CHAPTER-2

SOLUTION OF FUZZY TRANSPORTATION PROBLEMS
THROUGH OPTIMIZATION TECHNIQUES

2.1 INTRODUCTION

As the scope of transportation analysis proliferates and as the consequences of transportation decisions pose for-reaching impacts on many non-transportation aspects, the analysis of transportation must inevitably deal with the types of uncertainty that are different from the traditional form, which has been handled by probability theory.

This parts of the transportation research introduces fuzzy set theory as a paradigm to deal with some of the difficulties that are related to the concepts or numbers that have the vague boundaries. Fuzzy sets theory defines such concepts and numbers as fuzzy sets. There are many areas in which fuzzy set theory can be applied; they include inference, control, classification, transportation problem, decision-making and optimization.

In competitive market and global economics the pressure on organizations for finding the better ways to create and deliver value to customers stronger. Our problems is, how and when to send the products to the customers in the quantities they want in a cost-effective manner become more challenging. Transportation models provide a powerful role
to meet this challenge as reducing cost and improving service. Since, transportation problem is a linear programming problem and we might transport from sources to destinations by different transport ways to reduce costs or to meet time schedule. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients, supply demand and conveyance capacities are known exactly.

However, these parameter are not presented in a precise manner. All the parameter may be uncertain due to uncontrollable reduction. K.B. Haley(1962) developed the solution procedure of a solid transportation and made a comparison between the solid transportation problem and the classical transportation problem. For uncertainty of the information in making decision Zadeh(1965) introduces the notion of fuzziness.

The transportation problem is essentially a linear program and we have to develop a optimization techniques to apply the existing fuzzy linear programming to fuzzy multi-objective transportation problems. Bit A.K.(1993) apply fuzzy linear programming technique to multi-objective solid transportation problem. The method of B. Julien(1994) and M.A. Parra(1999) is applicable to find the possibility distributions of the objective value provided all the inequality constraints are “≥” or “≤” type, S. Chanas(1996) investigate the transportation problem with fuzzy supplies and demand and the transportation cost will be fuzzy as well.

In the present chapter we have to deal the basic Zadeh's extension principle and with the help of it we develop a solution procedure which is applicable for calculating the fuzzy objective value of the fuzzy multi-objective transportation problem. The fuzzy multi-objective transportation problem is transformed into a pair of two-level mathematical programs for describing the lower and upper bounds of the objective value at possibility level $\alpha$.

### 2.2 SOLUTION OF FUZZY TRANSPORTATION PROBLEM

Consider a set $\{1, 2, 3, \ldots, m\}$ of sources and assume a positive supply $s_i > 0$ to the supply node $i$ is given for each source $i$, and let a set $\{1, 2, 3, \ldots, n\}$ of destination and assume an unknown demand $d_j$ is associated with every destination $j$.

Let $s_i$ be the amount of supply of homogeneous product and $d_j$ be the amount of demand. Now let us suppose that $g_k$ be the units of this product which can be carries by $k$ different modes of transport said
conveyance, such as car, train, ship etc., and let $C_{ijk}^q$ be a penalty value represented by transportation cost per unit and $x_{ijk}$ be the unknown capacity which represents the transported from sources i to destination j by the $k$-th conveyance.

Our main goal of this theme is to determine the unknowns $x_{ijk}$ that will minimize the total transportation cost while satisfying all the supply and demand restrictions. These sources and destinations represented by a node. The arcs represents the routes linking the sources and the destinations. Arcs joining sources to destination by conveyance capacities.

![Network Diagram](image)

Figure 2.1 (Representation of transportation model)

A mathematical model of multi-objective transportation problem can be represented as
Chapter 2…

Minimize \( z_q = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} C_{ij}^q x_{ijk} \), \( q = 1, 2, 3, \ldots, Q \)

subject to constraints
\( \sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq s_i \), \( i = 1, 2, 3, \ldots, m \)

\( \sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq d_j \), \( j = 1, 2, 3, \ldots, n \) \( \ldots(2.1) \)

\( \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq g_k \), \( k = 1, 2, 3, \ldots, l \)

\( \forall x_{ijk} \geq 0 \)

For solving real-life transportation problems we should not use only objective knowledge (formulae and equations) or only subjective knowledge (linguistic information). We simply cannot and should not ignore the existence of linguistic information i.e. subjective knowledge. Fuzzy logic is an extremely suitable concept with, which to combine subjective knowledge and objective knowledge.

