Chapter 1

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1.1 Introduction

Differential equations play an innovative role in the development of science and machinery and social sciences. Differential equation is an effective tool in mathematics to describe the changes happening in every stages of nature. Numerous phenomena in these branches have mathematical models in expressions of differential equations. The consequence of differential equations lies in the plenty of the incidences and their effectiveness in understanding the sciences. While describing a physical phenomena by partial differential equations, that the future state of the system is determined by the present circumstances. On various incidents, the present state of a system depends on some previous history. If we consider this approach while modeling, we end up with another class of equations called delay differential equations.

For example, in economic schemes, various delays appear naturally between decisions and effects caused by some needed analysis within the time interval. In population dynamics, delays describe approximately maturation processes. In epidemiological and ecological models, the time delay appears to demonstrate the complicated model or is introduced to describe the result of a bad comprehension of the corresponding evolution. Finally, the traffic flow dynamics contain various discrete and distributed delays caused by mechanical processes or by human driver reactions. Distributed delay means that all values of delays lie inside a time interval with finite bounds and we can think that discrete delay is a particular case of distributed delay.

It is well known that the most of the partial differential equations with or without delay cannot be solvable in terms of elementary functions, so qualitative properties of solutions of such equations assume importance in the absence of closed form solutions. A neutral delay partial differential equation is a partial differential equation in which the highest order derivative of the unknown function appears in the equation both with and without delays. In general, the theory of neutral delay differential equations presents complications, and results which are true for non neutral equations may not be true for neutral equations. Neutral partial functional differential equations have numerous applications in electric networks. For instance, we are frequently used to the study of distributed networks containing loss-less transmission lines which rise in high speed computers where the loss-less transmission lines are used to interconnect switching circuits.

Nowadays, great effort has been devoted to examination of the impulsive neutral partial differential equations. Various techniques appeared for the investigation of solutions of impulsive neutral partial differential equations. Once the existence of a solution for differential equation is established, the next question in the study is: How does solution behave with the growth of time? These constitute the study of asymptotic behavior of solutions of differential equations, one such asymptotic property which has wide applications is the oscillatory behavior of solutions.

Various evolutionary processes from fields as diverse as population dynamics, orbital transfer of satellites, sampled-data systems and engineering are characterized by the fact that they experience unexpected changes are frequently negligible in comparison with that of the complete evolution. The period of these changes is often negligible in comparison with that of the entire evolution process and thus the abrupt changes can be well-approximated in terms of instantaneous changes of state named as impulses. These processes tend to be more properly modeled by impulsive partial differential equations, which permit for discontinuities in the evolution process. Impulsive partial differential equations are usually defined by a pair of equations, partial differential equation to be satisfied during the continuous portion of progress and a difference equation defining the discrete impulsive actions. The impulses occur when some spatio-temporal relation is satisfied. The theory of impulsive partial differential equations is much wealthier than the consequent theory of partial differential equations without impulse effects. Moreover, a simple impulsive partial differential equation may exhibit several new phenomena such as rhythmical beating, merging of solutions, and noncontinuability of solutions. The properties of the impulsive and the classical continuous models differ very much, so deep investigations of the impulsive models are required. So it is valuable to learn the theory of impulsive partial differential equations as a well deserved discipline, due to the increasing applications of impulsive partial differential equations in various fields.

The theoretical background of the second order and even order equations are nearly common and in this direction, we can study oscillatory behavior of even order equations. In this thesis, we initiate to find results about oscillatory behavior of solutions of even order impulsive partial differential equations with deviating arguments, because there are no theoretical results about oscillation for higher order impulsive partial differential equations in the earlier literature.

1.2 Background Works

The existence and location of the zeros of solutions of ordinary differential equations are of central significance in the theory of boundary value problems for such equations and the first essential result was the celebrated comparison theorem of Sturm [66]. Protter [55], Clark and Swanson [18] extended comparison theorems for elliptic differential equations and theorems for oscillation of elliptic differential equations done by Kreith [33] and Kuks [37]. These results were integrated in the monograph [65]. Also this monograph contains the arguments about comparison and oscillation theorems of Reid, Levin, Nehari, Hille, Wintner, Leighton, Hartman, Kneser, Courant and Hanan.

