CHAPTER 5

A NEW METHOD FOR IDENTIFICATION AND CONTROL OF WIENER-TYPE SYSTEM USING RFT

5.1 INTRODUCTION

An elaborate discussion on control schemes and the difficulties involved in the control of nonlinear system as discussed in the previous Chapters have led to the proposal of a new identification and control method in this Chapter. An iterative optimization algorithm like FMINSEARCH, FMINBND, FMINCON, FMINUNC and GA were used in the previous Chapters. A Least Square Estimation (LSE) optimization method is used in this chapter for both static and inverse static nonlinearity identification. A simple relay feedback method is proposed in the view point of practical applications to identify and control of the Wiener-type process.

Identification procedure consists of two steps. First, a linear dynamic subsystem is identified from the shape of the output curve. If the dynamics are not captured then the order of the subsystem is improved. The main idea behind the identification is that the positive and negative duration T is common for both the linear and the Wiener model. The linear subsystem parameters of the Wiener model are estimated by a well-established Chang et al (1992). Next, the static nonlinearity of the Wiener model is identified by the simple LSE method in polynomial form.

Inverse static nonlinearity is also identified by LSE method to control the nonlinear process in Wiener model based strategy. The proposed
method is tested for simulated stable and unstable examples and two industrially relevant case studies were also evaluated for the new method.

5.2 IDENTIFICATION OF LINEAR DYNAMIC SUBSYSTEM

In general, if the nonlinearity that follows the linear element is monotonic, it does not cause the duration of the positive and negative pulses to be unequal. The period of oscillations in a Wiener model is decided by the linear element and the presence of nonlinearity affects only the amplitude of the oscillations. The linear subsystem parameters can be identified as those of the linear system by period of oscillations alone.

FOPDT or two FOPDTs in series can be chosen as a linear subsystem and its transfer functions are given in Equations (5.1) and (5.2).

\[
G_1(s) = \frac{ke^{-Ls}}{(\tau_1s + 1)} \tag{5.1}
\]

\[
G_2(s) = \frac{ke^{-Ls}}{(\tau_2s + 1)^2} \tag{5.2}
\]

The linear subsystem steady state gain \(k\) is assumed to be unity because the overall gain of the nonlinear system can be accommodated in the static nonlinear subsystem. Dead time \(L\) can be found in two methods and is discussed in section 4.3 of Figure 4.6. The first method is from the initial stage of the RFT where the output starts to increase after the dead time and the second method is the time required to reach the peak amplitude from the initial condition and is applicable only for the FOPDT process (Thyagarajan and Yu, 2003). Chang et al (1992) have estimated time constants \(\tau_1\) and \(\tau_2\) using Equations (5.3) and (5.4).
\[ \tau_1 = \frac{\pi}{\omega_u \ln \left( 2 e^{(L/\tau_1)} - 1 \right)} \]  

(5.3)

\[ \tau_2 = \frac{2\pi \left[ M + (M - 1) e^{\left( \frac{\pi}{\tau_1 u} \right)} \right]}{\omega_u \left[ 1 + e^{\left( \frac{\pi}{\tau_1 u} \right)} \right] e^{\left( \frac{M\pi}{\tau_1 u} \right)} \left( 1 + e^{\left( \frac{\pi}{\tau_1 u} \right)} \right)^{-2}} \]  

(5.4)

where \( \omega_u = 2\pi/\rho_u \), \( M = 1 - (L \omega_u / \pi) \) and initial values for \( \tau_1 \) and \( \tau_2 \) are 0.1.

5.3 IDENTIFICATION OF STATIC NONLINEARITIES

The instrumental output \( v \) can be obtained by conducting suitable RFT for \( G(s) \). The equal number of data of \( y \) and \( v \) is taken to be \( y_{oc} \) and \( v_{oc} \) respectively. A second order polynomial is used to approximate the static nonlinearity and is given in Equation (5.5). The static nonlinearity is identified by mapping the instrumental output \( v_{oc} \) to the process output \( y_{oc} \). Coefficients \( c_1 \) and \( c_2 \) can be estimated as an optimization problem using the LSE method and is given in Equations (5.6) and (5.7).

\[ y(n) = c_1 v(n) + c_2 v(n)^2 \]  

(5.5)

\[ y_{oc}(n) = c_1 y_{oc}(n) + c_2 y_{oc}(n)^2 \]  