2.3 FUZZY OPTIMIZATION TECHNIQUE

Suppose the unit transportation cost \( c_{ij}^q \), supply \( s_i \), demand \( d_j \) and conveyance capacity \( g_k \) are approximately known and if any of the parameters \( c_{ij}^q \), \( s_i \), \( d_j \) or \( g_k \) is fuzzy then the transportation cost becomes fuzzy.
So we can reduce multi-objective transportation problem model (2.1) into the fuzzy multi-objective transportation problem model.

If all these parameters can be represented by convex fuzzy numbers $\tilde{c}_{ijk}^q$, $\tilde{s}_i$, $\tilde{d}_j$ and $\tilde{g}_k$ respectively with membership functions $\mu_{\tilde{c}_{ijk}^q}$, $\mu_{\tilde{s}_i}$, $\mu_{\tilde{d}_j}$ and $\mu_{\tilde{g}_k}$.

\[
\tilde{c}_{ijk}^q = \left\{ \left( c_{ijk}^q, \mu_{\tilde{c}_{ijk}^q} \right) \left| c_{ijk}^q \in S(\tilde{c}_{ijk}^q) \right\} 
\]

\[
\tilde{s}_i = \left\{ \left( s_i, \mu_{\tilde{s}_i} \right) \left| s_i \in S(\tilde{s}_i) \right\} 
\]

\[
\tilde{d}_j = \left\{ \left( d_j, \mu_{\tilde{d}_j} \right) \left| d_j \in S(\tilde{d}_j) \right\} 
\]

\[
\tilde{g}_k = \left\{ \left( g_k, \mu_{\tilde{g}_k} \right) \left| g_k \in S(\tilde{g}_k) \right\} 
\]

where, $S(\tilde{c}_{ijk}^q)$, $S(\tilde{s}_i)$, $S(\tilde{d}_j)$ and $S(\tilde{g}_k)$ are the supports of $\tilde{c}_{ijk}^q$, $\tilde{s}_i$, $\tilde{d}_j$ and $\tilde{g}_k$ which is convex fuzzy numbers as the universe of discourse.

Let the fuzzy objective function $Z_q = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \tilde{c}_{ijk}^q x_{ijk}$ which is to be minimized by

subject to constraints

\[
\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq \tilde{s}_i \quad \forall \quad i=1,2,3,\ldots,m
\]
\[
\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq \tilde{D}_j \quad \forall \quad j = 1, 2, 3, \ldots, n
\]
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \tilde{G}_k \quad \forall \quad k = 1, 2, 3, \ldots, l \quad \text{...(2.3)}
\]

Since all the supply, demand and conveyance capacities are taken to be convex fuzzy numbers because \( \tilde{Z} \) is a fuzzy number and we want to minimized transportation cost so we can use fuzzy optimization technique.

To transform the fuzzy multi-objective transportation problem by Zadeh's extension, principle, the membership function \( \mu_z \) can be defined by

\[
\mu_{z_q} = \text{Sup.Min.}\left\{ \mu_{\tilde{c}^q_{ijk}}(c_{ijk}), \mu_{\tilde{S}_i}(s_i), \mu_{\tilde{D}_j}(d_j), \mu_{\tilde{G}_k}(g_k) \right\} \forall i, j, k
\]
\[
= z(c, s, d, g) \quad \text{...(2.4)}
\]

Generally, the solution of fuzzy multi-objective transportation problem proposed by Julien(1994) and Parra(1999) can be applied to produce the possibility distribution of the objective value. However, in some cases their methods are not exact to obtain the correct results of the problems.

