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CERTIFICATE

I certify that the thesis entitle **"Oscillation and Asymptotic Behavior of Third Order Neutral Differential Equations with Mixed Deviating Arguments"** submitted for the award of the Degree of **Doctor of Philosophy in Mathematics** by **Mr. M. Sathish Kumar** is the record of research work done by him during the study under my guidance and supervision, and that the thesis has not formed the basis for the award of any degree, diploma, associateship, fellowship, titles in this or any other University or other similar Institution of higher learning.

Signature of the Supervisor

Place : Salem

Date : 04-07-2019.

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M. Sathish Kumar

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Plagiarism Declaration by the Candidate

I hereby declare that the thesis entitled "**Oscillation and Asymptotic Behavior of Third Order Neutral Differential Equations with Mixed Deviating Arguments"** submitted by me for the award of Ph.D. degree in **Mathematics** is my original contribution and it is not plagiarized or copied from any other thesis/books/ any other copy right materials.

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I hereby declare that the candidate **Mr. M. Sathish Kumar**, has carried out the Ph.D programme under my supervision during the period **February 2016** to **June 2019** and the thesis entitled "**Oscillation and Asymptotic Behavior of Third Order Neutral Differential Equations with Mixed Deviating Arguments"** submitted by him/her is verified and it is not plagiarized or copied from any other thesis/books/any other copy right materials.

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This is certify that Ph.D. candidate **M. Sathish Kumar** working under my supervision has published a research article in the **UGC-CARE GROUP A / GROUP B** listed journal named 1. International Journal of Applied and Computational Mathematics with Vol. 4 No: 78 Page No: 1-14 and year of publication 2018 published by Springer Nature India. 2. Ural Mathematical Journal with Vol. 3 No: 2 Page No: 122-129 and year of publication 2017 published by N.N. Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of Russian Academy of Sciences. 3. Malaya Journal of Matematik, paper accepted and year of publication 2019 published by University Press. The contents of the publication incorporate part of the results presented in his thesis.

Research Supervisor

Countersigned

Head of the Department

ABSTRACT

The research reported in this thesis deals with the oscillation and asymptotic of solutions of third order neutral differential equations with mixed deviating arguments. Here the study of the oscillation of solutions of third order nonlinear differential equation with several neutral terms by using comparison principle method. Next, we establish some sufficient conditions were the oscillatory and asymptotic behavior of solutions of third-order neutral differential equations with discrete and distributed delay by applying three Riccati substitution techniques, integral averaging techniques and comparison principles. Also, we discuss the oscillatory and asymptotic behavior of solutions of third-order nonlinear neutral differential equations of mixed type. Further we present some new results for the oscillation and asymptotic solutions of third-order neutral differential equations with distributed delay by using Riccati substitution and integral averaging techniques. Finally, we establish some sufficient conditions for the third-order retarded differential equations with a sublinear neutral term. Some examples were present in order to illustrate the main results at the chapter.

List of Publications

1. Oscillation of certain third order nonlinear differential equation with neutral terms, *Bangmod International Journal of Mathematics and Computer Science*, 3 (1-2) (2017), 53-60.

2. Oscillation theorems for third-order retarded differential equations with a sublinear neutral term, *International Journal of Pure and Applied Mathematics*, 114 (5) (2017), 63-71.

3. On the oscillation of a third order nonlinear differential equations with neutral type, *Ural Mathematical Journal (UMJ)*, 3 (2) (2017), 122-129.

4. Nonlinear oscillation of certain third order neutral differential equation with distributed delay, *Journal of Mahani Mathematical Research Center*, 7 (1-2) (2018), 1-12.

5. Some new oscillatory behavior of certain third-order nonlinear neutral differential equations of mixed type, *International Journal of Applied and Computational Mathematics*, 4 (78), 2018, 1-14.

6. On the oscillatory behavior of solutions of third order nonlinear neutral differential equations, *Malaya Journal of Mathematik*, 2019. (In Press).

7. Asymptotic behavior of solutions of third-order neutral differential equations with discrete and distributed delay, communicated to *Mathematical Biosciences and Engineering*.

8. Philos-type oscillation results for third-order differential equation with mixed neutral terms, communicated to *Communications, Series A1: Mathematics and Statistics*.

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