FUZZY COMPROMISE SOLUTION FOR TRANSPORTATION AND TRAFFIC PROBLEMS

4.1 INTRODUCTION

In the real situations, the transportation problems be a common problem which is occur in the distribution of goods from manufacturer to customer be described by the transportation problem. The transportation problem is well known because its model can be used when a company is trying to decide where to locate a new facility. Good financial decisions concerning facility location also consider to minimize total transportation and production cost for the entire system. In real transportation problems, several functions are generally considered which includes average delivery time of the commodities, minimum cost. Intensive investigations on multi-objective linear transportation problems have been made by several researchers. Aneja and Nair (1979) presented a bicriteria transportation problems. Lee and Moore (1973) studied the optimization of transportation problems with multi-objectives fuzzy set theory was proposed by Zadeh (1965) and has been found extensive in various fields. The applications of fuzzy sets to transportation problems include Chanas (1984), Bit. A.K. (1993) and Verdegy (1984) proposed the application of Zimmermann’s fuzzy programming and fuzzy

Grotschel, Martin (2001) formulated online optimization of complex transportation systems. Song, Ye Xin (2001) proposed the application of uncertain decision making under fuzzy information.

In this chapter we proposed a fuzzy compromise programming approach to linear transportation problem. In the fuzzy compromise programming approach proposed, various objectives are synthetically considered by marginally evaluating individual objectives and globally evaluating all objectives. This fuzzy compromise programming approach gives a compromise solution which is not only non-dominated but also optimal in the sense of that decision-maker’s global subjective evaluation value is maximum. And the transportation problems is depend upon the vehicle routing and traffic based problems, when we developed a model of company, workers, intuition appeared to be unexpectedly important part of the model. The manufacturer wants the distribution of the goods with minimum transportation cost and minimum time period. The time period in which the customer requests to be served is commonly expressed. In reality, it is often possible to extend it a little for the price of customer satisfaction. Experienced manufacturers know which customers can be served outside the time and which ones can’t.
Moreover, travel times are subject to changes, unfortunately, the company didn’t have any historical data concerning travel times, so this short of knowledge was based also on worker’s experience. Now soft time is incorporated into objective function and it is not expected by the decision maker, who wants to know all decision variables. Hence the need for a different approach is representation of uncertainty in the form of fuzzy numbers is a good alternative to traditional probability measures when historical data is missing and fuzzy values interpretation is easy for the decision maker.

Any transportation model is aimed providing an optimal solution subjected to minimization of costs and minimization of time when such a model is applied for local areas of developing countries, its needs to tackle other crucial objective like minimization of non local sources of energy which has direct implication in social life. This is for designing an appropriate scheme of energy supply consumption model which can assume maximization of the effects of specific locations.

The transport policy should aim at meeting the needs of economy with the minimum demands on sources so that the movements of passengers and goods are executed at the minimum possible real costs within a minimum possible time. A systematic arrangement of speedy transport is, therefore, an essential prerequisite for the faster economic development of the regions and of the country. So in the present analysis, the minimization of total cost and minimization of total time of
transportation have been the two major objectives of the model. Besides, another objectives namely, Minimization of non-local sources is also considered as important objectives in the task of energy allocation for local transportation. However, to assure the effects of all the three objectives functions on energy allocation, a compromised solution in a fuzzy environment has been considered to be useful.

4.2 NOTATIONS

\[ x_{ijk}^g : \] Goods carried over \( k \)th class of distance using \( i \)th source through \( j \)th mode of transport.

\[ x_{ijk}^p : \] Number of passengers travelling \( k \)th class of distance using \( i \)th source through \( j \)th mode of transport.

\[ C_{ijk}^p : \] Cost of travel for passengers over \( k \)th class of distance using \( i \)th source through \( j \)th mode of transport.

\[ C_{ijk}^g : \] Cost of transport of goods over \( k \)th class of distance using \( i \)th source through \( j \)th mode of transport.

\[ E_{ijk}^p : \] Energy required for passengers travelling over \( k \)th class of distance using \( i \)th source through \( j \)th mode of transport.

\[ E_{ijk}^g : \] Energy required for goods over \( k \)th class of distance using \( i \)th source through \( j \)th mode of transport.
\( T_{ijk}^P \): Time required for passengers to travel over \( k \)th class of distance using \( i \)th source through \( j \)th mode of transport.