### 2.4 NUMERICAL ILLUSTRATION

Consider the transportation problem of two supply nodes \( \tilde{S}_1 = (2, 3, 5), s_2 = 5 \) and two demand nodes \( d_1 = 4, \tilde{D}_2 = (1, 3, 6) \) where \( \tilde{S}_1 \)
and $\tilde{D}_2$ are triangular fuzzy numbers. This fuzzy transportation problem can be formulated as:

$$\tilde{Z} = \min x_{11} + 3x_{12} + 7x_{21} + 2x_{22}$$

st. $x_{11} + x_{12} \leq (2,3,5)$

$x_{21} + x_{22} \leq 5$

$x_{11} + x_{21} \geq 4$

$x_{12} + x_{22} \geq (1,3,6)$, $x_{11}, x_{12}, x_{21}, x_{22} \geq 0$

For $\alpha$-cuts of $\tilde{Z}$, there are objective value $Z^L_\alpha$ to the lower bound and objective value $Z^U_\alpha$ at $\alpha = 0$ to the upper bound. Right hand side value of the constraints with fuzzy numbers to the lower bound and upper bound we have,

$$Z^L_{\alpha=0} = \min x_{11} + 3x_{12} + 7x_{21} + 2x_{22}$$

st. $x_{11} + x_{12} \leq 2$

$x_{21} + x_{22} \leq 5$

$x_{11} + x_{21} \geq 4$

$x_{12} + x_{22} \geq 1$

$x_{11}, x_{12}, x_{21}, x_{22} \geq 0$

and $Z^U_{\alpha=0} = \min x_{11} + 3x_{12} + 7x_{21} + 2x_{22}$

st. $x_{11} + x_{12} \leq 5$
\[ x_{21} + x_{22} \leq 5 \]
\[ x_{11} + x_{21} \geq 4 \]
\[ x_{12} + x_{22} \geq 6 \]
\[ x_{11}, x_{12}, x_{21}, x_{22} \geq 0 \]

By Charne's penalty method or Big M method we obtained the optimal solution of these two transportation problem are feasible and objective values are \( Z^L_{\alpha}=18 \) and \( Z^U_{\alpha}=17 \), but it is contradiction, since upper values is always greater than lower values so we occurs supply and demand nodes, \( S_1 = (4 \text{ or } 5), s_2 = 5, d_1=4 \text{ and } D_2=1 \)

For lower bound condition

Min. \( Z = x_{11} + 3x_{12} + 7x_{21} + 2x_{22} \)

\[ \Rightarrow \quad \text{Max. } (-Z) = -x_{11} - 3x_{12} - 7x_{21} - 2x_{22} \]

subject to constraints

\[ x_{11} + x_{12} + s_1 = 4 \]
\[ x_{21} + x_{22} + s_2 = 5 \]
\[ x_{11} + x_{21} - s_3 + A_1 = 4 \]
\[ x_{21} + x_{22} - s_4 + A_2 = 1 \]
### Table 2.1 (For Finding the Optimal Solutions)

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Since, all \( Δ_j ≤ 0 \) ⇒ solution is optimal

\[
\Rightarrow x_{11} = 4, x_{12} = 0, x_{21} = 0, x_{22} = 1
\]

\[
\Rightarrow z = -4 - 3 \times 0 - 7 \times 0 - 2 \times 1 = -6
\]

Hence, \( \text{Min} Z^L_{x=0} = 6 \)

Again for upper bound condition,

\[
\text{Max. } Z = -x_{11} - 3x_{12} - 7x_{21} - 2x_{22}
\]
subject to constraints, \[ x_{11} + x_{12} + s_1 = 2 \]
\[ x_{21} + x_{22} + s_2 = 5 \]
\[ x_{11} + x_{21} - s_3 + A_1 = 4 \]
\[ x_{12} + x_{22} - s_4 + A_2 = 3 \]

### Table 2.2 (For Finding the Optimal Solutions)

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<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>x</td>
<td>2/1=2</td>
<td></td>
</tr>
</tbody>
</table>
Since, all $\Delta_j \leq 0 \Rightarrow$ solution is optimal

$\Rightarrow x_{11} = 2, x_{12} = 0, x_{21} = 2, x_{22} = 3$

Max. $Z = -2 - 0 - 14 - 6 = -22$

Hence, $\text{Min} Z_{\alpha=0}^U = 22$

2.5 DEVELOPMENT OF FUZZY OPTIMIZATION TECHNIQUE

Now we develop a technique for fuzzy multi-objective transportation problem for finding the minimizing transportation cost.