The oscillation theory of ordinary differential equations marks its commencement with the manuscript of Sturm [66] in 1836 appeared in which theorems of oscillation and comparison of the solutions of second order linear homogeneous ordinary differential equations were proved. The first oscillation result for differential equations with translated arguments were obtained by Fite [24] in 1921 and for partial differential equations by Hartman and Wintner [31] in 1955. To the best of authors knowledge, the work on impulsive delay differential equations was published and initiated in 1960 by Milman and Myshkis [53] and in 1989 by Gopalsamy and Zhang

[28]. Its consequences were contained in the monograph [38]. The literature on the oscillation of solutions of higher order ordinary differential equations has grown to such an extent that it is quite impossible to mention all the authors who contributed on this topic. For example, Li and Thandapani [43, 44], Tang et al. [67], Zhang et al. [82] have done extensive work in the above direction. The oscillation problem of ordinary differential equations has been studied by many authors with different methods, see for example [4, 5, 6, 7, 14, 15, 16, 17, 20, 26, 27, 34, 62], and the references cited therein.

On the other hand there are few papers which have been considered as higher order partial differential equations with distributed deviating arguments. We refer the reader to the papers [29, 42, 45, 46, 49]. The primary exertion for impulsive partial differential equations was started [23] in 1991. In recent years the oscillation of parabolic and hyperbolic equations with or without impulse effect has been widely studied in the literature. We refer the reader to the papers [11, 21, 22, 25, 35, 36, 40, 41, 47, 48, 50, 51, 57, 58, 59, 60, 63, 68, 69, 73, 75, 77, 79, 81] and the reference they are cited. In [78], population ecology, generic repression, control theory, climate models, coupled oscillators, viscoelastic materials, and structured population models are studied with distributed delay and boundary conditions of the type Dirichlet, Neumann and Robin. From the essence of these mathematical models, we formulated these even order problems. Distributed delay system models appear in hematopoiesis [1, 2], logistics [12], microorganism growth [56] and traffic flow [64]. The wide interest on qualitative studies of impulsive ordinary and partial functional differential equations is returned to their varieties of applications in various fields of science and technology [8, 9, 13, 39, 61, 80], and so it is desirable to study these equations scientifically.

1.3 Contribution of the Author

The above review of literature motivated the researcher to take up the study of oscillatory behavior of solutions of impulsive even order partial differential equations. We have established few important results on the following topics:

1. Oscillation of impulsive hyperbolic differential equations with distributed delay
2. On the oscillation of impulsive neutral partial differential equations with distributed deviating arguments and damping term
3. Oscillation of even order impulsive partial differential equations with deviating arguments
4. Oscillation of even order impulsive neutral partial differential equations with distributed deviating arguments
5. Oscillation of even order impulsive neutral partial differential equations with distributed delay and damping
6. On the oscillation of higher order impulsive neutral partial differential equations with distributed deviating arguments

Chapter 2 deals with oscillation of solutions of impulsive hyperbolic differential equations with distributed deviating arguments of the form

$$\left.\begin{array}{l} \dfrac{\partial^2 u}{\partial t^2} = a(t)\Delta u(x,t) + b(t)\Delta u(x,\tau(t)) - \sum\limits_{i=1}^{n} r_i(x,t)u(x,\sigma_i(t)) \\ \quad + \int_c^d q(x,t,\xi)f(u(x,g(t,\xi)))d\eta(\xi), \quad t \neq t_k, \quad (x,t) \in \Omega \times (0,+\infty) \equiv G, \\ u(x,t_k^+) = (1+\alpha_k)u(x,t_k) \\ u_t(x,t_k^+) = (1+\beta_k)u_t(x,t_k), \quad k = 1,2,\cdots, \end{array}\right\} \tag{1.3.1}$$

with the boundary condition

$$u = 0, \qquad (x,t) \in \partial\Omega \times (0,+\infty), \tag{1.3.2}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with a piecewise smooth boundary $\partial\Omega$ and Δ is the Laplacian in the Euclidean space $\mathbb{R}^N$.

Section 2.1 provides necessary introduction, formulation of the problem, definitions and Lemma to prove the main results and Section 2.2 contains the oscillation results for the boundary value problem (1.3.1)-(1.3.2). Example is given in Section 2.3 to illustrate the main results. The results obtained here improve and generalize some of the results presented in [11].