\( T_{ijk}^g \): Time required for transport of goods over \( k \)th class of distance using \( i \)th source through \( j \)th mode of transport.

\( D_k^P \): Demand for travel over \( k \)th class of distance of passengers.

\( D_k^g \): Demand for transport over \( k \)th class of distance of goods.

\( A_{ij}^P \): Available capacity in passenger-km using \( k \)th source through \( i \)th mode of transport

\( U_k \): Upper bounds level or highest acceptable level

\( L_k \): Lower bounds level or aspiration level.

\( A_{ij}^g \): Available capacity in passenger-km using \( i \)th source through \( j \)th mode of transport

\( M_k^P \): Maximum km travelled by any passenger for \( k \)th class of distance.
$M_k^g$: Maximum km traversed by any consignment of goods for $k$th class of distance.

$d_k^g$: Demand of goods to be transported by originating and terminating external traffic.

$A_j^p$: Availability of trips of $j$th mode for passenger transport in a year.

$A_j^g$: Availability of trips of $j$th mode for goods transport in a year.

$Q_j^p$: Average number of passenger can travel at a time through $j$th mode

$Q_j^g$: Average quantity of goods can travel at a time through $j$th mode

$A_i$: Availability of the source $i$

4.3 MATHEMATICAL FORMULATION OF THE MODEL

The mathematical form of the objective functions are:

Objective-1: Minimization of Total Cost of Direct Energy

$$O_1 = \sum \sum \sum C_{ijk}^p x_{ijk}^p + \sum \sum \sum C_{ijk}^g x_{ijk}^g$$  \hspace{1cm} ... (4.1)

Objective-2: Minimization of Non-Local Sources of Energy
Chapter 4...

\[ O_2 = \sum \sum \sum E_{ijk}^p x_{ijk}^p + \sum \sum \sum E_{ijk}^g x_{ijk}^g \text{ for } i = 1 \text{ and } 2 \] \hfill \ldots \text{(4.2)}

Objective-3: Minimization of Total Time

\[ O_3 = \sum \sum \sum T_{ijk}^p x_{ijk}^p + \sum \sum \sum T_{ijk}^g x_{ijk}^g \forall \ i, j, k \] \hfill \ldots \text{(4.3)}

The above Objective functions are optimized subject to the following sets of constraints.

\[ \sum \sum x_{ijk}^p = D_k^p \quad \forall \ k \] \hfill \ldots \text{(4.4)}

\[ \sum \sum x_{ijk}^g = D_k^g \quad \forall \ k \] \hfill \ldots \text{(4.5)}

\[ \sum M_k^p x_{ijk}^p \leq A_{ij}^p \quad \forall \ i, j \] \hfill \ldots \text{(4.6)}

\[ \sum M_k^g x_{ijk}^g \leq A_{ij}^g \quad \forall \ i, j \] \hfill \ldots \text{(4.7)}

\[ \sum \frac{x_{ijk}^p}{Q_j^p} \leq A_j^p \quad \forall \ j \] \hfill \ldots \text{(4.8)}

\[ \sum \frac{x_{ijk}^g}{Q_j^g} \leq A_j^g \quad \forall \ j \] \hfill \ldots \text{(4.9)}

\[ \sum \sum x_{ijk}^p \geq d_{jk}^g \quad \text{for } j = 1, 2 \text{ and } k = 1, 2 \] \hfill \ldots \text{(4.10)}

\[ x_{ijk}^g \geq d_{jk}^g \quad \text{for } j = 1 \text{ and } k = 3, 4 \] \hfill \ldots \text{(4.11)}

\[ \sum \sum \sum E_{ijk}^p x_{ijk}^p + \sum \sum \sum E_{ijk}^g x_{ijk}^g \leq A_i \text{ for } i = 1, 2 \] \hfill \ldots \text{(4.12)}
4.4 METHOD FOR OPTIMAL SOLUTION