Let us consider $\alpha$-cuts of $\tilde{C}_{ijk}^q, \tilde{S}_i, \tilde{D}_j$ and $\tilde{G}_k$ as follows:

$$\tilde{C}_{ijk}^q = \left\{ c_{ijk}^q \in S(\tilde{C}_{ijk}^q) \mid \mu_{\tilde{C}_{ijk}^q}(c_{ijk}) \geq \alpha \right\}$$

$$(S_i)_{\alpha} = \left\{ s_i \in S(\tilde{S}_i) \mid \mu_{\tilde{S}_i}(s_i) \geq \alpha \right\}$$

$$(D_j)_{\alpha} = \left\{ d_j \in S(\tilde{D}_j) \mid \mu_{\tilde{D}_j}(d_j) \geq \alpha \right\}$$

$$(G_k)_{\alpha} = \left\{ g_k \in S(\tilde{G}_k) \mid \mu_{\tilde{G}_k}(g_k) \geq \alpha \right\}$$

where, $c_{ijk}^q, s_i, d_j$ and $g_k$ be the fuzzy coefficients in the multi-objective functions, fuzzy supplies, fuzzy demands and conveyance capacity in the constraints respectively and all these fuzzy coefficients lie at possibility level $\alpha$.

To find the membership function $\mu_{\tilde{Z}}$, which is equivalent to finding the lower bound $Z_{\alpha}^L$ and upper bound $Z_{\alpha}^U$ of the $\alpha$-cuts of $\tilde{Z}$. 

55
Since, $\alpha$-cuts of a fuzzy set containing all the elements of the universe of discourse whose membership grades in that fuzzy set are greater than or equal to the specified value of $\alpha$.

So lower bound $Z^L_\alpha$ is the minimum of $Z \ (c, s, d, g)$ and upper bound $Z^U_\alpha$ is the maximum of $Z \ (c, s, d, g)$ which can be represented as

Min. \[ Z^L_\alpha = (C_{ijk}^q)^L \leq c_{ijk}^q \leq (C_{ijk}^q)^U \]

\[ (S_i)_\alpha^L \leq s_i \leq (S_i)_\alpha^U \]

\[ (D_j)_\alpha^L \leq d_j \leq (D_j)_\alpha^U \]

\[ (G_k)_\alpha^L \leq g_k \leq (G_k)_\alpha^U \]

\[ \forall i, j, k \]

and

Max. \[ Z^U_\alpha = (C_{ijk}^q)^L \leq c_{ijk}^q \leq (C_{ijk}^q)^U \]

\[ (S_i)_\alpha^L \leq s_i \leq (S_i)_\alpha^U \]

\[ (D_j)_\alpha^L \leq d_j \leq (D_j)_\alpha^U \]

\[ (G_k)_\alpha^L \leq g_k \leq (G_k)_\alpha^U \]

\[ \forall i, j, k \]

Min. \[ Z \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} C_{ijk}^q x_{ijk} \]

s.t. \[ \sum_{j=1}^{n} x_{ijk} \leq s_i, i = 1, 2, \ldots, m \]

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \geq d_j, j = 1, 2, \ldots, n \]

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq g_k, k = 1, 2, \ldots, l \]

\[ x_{ijk} \geq 0 \forall i, j, k \quad \ldots(2.6.1) \]

and

Min. \[ Z \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} C_{ijk}^q x_{ijk} \]

s.t. \[ \sum_{j=1}^{n} x_{ijk} \leq s_i, i = 1, 2, \ldots, m \]

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \geq d_j, j = 1, 2, \ldots, n \]

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq g_k, k = 1, 2, \ldots, l \]

\[ x_{ijk} \geq 0 \forall i, j, k \quad \ldots(2.6.2) \]
For feasible solutions the fuzzy multi-objective transportation problem ensure that the constraint

\[
\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} d_j \quad \text{and} \quad \sum_{k=1}^{l} g_k \geq \sum_{j=1}^{n} d_j \forall i, j, k \quad \ldots (2.7)
\]

and we get all \( c_{ijk}^q \) have been set of lower bounds of their \( \alpha \)-cuts

\[
\Rightarrow \mu \tilde{c}_{ijk}^q (c_{ijk}^q) = \alpha
\]

\[
\Rightarrow \mu \tilde{Z}(z) = \alpha
\]

i.e., \( Z_\alpha^L = \min \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} (C_{ijk}^q)_{\alpha} x_{ijk} \) is a linear program which can be easily solved.