(5.6)

\[ [c_1, c_2] = \min_{c_1, c_2} \sum_{n=1}^{p} [y(n) - y_{oc}(n)]^2 \]  

(5.7)

The inverse static nonlinearity is identified by fitting the process output \( y \) to the instrumental output \( v \). The polynomial given in Equation (5.8) is preferred to approximate the inverse static nonlinearity. The LSE method given in Equations (5.9) and (5.10) leads to the identification.

\[ v(n) = c_3 y(n) + c_4 y(n)^2 \]  

(5.8)
\[ v_{\text{est}}(n) = c_1 y_{\text{in}}(n) + c_4 y_{\text{in}}(n)^2 \]  
(5.9)

\[ [c_3, c_4] = \min_{c_1, c_4} \sum_{n=1}^{n} [v(n) - v_{\text{est}}(n)]^2 \]  
(5.10)

### 5.4 SYSTEM STUDY AND RESULTS

#### 5.4.1 Identification of Linear Subsystem

A Wiener-type second order system given in Equations (5.11) and (5.12) is chosen and RFT is conducted (Huang et al, 1998). The parameters of FOPDT are identified using the method discussed in section 5.2 and the result is given in Equation (5.13).

\[ G(s) = \frac{e^{-s}}{(5s+1)(s+1)} \]  
(5.11)

\[ y = 2[1 - e^{-0.694y}] \]  
(5.12)

\[ G(s) = \frac{e^{-1.4744s}}{(5.582ls+1)} \]  
(5.13)

The period of oscillations in a Wiener-type system is decided by the linear element and the presence of non-linearity affects only the amplitude of the oscillations (Huang, 2002). Figure 5.1 shows the responses of \( y \) and \( v \) of Equation (5.13) under RFT and their ultimate period is matching. The Figure 5.1 shows the mismatch in the dead time not in the ultimate period. The mismatch in the output (solid line) and estimated instrumental variable (dotted line) can be eliminated by careful determination of dead time and by increasing the order of the linear dynamic subsystem.

The linear subsystem order is improved to two FOPDT in series and is given in Equation (5.14). Figure 5.2 shows the matched response compared to of Figure 5.1. Figure 5.2 show the matched response for both
dead time and ultimate period but their amplitude differs. Thus the linear
dynamics of the Wiener-type system is validated.

\[ G(s) = \frac{e^s}{(1.8749s+1)(1.8749s+1)} \quad (5.14) \]
Hence, the identification algorithm is summarized as: Computing k and L are common for \( G_1(s) \) and \( G_2(s) \) of Equation (5.1) and (5.2).

(i) Assume \( k=1 \).

(ii) Calculate Dead time \( L \) using either a (or) b.
   a. Output starts to increase after the dead time.
   b. Time required to reach the peak amplitude from the initial condition.

(iii) Find \( p_u \) from the limit cycle.

(iv) Compute time constant.

(v) Conduct validation test. If the validation test is correct, the procedure ends. Otherwise steps (ii) through (iv) are performed until a validated model is obtained.

### 5.4.2 Identification of Static nonlinearities

The equal number of data \( y \) and \( v \) are taken to be \( y_{oc} \) and \( v_{oc} \) respectively. Method discussed in section 5.3 is used to obtain the parameters of the polynomials of Equations (5.5) and (5.8). MATLAB subroutine LSQCURVEFIT is used to obtain the nonlinearities and is given in Equations (5.15) and (5.16). The actual, identified static nonlinearity and their transformed inverse are shown in Figure 5.3. The matched response of actual and identified static nonlinearity is shown in blue and black color respectively. All points of \( y = v \) is transformed in to \( v = y \) in Figure 5.3 as magenta color. Table 5.1 indicates the tuning parameters (Yu, 1999) for linear PI controller using the ultimate information \( k_u \) and \( p_u \).

\[
y = 1.3461v - 0.39v^2 \quad (5.15)
\]

\[
v = 1.1483y + 0.2852y^2 \quad (5.16)
\]
Table 5.1 Tuning parameters and its values for PI controller

<table>
<thead>
<tr>
<th>Method</th>
<th>$k_c$</th>
<th>$\tau_i$ (sec)</th>
<th>$k_c$</th>
<th>$\tau_i$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ziegler and Nichols</td>
<td>$k_u/2.2$</td>
<td>$p_u/1.2$</td>
<td>0.1126</td>
<td>533.3</td>
</tr>
<tr>
<td>Tyreus and Luyben</td>
<td>$k_u/3.22$</td>
<td>$2.2p_u$</td>
<td>0.0769</td>
<td>1408</td>
</tr>
<tr>
<td>Shen and Yu</td>
<td>$k_u/3$</td>
<td>$2p_u$</td>
<td>0.0825</td>
<td>1280</td>
</tr>
</tbody>
</table>

Servo response of Equation (5.14), (5.15) and (5.16) for PI and Wiener model based controller is shown in Figure 5.4. ISE, IAE and the time domain specifications are given in Table 5.2 for the Ziegler and Nichols method. Wiener model based PI controller shows the minimum IAE, ISE, rise time and settling time compared to linear PI.