In chapter 3, we focus on oscillation of nonlinear impulsive second order neutral partial differential equations with distributed deviating arguments and damping term

$$\left.\begin{array}{l} \dfrac{\partial}{\partial t}\left[r(t)\dfrac{\partial}{\partial t}\left(u(x,t)+c(t)u(x,\tau(t))\right)\right] + p(t)\dfrac{\partial}{\partial t}\left(u(x,t)+c(t)u(x,\tau(t))\right) \\ \quad + \int_a^b q(x,t,\xi)f(u(x,g(t,\xi)))d\eta(\xi) = a(t)\Delta u(x,t) \\ \quad\quad - \int_a^b b(t,\xi)\Delta u(x,h(t,\xi))d\eta(\xi), \quad t \neq t_k, \quad (x,t) \in \Omega \times \mathbb{R}^+ \equiv G \\ u(x,t_k^+) = \alpha_k\left(x,t_k,u(x,t_k)\right), \\ u_t(x,t_k^+) = \beta_k\left(x,t_k,u_t(x,t_k)\right), \quad t = t_k, \quad k = 1,2,\cdots, \end{array}\right\} \tag{1.3.3}$$

Equation (1.3.3) is supplemented by the following Dirichlet and Robin boundary conditions,

$$u = 0, \qquad (x,t) \in \partial\Omega \times \mathbb{R}^+ \tag{1.3.4}$$

$$\frac{\partial u}{\partial \gamma} + \mu(x,t)u = 0, \qquad (x,t) \in \partial\Omega \times \mathbb{R}^+ \tag{1.3.5}$$

where γ is the unit exterior normal vector to $\partial\Omega$ and $\mu(x,t) \in C(\partial\Omega \times \mathbb{R}^+,\ \mathbb{R}^+)$.

Section 3.1 presents introduction of chapter 3 and provides essential definitions and notations. Section 3.2 focuses oscillatory results for the problem (1.3.3) with (1.3.4) and section 3.3 investigates the oscillatory properties of solutions of (1.3.3)-(1.3.5). Example is provided in section 3.4.

In chapter 4, we are concerned with the oscillatory behavior of solutions of the even order impulsive partial differential equation of the form

$$\left.\begin{aligned} &\frac{\partial^m}{\partial t^m}u(x,t) + \sum_{j=1}^{l} Q_j(x,t)u(x,\rho_j(t)) \\ &\quad = a(t)\Delta u(x,t) + \sum_{s=1}^{n} p_s(t)\Delta u(x,\tau_s(t)), \qquad t \neq t_k, \quad (x,t) \in \Omega \times (0,+\infty) \equiv G, \\ &\frac{\partial^{(i)}u(x,t_k^+)}{\partial t^{(i)}} = (1+\alpha_k^{(i)})\frac{\partial^{(i)}u(x,t_k)}{\partial t^{(i)}},\ t = t_k,\ k = 1,2,\cdots,\ i = 0,1,2,\cdots,m-1, \end{aligned}\right\} \tag{1.3.6}$$

with the boundary condition

$$u = 0, \qquad (x,t) \in \partial\Omega \times (0,+\infty), \tag{1.3.7}$$

Section 4.1 covers essential introduction, hypotheses for the problem (1.3.6)-(1.3.7) with definitions and preliminary lemmas. In section 4.2, we have establish sufficient conditions for the oscillation of all solutions of the problem (1.3.6)-(1.3.7) and couple of examples are provided in Section 4.3.

In chapter 5, we have focused on the results for oscillation of the following even order nonlinear impulsive neutral partial functional differential equation with continuous distributed deviating arguments

$$\left.\begin{aligned} &\frac{\partial^m}{\partial t^m}\left[u(x,t)+c(t)u(x,\tau(t))\right]+\int_a^b q(x,t,\xi)f(u(x,\sigma(t,\xi)))d\eta(\xi)=a(t)\Delta u(x,t)\\ &\qquad -\int_a^b b(t,\xi)\Delta u(x,\rho(t,\xi))d\eta(\xi), \quad t\neq t_k,\ \ (x,t)\in\Omega\times(0,+\infty)\equiv G\\ &\frac{\partial^{(i)}u(x,t_k^+)}{\partial t^{(i)}} = I_k^{(i)}\left(x,t_k,\frac{\partial^{(i)}u(x,t_k)}{\partial t^{(i)}}\right),\ k=1,2,\cdots,\ i=0,1,2,\cdots,m-1 \end{aligned}\right\} \tag{1.3.8}$$

Equation (1.3.8) is enhancement with the subsequent Dirichlet and Robin boundary conditions,

$$u = 0, \qquad (x,t)\in\partial\Omega\times(0,+\infty) \tag{1.3.9}$$

$$\frac{\partial u}{\partial\gamma}+\mu(x,t)u = 0, \qquad (x,t)\in\partial\Omega\times(0,+\infty) \tag{1.3.10}$$

where γ is the outer surface normal vector to $\partial\Omega$ and $\mu(x,t)\in C\left(\partial\Omega\times[0,\infty),[0,\infty)\right)$.