Now we take the help of duality of linear programming that the primal model and the dual model be the same objective value.

\[
\begin{aligned}
\text{Max. } & Z_\alpha^U = (C_{ijk}^q)_{\alpha} \leq c_{ijk}^q \leq (C_{ijk}^q)_{\alpha} \\
\text{s.t. } & (S_i)_{\alpha} \leq s_i \leq (S_i)_{\alpha} \\
& (D_j)_{\alpha} \leq d_j \leq (D_j)_{\alpha} \\
& (G_k)_{\alpha} \leq g_k \leq (G_k)_{\alpha} \\
& -u_i + v_j - w_k \leq c_{ijk}^q \\
& \forall i = 1, 2, \ldots, m \\
& \quad j = 1, 2, \ldots, n \\
& \quad k = 1, 2, \ldots, l \\
& u_i, v_j, w_k \geq 0 \forall i, j, k \quad \ldots (2.8)
\end{aligned}
\]
Since, \((C_{ijk}^q)_\alpha^L \leq e_{ijk}^q \leq (C_{ijk}^q)_\alpha^U \) \(\forall i,j,k\) in model (2.8) one can derive the upper bound to the objective value of \(e_{ijk}^q\) to \((C_{ijk}^q)_\alpha^U\) upper bounds.

Hence, \(Z_{\alpha}^U = \max. - \sum_{i=1}^{m} s_i u_i + \sum_{j=1}^{n} d_j v_j - \sum_{k=1}^{l} g_k w_k\)

s.t. \(-u_i + v_j - w_k \leq (C_{ijk}^q)_\alpha^U\) \(...(2.9)\)

\[\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} d_j \text{ and } \sum_{k=1}^{l} g_k \geq \sum_{j=1}^{n} d_j\]

If the total supply and total conveyance capacity are greater than the demand at all \(\alpha\)-values

i.e., \(\sum_{i=1}^{m} (s_i)_\alpha^L - (s_i)_\alpha^U \geq \sum_{j=1}^{n} (D_j)_\alpha^L - (D_j)_\alpha^U\)

and \(\sum_{k=1}^{l} (G_k)_\alpha^L - (G_k)_\alpha^U \geq \sum_{j=1}^{n} (D_j)_\alpha^L - (D_j)_\alpha^U\)

then subject to constraints will be \(\sum_{i=1}^{m} (s_i) \geq \sum_{j=1}^{n} d_j\) and

\(\sum_{k=1}^{l} g_k \geq \sum_{j=1}^{n} d_j\) and the transportation problem will be feasible if the lower bounds of the fuzzy total demand is smaller than both of the upper bound of the fuzzy total supply and upper bound of the total conveyance capacity which is:

\[\sum_{j=1}^{n} (D_j)_\alpha^L \leq \sum_{i=1}^{m} (S_i)_\alpha^U \text{ and } \sum_{j=1}^{n} (D_j)_\alpha^L \leq \sum_{k=1}^{l} (G_k)_\alpha^U\]
2.5.1 Illustration: To determine the fuzzy programming algorithm, we consider a multi-objective standard solid transportation problem having the following characteristics.

Supplies: \[ s_1 = 24, \quad s_2 = 8, \quad s_3 = 18, \quad s_4 = 10 \]

Demands: \[ d_1 = 11, \quad d_2 = 19, \quad d_3 = 21, \quad d_4 = 9 \]

Conveyance capacities: \[ g_1 = 17, \quad g_2 = 31, \quad g_3 = 12 \]

Penalties:

Destinations: \[ W_1 \quad W_2 \quad W_3 \quad W_4 \]

Conveyances: \[ C \quad C \quad C \quad C \quad C \quad C \quad C \quad C \quad C \quad C \quad C \]

