CHAPTER 4

IDENTIFICATION AND CONTROL OF WIENER-TYPE SYSTEM USING RFT

4.1 INTRODUCTION

In the previous two chapters a Wiener model based NPID and Two NN model based control approaches are simulated for CSTR process. The results show the improved performance in dynamic response. But still the control design methodology has to be simplified for the operators need.

In the 1950s, relays were mainly used as amplifiers, but such applications are now obsolete, because of the development of electronic technology. Relay feedback was applied to adaptive control in 1960’s. One prominent example of such applications is the self-oscillating adaptive controller developed by Minneapolis Honeywell, which uses relay feedback to attain a desired amplitude margin. This system is tested extensively for flight control systems, and it is also used in several missiles.

Astrom and Hagglund (1984) applied the relay feedback method to autotune PID controllers for process control and triggered a resurgence of interest in relay methods. This includes complex systems, time-delay systems, non-minimum phase systems, unstable systems, multivariable and nonlinear systems. Nowadays relay feedback methods are used in industries for automatic tuning for its simplicity of use. Many industrial control products are now equipped with relay feedback functionality, to enable automatic tuning. Extension of the simple approach to performance assessment, nonlinearity modeling, and robust control has also been proposed and developed. Park et al
and Jeng et al (2005) have estimated the static nonlinearities by Least Square Estimation (LSE).

In this chapter, the control of the heat exchanger process is implemented in real-time using two RFT methods. Real-time heat exchanger process and RFT are discussed. The identification of static nonlinearity and linear dynamic subsystems for the heat exchanger is discussed by the method of Huang et al (1998).

Symmetric and asymmetric stage of RFT is used to obtain the Wiener model parameters. A linear dynamic subsystem is identified as FOPDT from the shape of the curve. The key term of the identification procedure is to estimate the invertible function of the static nonlinear subsystem. Genetic Algorithm (GA) based searching is used to avoid the initial assumption during optimization. Five different degrees of complexities are given to facilitate the understandings of these two RFT methods.

4.2 REAL-TIME HEAT EXCHANGER PROCESS

The laboratory model, data acquisition modules and hardware elements for the feedback control of heat exchanger are shown in Figures 4.1, 4.2 and 4.3. Heat exchanger is used to transfer heat energy from the hotter to the colder side. Control of the heat exchanger is complex due to its nonlinear dynamics and particularly the variable steady state gain and time constant of the process fluid.

The preferred configuration for SISO temperature control is discussed here. The process fluid and the cooling fluid are chosen as water. The process flow and the cooling liquid flow are in the opposite direction called as counter-flow mode. The process liquid is pumped from pump2 to heater S2 and S1 through HV3 (Hand Valve) and rotameter2. The heated
water is allowed to enter into the heat exchanger through HV8. The cooling fluid is allowed to flow in the opposite direction through the heat exchanger through pump1, control valve, HV4 and rotameter1. The process fluid temperature is measured using the temperature sensor in the counter current output at A1 and A2. HV1, HV2, HV7 and HV9 are closed during this counter current mode. By varying the flow rate of the inlet cooling fluid (u), the outlet temperature of the process fluid (y) is controlled. Heat exchanger dynamics are given in APPENDIX 2. Steady state response of laboratory heat exchanger is shown in Figure 4.4.

A personal computer is used as a controller that generates the control signal in the form of 4-20 mA range signal by reading the current values through the Analog to Digital Converter (ADC). This control signal is converted back into 4-20 mA by the Digital to Analog Converter (DAC), and in turn drives the current to pressure converter and thereby the pneumatic valve. The real-time implementation is done through the data acquisition modules given in APPENDIX 3. The main aim is to produce the control signal u in order to obtain the desired outlet process fluid temperature y, in an optimal way.

Figure 4.1 Laboratory model of heat exchanger
Figure 4.2 Schematic diagram of data acquisition modules

Figure 4.3 Hardware elements for the feedback control of heat exchanger
4.3 RELAY FEEDBACK TEST

