigate the Near Super Meanness of the following graphs:

i) Standard graphs : Path $P_n$, Cycle $C_n$, $K_{2,n}$, Bi-star $B_{n,n}$, $K_n$ (n > 4) and $<B_{n,n} : W>$
ii) Special class of graphs: Comb $P_n^+, P_n^2$, Triangular snake $T_n$, H- graph and $P_{n(m)}$

iii) Cycle related graphs: Parachute $P_{2,n-2}$, $C_m \odot P_n$, $C_m \odot \bigcirc P_n$ and Prism $C_3 \times P_m$

M. Sundaram and R. Ponraj introduced the concept of Mean Number of a graph [28]. Further, R. Vasuki [32] defined Super Mean Number of a graph. It motivates us to define Near Super Mean Number of a graph as follows:

Let $f : V(G) \rightarrow \{0, 1, 2, \ldots, n-1, n+1\}$ be a function such that for each edge $e = uv$, the induced edge label $f'(e) = \left\lceil \frac{f(u) + f(v)}{2} \right\rceil$ is defined, where $\lceil x \rceil$ denotes the least integer which is greater than or equal to $x$ and $f(V(G)) \cup \{f'(e) : e \in E(G)\} \subseteq \{0, 1, 2, \ldots, n+1\}$. If $n$ is the smallest positive integer satisfying all the above conditions and if the labels of $V(G) \cup E(G)$ are distinct, then $n$ is called the Near Super Mean Number of a graph $G$ and it is denoted by $NS_m(G)$.

In Chapter 4,

i) We observed that $NS_m(G) \leq 11 \left(2^{p-4}\right) - 1$, $(p > 4)$ for any graph $G$ with $p$ vertices and $q$ edges.

ii) We find the upper bound for Near Super Mean Number of the following graphs.

(a). For a Fan, $NS_m(P_n + K_1) \leq 4n - 4$, $n > 4$

(b). For a Double fan, $NS_m(P_n + 2K_1) \leq 4n + 4$, $n \geq 4$. 

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Graphoidal Cover was introduced by B.D. Acharya and E. Sampathkumar in [1] and it was further studied by C. Packiam and S. Arumugam in [20]. It is defined as the internally disjoint union of paths covering all the edges of G exactly once. The concept of Magical Graphoidal total labeling of a graph is introduced as follows: Let G = (V, E) be a graph and let ψ be a Graphoidal Cover of G. Define f: V ∪ E → {1, 2, ..., p+ q} such that for every path P = (v_0v_1v_2 ... v_n) in ψ with f^*(P) = f(v_0) + f(v_n) + \sum_{i=1}^{n} f(v_{i-1}v_i) = k, a constant, where f^* is the induced labeling on ψ. Then, we say that G admits ψ - Magic Graphoidal total labeling.

A graph G is called Magic Graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ - Magic Graphoidal total labeling.
In Chapter 5,

(i) We prove some basic results in Magic Graphoidal

(ii) We prove that the following standard graphs $C_n$, $K_{1,n}$, Parachute $P_{2,n-2}$ and $B_{2,n}$ are Magic Graphoidal

(iii) Some Special Type of Unicyclic graphs: Crown $C_n^+$, Dragan $C_n \Theta P_n$, Armed crown $C_m \Theta P_n$ are Magic Graphoidal [13].

(iv) Join of two graph: $K_2 + mK_1$ is Magic Graphoidal.

(v) Product graph: $C_m \times K_2$ is Magic Graphoidal.

(vi) Special Type of Trees such as $[P_n; S_1]$, $[P_n; S_2]$, $T_{(n)}$, $P_m \Theta 2K_1$ and $P_m \Theta K_{1,3}$ are Magic Graphoidal.

(vii) $\psi$ - Magic Graphoidal total labeling of Some Graphs: Grab $P_{a,b}$, Pencil $P_{a \times b}$, Fan $P_n + K_1$, (n-even, n>4), Double Fan $P_n + 2K_1$ and Book $K_{1,n} \times K_2$. 