The three objective functions (4.1) to (4.3) can be solved one after another subject to the sets of constraints (4.4) to (4.12). While solving any single objective function independently, the values of other objective functions may largely deviate from their respective optimum values obtainable. The maximum deviation creates the aspiration level of corresponding objective function. This calls for attaining a compromised solution incorporating all the constraints (4.4) to (4.12) and simultaneously introducing the three additional constraints depicting the aspiration levels of three objective functions. The Multi Objective Fuzzy Linear Programming is such a technique which can be considered as—

Let $x^1, x^2$ and $x^3$ represent the set of solutions for the objective functions $O_1, O_2,$ and $O_3$ i.e.

$$U_t = \text{Min}(O_t(x^t)) \text{ and } \quad \ldots(4.13)$$

$$L_t = \text{Max}(O_t(x^t)) \text{ for } t \neq s$$

Where $U_t$ and $L_t$ indicate upper and lower acceptable values for $t$th objective function respectively.

Then $\mu_t$ be the membership function of $t^{th}$ objective is defined by

$$\mu_t(x) = \begin{cases} 0 & \text{when } O_t \geq L_t \\ \frac{L_t - O_t}{L_t - U_t} & \text{when } U_t \leq O_t \leq L_t \\ 1 & \text{when } O_t \leq U_t \end{cases} \quad \ldots(4.14)$$
Chapter 4...

Now a dummy variable $\lambda$ is introduced for fuzzy solution such that $\lambda \leq \mu_t$, $\forall t$. Therefore three more constraints have been formed with these aspiration levels.

Then Fuzzy Linear Programming becomes

Maximize $\lambda$ \hspace{1cm} \text{...(4.15)}

Subject to:

$O_t + (L_t - U_t)\lambda \leq L_t$

$\lambda \geq 0$ and all other constraints used in individual objectives including non-negativity of decision variables.

Then, applying the simplex method the above problem is solved where all the objectives are considered together with the initiated nine sets of constraints to achieve a best compromised solution.

4.5 FUZZY COMPROMISE PROGRAMMING

(i) Non-dominated (efficient) solution

Let $\bar{x} = \{x_{ij}\} \in X$ be a feasible solution and called the non-dominated solution of the multi-objective transportation problem

\[
\text{Min. } Z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij} \hspace{1cm} \text{...(4.16)}
\]

s.t. $\sum_{i=1}^{m} x_{ij} = b_j \forall j = 1, 2, \ldots, n$
\[ \sum_{j=1}^{n} x_{ij} = a_i \quad \forall i = 1, 2, \ldots, m \]

\[ x_{ij} \geq 0 \quad \forall i, j = 1, 2, \ldots, m, n \]

if there exists no other feasible solution \( x = (x_{ij}) \in X \)

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \leq \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \bar{x}_{ij} \quad \forall k = 1, 2, \ldots, K \]

such that \[ \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \leq \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \bar{x}_{ij} \quad \forall \, k = 1, 2, \ldots, K \]
\[ \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} < \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \bar{x}_{ij} \quad \text{for at least one } K. \]

(ii) **Optimal Compromise Solution**

An optimal compromise solution of the multi-objective transportation problem (4.16) is a solution \( x = \{x_{ij}\} \in X \) which is preferred by the decision-maker to all other solutions (decision-maker’s global subjective evaluation value) taking into consideration all criteria contained in the multi-objective functions.

Hence, an optimal compromise solution has to be a non-dominated solution. Applying the fuzzy compromise programming approach determined to the multi-objective linear transportation problem we obtained an optimal compromise solution by the ordinary technique of linear programming or other.
Consider the multi-objective programming problem

\[
\text{Minimize } Z(x) = \sum_{k=1}^{K} [Z_k(x)]^T \quad \ldots (4.18)
\]

Subject to conditions \( x \in X \)

It is a set of conflicting goals that cannot be achieved simultaneously since every objective function is optimal. So we try to find an optimal compromise solution at which the global evaluation of the synthetic membership degree of optimum for all objectives is maximum.

Since, \( x \in R^n \) is an n-dimensional decision variable and \( X \) be the set of feasible solutions such that global evaluation employed reflects the decision maker’s consideration of all criteria belonging in the multi-objective functions now we will present the marginal evaluation for each objective and to aggregate these marginal evaluations into the global evaluation of the synthetic membership degree of optimum for several objectives. The compromise linear programming must be made based on intuition.