Astrom and Hagglund (1984) suggest the relay feedback test to generate sustained oscillations as an alternative to the conventional continuous cycling technique. It is very effective in determining the ultimate gain and ultimate frequency. A RFT and its responses are shown in Figure 4.5 and 4.6. The distinct advantages of the relay feedback are it identifies process information around the important frequency, the ultimate frequency. RFT is a closed loop test therefore the process will not drift away from the nominal operating point. For processes with a long time constant, it is a more time-efficient method than conventional step or pulse testing. The experimental time roughly equals to 2 to 4 times of the ultimate period.
When the output ($y$) lags behind the input ($u$) by $-\pi$ radians, the closed-loop system may oscillate with constant cycle $T$. As the output $y$ starts increasing after a time delay $L$, the relay switches to the opposite position $u = -h$. Since the phase lag is $-\pi$, a limit cycle with a period ($p_u$) exists and the frequency of the limit cycle is the ultimate frequency ($\omega_u = 2\pi / p_u$). From the Fourier series expansion, the amplitude $a_1$ can be considered as the result of the primary harmonic of the relay output. The ultimate gain is approximated by Astrom and Hagglund (1984) as $k_u = 4h / \pi a_1$. If an ideal and symmetrical RFT are conducted for a Wiener type open loop process then the positive and negative ($T_+ = T_-$) durations are equal in the constant cycling zone.
4.4 IDENTIFICATION OF WIENER MODEL BY HUANG et al (1998) METHOD

Identification of static nonlinearity and linear dynamic subsystems by Huang et al (1998) method is summarised in section 4.4.1 and 4.4.2 respectively.

Haber and Unbehauen (1990) have summarised different structure selection methods based on step and impulse tests, frequency response measurements, correlation analysis and repeated reproducible tests. Menold et al (1997) have also studied structure identification methods, which have different complexities in computations and emphasize the use of statistical methods.

4.4.1 Identification of Static nonlinearity

Symmetric RFT is conducted for the heat exchanger process and the output data is used for the identification of static nonlinearity. For each cycle, the time period of positive and negative output (y) is designated as $T_+$ and $T_-$. If both the outputs are equal, then the nonlinear process is structured as the Wiener type. Otherwise, it is of the Hammerstein type. The static nonlinearity is usually expressed as a nonlinear algebraic function. The nonlinear function $f(v;\theta)$ in the operating range can be approximated by a given function with parameter, $\theta$, which is to be determined.

The approximate inverse function is constructed as follows. Using the nonlinear function $f(v;\theta)$ and discretizing variable $v$ into a sequence of points $\{v_i, i=1,2,...,m\}$, and a set of data pairs are grouped as $S$ in Equation (4.1). Then, a sequence for the inverse of $f(v)$ is denoted as $S^{-1}$ and is given in Equation (4.2).
\[ S = \{ [x_i = v_i, \ y_i = f(v_i; \theta)] \}, \quad i = 1, 2, ..., m \]  \hspace{1cm} (4.1)

\[ S^\prime = \{ [x_i = f(v_i; \theta), \ y_i = v_i] \}, \quad i = 1, 2, ..., m \]  \hspace{1cm} (4.2)

From the sequence of points of \( S^\prime \), a piecewise linear function curve is constructed \( (z(y; \theta)) \). The above manipulation is implemented to construct an approximate inverse and is shown in Figure 4.7.

![Figure 4.7 Construction of the static nonlinearity](image)

**Figure 4.7 Construction of the static nonlinearity**

The instrumental output corresponding to an assumed static nonlinearity can be computed as the inverse of output at each sampling instant. A set of instrumental output is computed as given in Equation (4.3) for each assigned parameter of \( \theta \),

\[ \hat{v}(\theta) = \{ \hat{v}_i = z(y_i; \theta), \quad i = 1, 2, ..., m \} \]  \hspace{1cm} (4.3)

\( A_a(k) \) and \( h_a(k) \) are the area and the positive peak of the instrumental output in the period of \( T_+ \), which starts from \( t_k \), where \( t_k \) is the
starting time of constant cycle after one or two cycles from the RFT output (y). Input output response is shown in Figure 4.6. Similarly, $A_b(k)$ and $h_b(k)$ designates those of $T$ which starts from the same $t_k$. Calculation of $A_a(k)$, $A_b(k)$, $h_a(k)$ and $h_b(k)$ are given in Equation (4.4) to (4.7).

\[
A_a(k) = \int_{t_k}^{t_k + T_{a}} \hat{v}[t;\theta] \, dt, \quad \text{sgn}[\hat{v}] \geq 0 \tag{4.4}
\]

\[
A_b(k) = \int_{t_k}^{t_k + T_{b}} \hat{v}[t;\theta] \, dt, \quad \text{sgn}[\hat{v}] \geq 0 \tag{4.5}
\]

\[
h_a(k) = \max_{t \in [t_k, t_k + T_{a}]} \left\{ \hat{v}[t;\theta] \right\} \tag{4.6}
\]

\[
h_b(k) = \max_{t \in [t_k, t_k + T_{b}]} \left\{ \hat{v}[t;\theta] \right\} \tag{4.7}
\]

If the inverse of $f(v)$ is exists, then the instrumental output would be the same as $v$. The identification for static nonlinearity is formulated as an optimization problem given in Equation (4.8), which makes the instrumental output to have, $A_a = A_b$ and $h_a = h_b$.

\[
\theta = \max_{\theta} \sum_{k=1}^{n} \left[ \frac{A_a(k;\theta)}{A_b(k;\theta)} - 1 \right]^2 + \omega \left[ \frac{h_a(k;\theta)}{h_b(k;\theta)} - 1 \right]^2 \tag{4.8}
\]

where $\omega$ is the weighting factor and is considered as unity. Many standard searching algorithms can be used to find the optimal parameter $\theta$.