![Figure 5.3 Static nonlinearity functions for two FOPDTs in series](image-url)
Figure 5.4 Servo responses of two FOPDTs in series

Table 5.2 Performance criteria for FOPDTs in series

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>IAE</th>
<th>ISE</th>
<th>Rise Time (sec)</th>
<th>Settling Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI control</td>
<td>532.94</td>
<td>213.67</td>
<td>1913</td>
<td>2464</td>
</tr>
<tr>
<td>Wiener model based PI control</td>
<td>351.29</td>
<td>132.47</td>
<td>1239</td>
<td>1669</td>
</tr>
</tbody>
</table>

The flow chart for identification and control of Wiener-type system is shown in Figure 5.5.
Figure 5.5  Flow chart for a new identification and control method
The algorithm for identification and control of Wiener-type system is given below:

(i) Conduct a RFT for the nonlinear system
(ii) Measure $p_u$, $k$, and $L$ from the input $u$ or the output $y$
(iii) Specify a class of models including the structure that represent the linear system to be identified. Assume $k=1$
(iv) Compute $\omega_u$ and $\tau$. Perform parameter identification to select the specified class that best fits the statistical data.
(v) Perform the validation test to see if the model chosen adequately represents the system.
(vi) If the validation test is correct, the procedure ends. Otherwise steps (iii) and (iv) are performed until a validated model is obtained.
(vii) Conduct a RFT for the validated linear subsystem and obtain $y$.
(viii) Compute the static and inverse static nonlinearities.
(ix) Compute PID parameters from the $K_u$ and $p_u$ or from the linear system.
(x) Control the nonlinear system using the linear controller via nonlinear control strategy

Five cases of stable system are tabulated in Table 5.3 to compare and test the effectiveness of the existing and proposed identification methods. The static nonlinearity is assumed as constant for all cases. Four cases of unstable systems are simulated and tabulated in Table 5.4. Figure 5.6 shows the estimated static nonlinearity functions for the cases 5.1 to 5.5 by the proposed method. Figure 5.7 shows the matched response under RFT for actual system and the identified linear subsystem for case 5.6. The nonlinear fit of actual and the identified static and inverse static nonlinearity is shown in
Figure 5.8. These nine cases show the effectiveness of the estimated proposed method by its simplicity. The stable system given in Table 5.3 proves the proposed identification values close to existing method.

Figure 5.6  Static nonlinearity functions for the proposed method cases 5.1 to 5.5
Table 5.3 Comparisons of proposed method with the Existing methods

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Static Nonlinearity: $y=2(1-e^{-0.693v})$</th>
<th>Identification of Wiener-type system by</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Linear subsystem</td>
</tr>
<tr>
<td>5.1</td>
<td>$\frac{e^{-s}}{(5s+1)(s+1)}$</td>
<td>$1.8461v - 0.8017v^2$</td>
</tr>
<tr>
<td>5.2</td>
<td>$\frac{e^{-s}}{5s+1}$</td>
<td>$1.8457v - 0.8027v^2$</td>
</tr>
<tr>
<td>5.3</td>
<td>$\frac{e^{-s}}{(3s+1)(s+1)^2}$</td>
<td>$1.8508v - 0.7911v^2$</td>
</tr>
<tr>
<td>5.4</td>
<td>$\frac{e^{-s}}{(s+1)^2(2s+1)^2}$</td>
<td>$1.8563v - 0.7796v^2$</td>
</tr>
<tr>
<td>5.5</td>
<td>$\frac{e^{-s}}{(s+1)^2(2s+1)^3}$</td>
<td>$1.86v - 0.7719v^2$</td>
</tr>
</tbody>
</table>
Figure 5.7  Comparison of Wiener system and it’s identified FOPDT under RFT for case 5.6

Figure 5.8 Static nonlinearity functions for case 5.6
Table 5.4 Identified Wiener Models from a Single RFT