Section 5.1 provides introduction to impulsive even order neutral partial differential equations, preliminary definitions and Lemmas. In Section 5.2, we provide sufficient conditions for oscillation of all solutions of the problem (1.3.8) with Dirichlet boundary condition and Section 5.3 contains oscillatory results with Robin boundary condition. Some examples are provided in Section 5.4. The results obtained in this chapter generalize few of the results of [75].

In chapter 6, we will studied the following even order impulsive neutral partial functional differential equation with continuous distributed deviating arguments and damping

$$\left.\begin{aligned} &\frac{\partial}{\partial t}\left[r(t)\frac{\partial^{m-1}}{\partial t^{m-1}}\left(u(x,t)+c(t)u(x,\tau(t))\right)\right]+p(t)\frac{\partial^{m-1}}{\partial t^{m-1}}\left(u(x,t)+c(t)u(x,\tau(t))\right)\\ &+\int_a^b q(t,\xi)u(x,\sigma(t,\xi))d\eta(\xi)=a(t)\Delta u(x,t)+\int_a^b b(t,\xi)\Delta u(x,\rho(t,\xi))d\eta(\xi),\\ &\qquad\qquad t\neq t_k,\ \ (x,t)\in\Omega\times(0,+\infty)\equiv G\\ &\frac{\partial^{(i)}u(x,t_k^+)}{\partial t^{(i)}} = I_k^{(i)}\left(x,t_k,\frac{\partial^{(i)}u(x,t_k)}{\partial t^{(i)}}\right),\quad k=1,2,\cdots,\ i=0,1,2,\cdots,m-1 \end{aligned}\right\} \tag{1.3.11}$$

Equation (1.3.11) is enhancement with the following Robin boundary condition,

$$\alpha(x)\frac{\partial u(x,t)}{\partial \gamma} + \beta(x)u(x,t) = 0, \qquad (x,t) \in \partial\Omega \times (0,+\infty) \tag{1.3.12}$$

where γ is the outer surface normal vector to $\partial\Omega$ and $\alpha, \beta \in C(\partial\Omega, \mathbb{R}^+)$, $\alpha^2(x) + \beta^2(x) \neq 0$.

Section 6.1 provides introduction and motivation, notations and preparatory Lemmas. In Section 6.2, we establish necessary and sufficient conditions for oscillation of all solutions of (1.3.11) with Robin boundary condition (1.3.12) and two examples are given in Section 6.3.

In chapter 7, we consider the following impulsive system with damping of the form

$$\left.\begin{array}{l}
\dfrac{\partial}{\partial t}\left[r(t)\dfrac{\partial^{m-1}}{\partial t^{m-1}}\left(u(x,t) + \int_a^b g(t,\xi)u(x,\tau(t,\xi))d\eta(\xi)\right)\right] \\
+p(t)\dfrac{\partial^{m-1}}{\partial t^{m-1}}\left(u(x,t) + \int_a^b g(t,\xi)u(x,\tau(t,\xi))d\eta(\xi)\right) + \int_a^b q(t,\xi)u(x,\sigma(t,\xi))d\eta(\xi) \\
= a(t)\Delta u(x,t) + \sum\limits_{j=1}^{n} b_j(t)\Delta u(x,\rho_j(t)), \quad t \neq t_k, \quad (x,t) \in \Omega \times (0,+\infty) \equiv G \\
\dfrac{\partial^{(i)}u(x,t_k^+)}{\partial t^{(i)}} = I_k^{(i)}\left(x, t_k, \dfrac{\partial^{(i)}u(x,t_k)}{\partial t^{(i)}}\right), \quad k = 1,2,\cdots, \; i = 0,1,2,\cdots,m-1
\end{array}\right\} \tag{1.3.13}$$

Equation (1.3.13) is supplemented by the following Robin boundary condition,

$$\alpha(x)\frac{\partial u(x,t)}{\partial \gamma} + \beta(x)u(x,t) = 0, \qquad (x,t) \in \partial\Omega \times (0,+\infty) \tag{1.3.14}$$

where γ is the outer surface normal vector to $\partial\Omega$ and $\alpha, \beta \in C(\partial\Omega, [0,+\infty))$, $\alpha^2(x) + \beta^2(x) \neq 0$.

Section 7.1 provides introduction and preparatory Lemmas. In Section 7.2, we have derived necessary and sufficient conditions for oscillation of all solutions of (1.3.13) with Robin boundary condition (1.3.14) and pair of examples are given in Section 7.3.