RFT is conducted at the operating point of 42°C and their input and output response of four positive ($T_+$) and negative ($T_-$) durations are shown in Figure 4.8. The input is switched between 13 to 100 lph and the corresponding output is 40 to 44°C.
The following configuration is used during RFT.

- $h_1$ - 44°C,
- $h_2$ - 40°C,
- Switching point - 42°C,
- Room temperature - 34.5°C,
- Sampling time - 5 sec.

The second duration of $T_+$ and $T_-$ are 280 sec and 355 sec respectively. The heat exchanger is assumed as Wiener model by the theory of structure identification (Huang et al, 2002) and for the implementation Huang et al (1998) method is suggested. The static nonlinear subsystem is estimated in the form of second order polynomial. Four constant cycles of samples are used to estimate the unknown parameters $c_1$ and $c_2$ of Equation (4.9) and the values are $c_1 = 35$ and $c_2 = 8.9$. MATLAB (FMINUNC) searching algorithm is used to solve the optimization problem, given in Equation (4.8).

\[ y(n) = c_1 v(n) + c_2 v(n)^2 \]  

(4.9)
4.4.2 Identification of Linear Dynamic Subsystem

Figure 4.9 RFT response of a typical linear FOPDT process

Identification of the linear subsystem in a Wiener model is the same as those of a linear system (Luyben and Eskinat, 1994). Data from the asymmetric relay test is used to identify the parameters of the linear subsystem. The linear subsystem is given Equation (4.10), where $L$ and $\tau$ are the dead time and time constant of the linear model. The typical FOPDT output response under RFT is shown in Figure 4.9.

Dead time can be found in two methods. First method is the time required to reach the peak amplitude from the set point and is applicable only for the FOPDT process, i.e., $L_a$ or $L_b$. Second method is in terms of mean values of positive and negative duration and is given in Equation (4.11)

$$ G(s) = \frac{ke^{-Ls}}{\tau s + 1} \quad (4.10) $$

$$ L = \frac{L_a + L_b}{2} \quad (4.11) $$

where, $L_\Delta = \frac{\sum_{i=1}^{n} L_{ai}}{n}$ and $L_\epsilon = \frac{\sum_{i=1}^{n} L_{bi}}{n} \quad (4.12)$
Similarly to estimate time constant, a method is given in Equation (4.14) in terms of mean value

\[
\tau = \frac{\tau_a + \tau_b}{2}
\]  

(4.13)

where,

\[
\hat{\tau}_a = \frac{L_a}{\ln(1-a_{\text{in}}/h_1)} , \quad \hat{\tau}_b = \frac{L_b}{\ln(1-a_{\text{in}}/h_2)}
\]

\[
\hat{a}_{1+} = \frac{\sum_{i=1}^{n} a_{i+1}}{n} \text{ and } \hat{a}_{1-} = \frac{\sum_{i=1}^{n} a_{1-i}}{n}
\]  

(4.14)

An asymmetrical RFT is conducted for heat exchanger around the operating point of 42°C and their input and output responses are shown in Figure 4.10. The preferred configuration during RFT are \( h_1 = 44.2 \)°C, \( h_2 = 40 \)°C.

**Figure 4.10** Asymmetric RFT response of the heat exchanger process
Five constant cycles of data are taken to identify FOPDT model as per Equation 4.11 and 4.13 and is derived in Equation (4.15).

\[ G(s) = \frac{e^{-20s}}{836s + 1} \] (4.15)

The linear subsystem steady state gain \( k \) is assumed to be unity because the overall gain can be accommodated in the nonlinear static subsystem.


Parameters of the inverse nonlinear function are determined via a simple optimization procedure which aims to obtain a symmetric cycling instrumental output. Therefore, the loop can be made approximately linear. Figure 4.11 shows the structure of the Wiener type system to estimate the inverse static nonlinearity.