<table>
<thead>
<tr>
<th>case</th>
<th>Linear subsystem [Common Static nonlinearity is $y=2(1-e^{-0.693v})$]</th>
<th>Proposed method</th>
<th>Linear Subsystem identified as</th>
<th>from</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.6</td>
<td>$\frac{2e^{-3s}}{(5s-1)}$</td>
<td>1.437v - 0.4888v^2</td>
<td>0.7718y + 0.1683y^2</td>
<td>$\frac{e^{-2.9999s}}{5.0001s-1}$</td>
</tr>
<tr>
<td>5.7</td>
<td>$\frac{2.4(s+5)e^{-s}}{(s+1)^3(s+3)(s+4)}$</td>
<td>0.0033v - 0.0816v^2</td>
<td>0.0072y + 0.0253y^2</td>
<td>$\frac{e^{-1.87s}}{(2.54s+1)}$</td>
</tr>
<tr>
<td>5.8</td>
<td>$\frac{e^{-0.4s}}{(s-1)}$</td>
<td>-0.224v - 0.2685v^2</td>
<td>-0.1211y - 0.0148y^2</td>
<td>$\frac{e^{-0.4s}}{(0.9815s-1)}$</td>
</tr>
<tr>
<td>5.9</td>
<td>$\frac{e^{-0.5s}}{(2s-1)(0.5s+1)}$</td>
<td>-0.017v - 0.1864v^2</td>
<td>-0.0094y + 0.0223y^2</td>
<td>$\frac{e^{-1.36s}}{(2.74s-1)}$</td>
</tr>
</tbody>
</table>

5.5 EXPERIMENTAL SYSTEM STUDY AND RESULTS

5.5.1 Heat Exchanger process

Steady state curve of physical heat exchanger is shown in Figure 4.4. RFT is conducted around the operating point of 42°C and their input and output responses of four positive ($T_+$) and negative ($T_-$) durations are shown in Figure 4.8. The heat exchanger is assumed as Wiener model by the theory of structure identification and the proposed method is suggested. The linear and nonlinear parts are identified using the methods discussed in sections 5.2 and 5.3 and the results are given in Equation (5.17), (5.18) and (5.19).

$$G(s) = \frac{e^{-20s}}{(8468s+1)}$$ (5.17)

$$y = 1.2510v + 39.8218v^2$$ (5.18)

$$v = -0.5480y + 0.0130y^2$$ (5.19)
The Wiener model is obtained using the proposed method. The Wiener model-based nonlinear control strategy shown in Figure 2.11 is used to compensate the nonlinear dynamics of the process, so that the controller parameters can be determined by the tuning rules developed for linear processes.

The nonlinear control strategy compensates the static nonlinearity by its inverse. $k_u$ and $p_u$ are the two important parameters obtained through RFT. PI parameters are obtained directly from these two parameters. Servo response of the real-time process are shown in Figure 5.9 for $k_c = 20$ and $\tau_i = 98.33$ sec. There manipulated variable is shown in Figure 5.10. The performance of Wiener model based controller over the PI controller is given in Table 5.5. The Wiener model based PI shows the improved performance compared to linear PI controller in offset error and settling time.

![Figure 5.9](image)

*Figure 5.9 Servo responses in nonlinear control strategy for negative step change from 45.25°C to 41.25°C.*
Figure 5.10 Manipulated variables for the negative step change from 45.25°C to 41.25°C.

Table 5.5 Performance criteria of heat exchanger

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>IAE</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI control</td>
<td>648.46</td>
<td>167.4</td>
</tr>
<tr>
<td>Wiener Model based PI control</td>
<td>614.59</td>
<td>137.0</td>
</tr>
</tbody>
</table>

5.5.2 Three-tank system

The three-tank benchmark system is regarded as a valuable experimental set-up and popularly used in all the control laboratories for investigating non-linear feedback system. A schematic diagram of the three-tank system is shown in Figure 5.11. The experimental setup of three-tank system is shown in Figure 5.12. It consists of three cylindrical tanks of same diameter, with circular pipes interconnecting the tanks. Hand valves are fixed between the tanks as indicated in Figure 5.11.
Figure 5.11 Schematic diagram of three-tank system

Figure 5.12 Experimental setup of three-tank system
M and M1 are hand valves located between tanks 1 and 2 and tanks 2 and 3 respectively. The outlet valve present in tank 3 provides the outflow from the system. There is another set of three valves L1, L2 and L3 to create leakage in the respective tanks. Two peristaltic pumps 1 and 2 are available to adjust the incoming flow rate of water $Q_1$ and $Q_2$ to the tanks 1 and 3 respectively. The peristaltic pump is a single channel pump in which the flow rate is decided by the speed of the pump. The pump can be driven either internally using 10 turn potentiometer or externally using 4 to 20 mA signal.