Consider \( U_k \) and \( L_k \) i.e. \( U_k > L_k \) be two values which can be represented as upper and lower bonds of the objective function \( Z_k \) in the multi-objective programming problem where \( (k = 1, 2, 3 \ldots K) \). For objective function \( Z_k \) we can find a marginal evaluation with upper and lower bounds.
Taking a mapping \( \phi_k(x) \in [0,1] \), where \( x \in X \) for objective value \( Z_k \), since \( \phi_k \) be the fuzzy subset for objective function \( Z_k \) on feasible solution \( X \).

For \( k = 1, 2, 3, 4 \ldots K \)

We have \( \phi_k(x) = \begin{cases} 
1, & \text{if } Z_k(x) \leq L_k \\
Z_k(x) - U_k, & \text{if } L_k < Z_k(x) < U_k \\
0, & \text{if } Z_k(x) \geq U_k 
\end{cases} \) \ldots(4.19)

where, \( L_k \neq U_k \)

If we put \( L_k = U_k \Rightarrow \phi_k(x) = 1 \) for any value of \( k \).

We let \( x_1, x_2, x_3, \ldots \alpha x_k \) be the ideal solution of multi-objective programming problem.

Hence, for a single objective programming problem, the optimal solution is \( \text{Min.} Z_k(x) \)

For all \( k \) optimal solution \( Z_k^* = Z_k \alpha x_k \)

If lowest level \( L_k \) and highest acceptable level \( U_k \) can be represented as

\[
U_k = \max_{1 \leq j \leq k} \{Z_k x_{(j)}\} \quad \forall \ k = 1, 2, 3, \ldots K \quad \ldots(4.20)
\]

and \( L_k = Z_k^* = Z_k (x_{(j)}^*) \)

Now let us suppose that \( \xi \) be an aggregating operator and global evaluation
\[ \phi_k(x) : X \rightarrow [0, 1] \text{ for all } Z_k \]

Max. \( \phi(x) = \xi\{\xi_1(x), \xi_2(x), \xi_3(x), \ldots \xi_k(x)\} \) \hspace{1cm} ...(4.21)

for fuzzy compromise programming problem and use ordinary optimization technique, we have \( \phi(x^\alpha) = \text{Max. } \phi(x) \) \hspace{1cm} \text{for } x \in X \)

since, the multi-objective linear transportation problem

Min. \( Z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij} \) is a multi-objective linear programming problem, therefore we can solve it with the fuzzy compromise programming approach.

For each \( Z_k \) (where \( k = 1, 2, 3, \ldots K \))

Assume an optimal solution \( x_{k}^\alpha = x_{(ij)k}^\alpha \) for all \( X \).

Hence \( \phi_k\{x_{ij}\} = \begin{cases} 1, & \text{if } Z_k(x_{ij}) \leq L_k \ \\ \frac{Z_k(x_{ij}) - U_k}{L_k - U_k}, & \text{if } L_k < Z_k(x_{ij}) < U_k \ \\ 0, & \text{if } Z_k(x_{ij}) \geq U_k \end{cases} \) \hspace{1cm} ...(4.22)

[where, \( U_k = \text{Max.} \{Z_k(x_{ij})\} \) and \( L_k = Z_k(x_{ij}^\alpha) \forall (k = 1, 2, 3, \ldots, K) \) ]

4.6 \textbf{VEHICLE ROUTING AND TRAFFIC SIGNAL CONTROL PROBLEM FOR TRANSPORTATION}

Fuzzy control has emerged as one of the most promising areas for research in the application of fuzzy set theory, especially in areas that lack of quantitative data regarding input-output relations such as traffic
signal control. The theory of fuzzy sets is based on concepts graded to handle uncertainties and imprecision in a particular domain of knowledge. The graded concepts are useful, since real situations are very often neither crisp nor deterministic, and cannot be described precisely. Hence, fuzzy sets are manipulated by the set theoretic operations of union, intersection and complement with their membership functions. The use of fuzzy sets provides a systematic way of manipulating vague and imprecise concepts by introducing linguistic variables, fuzzy relations and fuzzy logic.