![Figure 4.11 Parameter estimation of the inverse static nonlinearity](image)

This subsystem is estimated in the form second order polynomial as given Equation (4.16).

\[ v(n) = c_3 y(n) + c_4 y(n)^2 \] (4.16)
The unknown parameters, $c_3$ and $c_4$ are determined by the optimization technique. The parameter $\hat{v}$ of the Equations (4.4) to (4.7) is changed into estimated $\hat{y}$. Many standard-searching algorithms like FMINSEARCH, FMINBND, FMINCON and FMINUNC can be used to find an optimal parameter $\theta$ of Equation (4.8). The MATLAB subroutine FMINSEARCH is used to compute the converged polynomial parameters $c_3 = 35.84$ and $c_4 = -7.0$ from the initial values of $c_3 = 1.9$ and $c_4 = 0.39$. Uniform positive and negative duration of instrumental variable is given in Figure 4.12.

![Figure 4.12 Estimated instrumental Variable (v)](image)

**Figure 4.12 Estimated instrumental Variable (v)**

4.6 **WIENER MODEL BASED CONTROL**

The Wiener model based control system given in Figure 2.11 nullifies the presence of nonlinearity and produces a linear system with unity gain. This resulting linearised system will provide a symmetric oscillation under RFT. Section 4.3 and 4.4 has to be revaluated, if there is a considerable change in the operating point. The $k_u$ and $p_u$ are estimated form the RFT and
Figure 4.13  Step responses in hot water temperature of real-time heat exchanger from 43.5°C to 46°C.

Figure 4.14  Manipulated variable of hot water temperature from 43.5°C to 46°C.
the PI controller is designed. The values of Proportional gain \( k_c \) is 
\((1.2\tau)/(kL)\), the Integral time \( \tau_i \) is \(2*L\) and the Derivative time \( \tau_d \) is \(0.5*L\) 
are obtained by Zeigler-Nichols method.

Closed loop step response of the real-time process is shown in 
Figure 4.13 for \( k_c = 35.82 \) sec and \( \tau_i = 56 \) sec. In Figure 4.13 the disturbance 
is eliminated by the Wiener model based PI controller than the PI controller. 
Their manipulated variable is shown in Figure 4.14.

4.7 SYSTEM STUDY AND RESULTS

Table 4.1 Simulation results of different Wiener-type system

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>( \frac{e^s}{5s+1} )</td>
<td>1.8457v-0.8027v^2 ( e^{-0.9935s} ) ( 3.975s+1 )</td>
<td>1.5653v+0.3887v^2</td>
</tr>
<tr>
<td>4.2</td>
<td>( \frac{e^s}{(5s+1)(s+1)} )</td>
<td>1.8461v-0.8017v^2 ( e^{-1.4755s} ) ( 5.5859s+1 )</td>
<td>1.5609v+0.3868v^2</td>
</tr>
<tr>
<td>4.3</td>
<td>( \frac{e^s}{(3s+1)(s+1)^2} )</td>
<td>1.8508v -0.7911v^2 ( e^{-2.08s} ) ( 3.762s+1 )</td>
<td>1.5757v+0.3823v^2</td>
</tr>
<tr>
<td>4.4</td>
<td>( \frac{e^s}{(s+1)(2s+1)^2} )</td>
<td>1.8563v-0.7796v^2 ( e^{-3.17s} ) ( 3.6s+1 )</td>
<td>1.7942v+0.0844v^2</td>
</tr>
<tr>
<td>4.5</td>
<td>( \frac{e^s}{(s+1)(2s+1)^3} )</td>
<td>1.86v-0.7719v^2 ( e^{-4.3s} ) ( 4.268s+1 )</td>
<td>1.7984v+0.0846v^2</td>
</tr>
</tbody>
</table>

Data of four constant cycles in each stage are sampled for 
identification. The second method is an on-line method and is not easy to
implement in real-time process. Whereas the first method is easy to implement in real-time but the linear system is identified by the asymmetrical RFT. Normally 95% of the controller in control loops is PID and on-off, the collection of asymmetrical RFT is not possible. All five systems results in slight deviation to estimate $c_1$, $c_2$, $\tau$ and $L$. The method discussed in section 4.4 will be more suitable to identify in real-time because of the off-line procedure.

Five types of Wiener model are given in Table 4.1, to test the effectiveness of the identification procedure. The actual and identified nonlinearities are shown in Figure 4.15 for the five cases in Table 4.1. The actual nonlinearity is converged toward the identified nonlinearity in Figure 4.15.