The speed of the pump is calibrated in terms of outflow, i.e., the outflow per minute is correlated to the number of revolutions of the pump per minute. The three-tanks are equipped with DPT for measuring the levels of liquid namely $H_1$, $H_2$ and $H_3$. 4 to 20 mA current output is converted into 0.4 to 2 V and then given to a data acquisition card which converts analog input signal to digital signal. This digital signal is given to a computer through parallel port. The program is written in ‘C’ language to acquire the data and store it in the notepad text format.

The controller adjusts the inflow $Q_1$ to tank 1 and controls the level in tank 3 at the desired value. The inflow $Q_2$ is zero. The controller uses the outflow of the pump 1 as manipulated variable (u) to maintain the level (y) in tank 3. The specifications of the Tank, DTP and pump are given in APPENDIX 4. RFT is conducted around the operating point of 20 cm and their input and output are shown in Figures 5.13 and 5.14 respectively. The three-tank process is assumed as Wiener model by the theory of structure identification and the proposed method is suggested regardless of the duration.
Figure 5.13 Input response of three-tank process under RFT [Relay height: \(H_1 = 0.15\) m, \(H_2 = 0.25\) m]

Figure 5.14 Output response of three-tank process under RFT
Linear and nonlinear parts are derived using the methods discussed in sections 5.2 and 5.3 and the results are given in Equations (5.20), (5.21) and (5.22).

\[ G(s) = \frac{e^{-3s}}{(16s + 1)} \]  
\[ y = 19.6v + 1.233v' \]  
\[ v = -0.0367y + 0.0018y' \]

PI controller is designed using the \(k_u\) and \(p_u\) by Ziegler and Nichols method. The nonlinear control strategy compensates the static nonlinearity by its inverse. Servo response of the real-time three-tank process is shown in Figure 5.15 for \(k_c = 0.12\) and \(\tau_i = 16.45\) sec. The performance of Wiener model based controller over the PI controller is given in Table 5.6. The Wiener model based control strategy results in less value of IAE, ISE, rise time and settling time are compared to liner PI. The Wiener model based PI shows the improved performance compared to linear PI controller in dynamic response.

**Table 5.6 Performance criteria of three-tank system**

<table>
<thead>
<tr>
<th>Control</th>
<th>IAE</th>
<th>ISE</th>
<th>Rise Time (sec)</th>
<th>Settling Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>48.03</td>
<td>1.6692</td>
<td>200</td>
<td>230</td>
</tr>
<tr>
<td>Wiener Model based PI</td>
<td>44.67</td>
<td>1.4983</td>
<td>190</td>
<td>220</td>
</tr>
</tbody>
</table>
CONCLUSION

A new method for identification and control of Wiener-type system using a RFT is proposed and is validated for nine simulation and two experimental examples. The identification procedure is divided into two parts. In the first part, the linear subsystem is assumed as FOPDTs and their parameters are identified. In the second part, the parameters of the nonlinear subsystem and its inverse are determined via a simple optimization procedure. By this approach, the identification of the linear and non-linear parts is fully decoupled. With the obtained process model, the linear PI controller is designed regardless of the nonlinear process. Case studies and the experimental results show the effectiveness of this proposed method.

The disadvantage of the proposed method is the estimation of dead time and time constant. If the estimation procedure is not performed well, then the computation of linear subsystem-dynamics is affected. This can be
avoided by conducting closed loop step test methods and finding the average value of dead time and time constant.

The advantages of the proposed identification are

(i) This method can be easily implemented in real-time.
(ii) RFT can be conducted in ON-OFF and PID control loops.
(iii) The dead time and RFT response (three cycles are enough) alone are needed to identify and control Wiener-type process.
(iv) The commonly operated PID controller can still be used by the proposed approach.
(v) PID tuning can also be done through RFT
(vi) Identification and control is done in a single RFT test.
(vii) LSE is a most accepted method for mapping among researchers.
(viii) RFT is Applicable for Stable, Unstable, Pole or Zero on Right of S-Plane, nonlinear types of systems