Transport planners have traditionally concentrated on the movement of vehicles as the major target of traffic management. The introduction of the earliest traffic signals was to ensure safety at intersections by keeping conflicting traffic problems. In these days, the traffic signal design can be viewed through measures of performance of intersection operation criteria or desirable outcomes. Hence a decrease in delay, number of stops, fuel consumption, pollutant emissions, vehicle operating costs and queue lengths, as well as an increase in consideration for public transport vehicles and also in safety, are all desirable, the main performance measure for judging the efficiency of traffic signal control systems has been reduction of vehicle delay and stops. Another considerations, such as convenience and bus priorities, have usually been incorporated into the systems later in their development as pragmatic
features required for on-street operation but have not been used as performance measures during the development and optimization phases.

The signal group control at local level normally represents a decentralized control strategy. This approach makes it more difficult to handle mathematically than the traditional fixed stage control.

Fuzzy control has been developed in the context of fuzzy inference. Fuzzy inference is the inference process based on the multi-value logic of inference, the truth values of input and the rules of the inference process are not singular but they are multi-valued, the truth of the conclusion is given a value between 0 and 1.

In transportation problems, the fuzzy logic linguistic and in exact data to be manipulated as a useful tool in designing signal timings. It also provides a means of converting a linguistic control strategy. The motivation for designing a fuzzy controller is that there is a fairly direct relationship between the loose linguistic expressions of a traffic control strategy and its manual implementation. Controlling traffic signal timing involves making continuous evaluations.

4.7 FUZZY TRAFFIC CONTROL FOR TRANSPORTATION PROBLEM

The signal group and changing to the most appropriate phase extend to the regular intervals while information and evaluation are the most appropriate option i.e. most practical control problems, this control
process involves such types of elements as input, processor, output. The processor is the rule base that, given the input provides the decisions as to whether to continue the signal group and output is the predicted consequence of the control prescribed by the processor. The current traffic control situation with signal status, crisp input and traffic situation Modelling. The fuzzification interface, fuzzy decision-making, defuzzification and signal control are involves for making the controlling traffic transportation problems.

![Diagram](image)

Figure 4.1 (fuzzy traffic control)

Public transport priorities are usually transporter-made using a number of special principles. A public transport priority is given when a public transport vehicle is detected and our idea of the fuzzy approach to public transport priorities when time ($t > 0$) is a fuzzy variable. Then the mathematical model of the given problem is based on the multiple vehicle pickup and delivery problem with time ‘$t$’.
Let us consider $\mu_{c}(t) = 1$ be the satisfaction level of the customer if the service starts at time ‘$t$’.

When $\mu_{c}(t) = 1$, the customer is fully satisfied and if $\mu_{c}(t) = 0$ then this is the dissatisfaction of the customer and we say $\mu_{c}$ is the membership function of the set of service starts times which satisfying the customer. At each time ‘$t$’, the fuzzy linear programming is $a_i \leq t_i \leq b_i$ where $\tilde{c}_i = (a_i, b_i, \overline{b}_i, \overline{a}_i)$ be the trapezoidal fuzzy number then the satisfaction level $\tilde{c}_i$ by the value $\tilde{T}_i$ is equal to the maximum level of both fuzzy values at the same time and let the value $\delta(-\infty, t_i)$ describe the time $t$ from which service has already started.

$$\sup_{t_i} \min \{\mu_{c_i}(t_i), \delta(-\infty, t_i)\} = \sup_{t_i} \min \{\mu_{c_i}(t_i), \inf_{t > t_i} \mu_{\tilde{T}_i}(t)\}$$

![Figure 4.2 (Fuzzy constraint by a fuzzy value)](image)

Figure 4.2 (Fuzzy constraint by a fuzzy value)
4.8 CONCLUSION

In the present chapter deals a very limited scope for altering the transport modes identified for different distance ranges in the solutions. For introducing new modes as, bus or truck, this model will be of immense help in selecting origin and destination. In the search for a best result we proposed to find an optimal value of transportation problem from both model uncertainty and fuzzy constraints. The full optimization method considering both objectives (cost and satisfaction). The present result is also an important role as in multi-objective space there might be no way to point the best solution. In real world traffic conditions are changing all the time and we develop an effective control process for traffic signal timings and for multi-objective linear transportation problem with k objective functions using the fuzzy compromise programming approach we find a compromise solution not only non-dominated but also optimal solution for different objectives.