![Figure 4.15 Nonlinear fit for the simulated examples of cases 4.1 to 4.5](image)
The derived static nonlinearity and FOPDT transfer function by the Haung et al (1998) and the inverse static nonlinearity by Lee et al (2004) for the laboratory heat exchanger process is summarized below in Table 4.2

Table 4.2 Results of laboratory heat exchanger as Wiener-type system

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 35v + 8.9v^2$</td>
<td>$G(s) = \frac{e^{-20s}}{836s + 1}$</td>
<td>$v = 35.84y - 7y^2$</td>
</tr>
</tbody>
</table>

4.8 GENETIC ALGORITHM

Genetic Algorithms are search algorithms based on natural selection and natural genetics organized by the evolutinal pressure in a biological system. In genetic algorithms, an object to be tuned is encoded to a string using binary and real numbers, and a population is generated as a set of strings. Each string is determined to survive or die depending on its fitness value. The values are calculated based on each search result. Only surviving strings are copied to the next generation. This process is called “Reproduction” operation.

One of the drawbacks of conventional optimization is initial value assumption. This is over come by Genetic Algorithm (GA) in the present real-time work. Figure 4.16 shows the nonlinearity fit for various initial value assumptions using MATLAB function FMINUNC for case 4.1.
The following GA parameters are used in the search

- population size \( = 20 \)
- string length \( = 8 \) bits
- crossover probability \( = 0.8 \)
- mutation probability \( = 0.001 \)
- convergence tolerance \( = 0.01 \)
- maximum number of generations \( = 200 \)
- problem type \( = \) unconstrained

The initial assumptions \([1 -1], [1 -2], [-3 5], [1 -100] \) and \([1 -10] \) are converging towards the actual nonlinearity of Equation (5) and \([1 -10] \) is not converging. This assumption is not need for GA based optimization. A GA search result for the actual nonlinearity is tabulated in Table 4.3. Figure 4.17 shows the converged coefficients of polynomials towards the actual. Exit condition of GA is defined as the magnitude of step smaller than machine precision and constraint violation.
Table 4.3 GA Results for case 4.1 to 4.5

<table>
<thead>
<tr>
<th>Case no.</th>
<th>Final $\theta$</th>
<th>$A_a$</th>
<th>$A_b$</th>
<th>$h_a$</th>
<th>$h_b$</th>
<th>$c_1$ and $c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>final</td>
</tr>
<tr>
<td>4.1</td>
<td>0.1274</td>
<td>27.0988</td>
<td>48.0891</td>
<td>0.2464</td>
<td>0.5848</td>
<td>0.751, 0.0226, 0.872, 0.0239</td>
</tr>
<tr>
<td>4.2</td>
<td>0.0613</td>
<td>52.2996</td>
<td>408.3171</td>
<td>0.2668</td>
<td>2.4659</td>
<td>0.895, 0.0076, 1.286, -0.124</td>
</tr>
<tr>
<td>4.3</td>
<td>0.0010</td>
<td>115.7879</td>
<td>38.7773</td>
<td>0.4662</td>
<td>0.1196</td>
<td>1.495, -2.509, 2.732, -1.754</td>
</tr>
<tr>
<td>4.4</td>
<td>0.2746</td>
<td>89.2663</td>
<td>104.5021</td>
<td>0.2383</td>
<td>0.2918</td>
<td>0.859, 0.1496, 2.485, -0.563</td>
</tr>
<tr>
<td>4.5</td>
<td>1.0174</td>
<td>0.2928</td>
<td>7.4598</td>
<td>7.9047</td>
<td>0.1810</td>
<td>-0.23, 0.2675, 0.605, 0.0128</td>
</tr>
</tbody>
</table>

Figure 4.17 Nonlinear fit by GA
4.9 CONCLUSION

A real-time heat exchanger is modeled using symmetric and asymmetric RFT. The identification and control is categorized into three aspects. In the first case, symmetric RFT data is used to determine the parameters of the nonlinear static subsystem via a simple optimization procedure, whereas in the second case, the transfer function is identified as the FOPDT model using the asymmetric RFT data. Identification of the linear and nonlinear parts are fully decoupled by this approach. In the third case, the inverse static nonlinear subsystem is identified from Wiener model.

The advantage of using the Wiener type model is that it can be easily adapted to use existing linear methods for closed loop control. The performance of the Wiener model based control strategy is evaluated in a real-time heat exchanger and the results are compared with those of a PI controller.

Many searching algorithms like the standard FMINSEARCH, FMINBND, FMINCON and FMINUNC can be used to solve the optimization. The conventional approaches fail in the initial value assumptions. This is still a trial and error procedure. GA based searching is compared with the FMINUNC. GA avoids the initial assumption during optimization.

The problems addressed for the previous chapters are eliminated by the RFT based identification and control. But still a better method is need to implement the control method in real-time. If the iterative procedures during optimization of static nonlinear subsystems are avoided then this RFT method can be implemented easily in real-time.