Sources:

\[
C^1 = \begin{bmatrix}
0_1 & 15 & 18 & 17 & 12 & 22 & 13 & 10 & 4 & 12 & 8 & 11 & 13 \\
0_2 & 17 & 20 & 19 & 21 & 21 & 22 & 21 & 19 & 18 & 30 & 10 & 23 \\
0_3 & 14 & 11 & 12 & 25 & 34 & 33 & 20 & 16 & 15 & 21 & 23 & 22 \\
0_4 & 22 & 18 & 13 & 24 & 35 & 32 & 18 & 21 & 14 & 13 & 23 & 20
\end{bmatrix}
\]

\[
C^2 = \begin{bmatrix}
0_1 & 6 & 7 & 8 & 10 & 6 & 5 & 11 & 3 & 7 & 10 & 9 & 6 \\
0_2 & 13 & 8 & 11 & 12 & 2 & 9 & 20 & 15 & 13 & 17 & 15 & 13 \\
0_3 & 5 & 6 & 7 & 11 & 9 & 7 & 10 & 5 & 2 & 15 & 14 & 18 \\
0_4 & 13 & 6 & 6 & 17 & 11 & 18 & 12 & 16 & 12 & 18 & 14 & 7
\end{bmatrix}
\]

These penalties can be expressed in a three dimensional table

\[
\text{Min. } Z_q = \sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{k=1}^{3} C^q_{ijk} x_{ijk}, q = 1, 2
\]
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\[
\begin{align*}
\text{s.t.} & \quad \sum_{j=1}^{4} \sum_{k=1}^{3} x_{jk} = 24, \quad \sum_{j=1}^{4} \sum_{k=1}^{3} x_{2jk} = 8, \quad \sum_{j=1}^{4} \sum_{k=1}^{3} x_{3jk} = 18 \\
& \quad \sum_{j=1}^{4} \sum_{k=1}^{3} x_{4jk} = 10, \quad \sum_{i=1}^{4} \sum_{k=1}^{3} x_{ijk} = 11, \quad \sum_{i=1}^{4} \sum_{k=1}^{3} x_{i2k} = 19, \\
& \quad \sum_{i=1}^{4} \sum_{k=1}^{3} x_{i3k} = 21, \quad \sum_{i=1}^{4} \sum_{j=1}^{4} x_{i4j} = 9, \quad \sum_{i=1}^{4} \sum_{j=1}^{4} x_{ij1} = 17, \\
& \quad \sum_{i=1}^{4} \sum_{j=1}^{4} x_{ij2} = 31, \quad \sum_{i=1}^{4} \sum_{j=1}^{4} x_{ij3} = 12 \\
x_{ijk} \geq 0 \forall i=1,2,3,4, \quad j=1,2,3,4, \quad k=1,2,3,4
\end{align*}
\]

where, \( C^1 = [C_{ijk}^1] \) and \( C^2 = [C_{ijk}^2] \)

Now applying additive utility function method to the multi-objective solid transportation problem, we get the non-zero decision variables

\[
x_{121} = 8, \quad x_{123} = 3, \quad x_{132} = 13, \quad x_{222} = 8, \quad x_{312} = 10, \quad x_{333} = 8, \\
x_{413} = 1, \quad x_{441} = 9
\]

The objective values is \( Z_1 = 715 \) and \( Z_2 = 394 \)

\[\Rightarrow \] it is a compromise solution but not to be optimal solution.

According to Diaz method (1978) we get the non-zero decision variables.

\[
x_{123} = 3, \quad x_{132} = 21, \quad x_{222} = 8, \quad x_{311} = 9, \quad x_{312} = 1, \quad x_{321} = 8, \\
x_{412} = 1, \quad x_{443} = 9
\]
and the objective value $Z_1 = 826$, $Z_2 = 302$ but it is not to be optimal solution, so by using fuzzy programming method, we obtain

$$U_1 = 866, \quad L_1 = 703, \quad U_2 = 537, \quad L_2 = 293$$

and compromise solution

$$x_{121} = 10.143, \quad x_{132} = 13.857, \quad x_{222} = 8$$

$$x_{311} = 1.714, \quad x_{312} = 9.143, \quad x_{333} = 7.143, \quad x_{421} = 0.857$$

$$x_{419} = 0.143, \quad x_{441} = 4.286, \quad x_{443} = 4.714, \quad \lambda = 0.716$$

and the values of objective functions are

$$Z_1 = 749.2853 \text{ and } Z_2 = 362.2860$$

Therefore, we have the best compromise solution $[749.2853, 362.2860]$ by fuzzy linear programming method.

### 2.6 CONCLUSION

Fuzzy linear programming is a more suitable technique for the multi-objective transportation problem. For the larger problem, it is not easy to find the compromise solution by Ringuest(1987) and Diaz(1978) but with the help of fuzzy linear programming technique we get a suitable compromise solution which is optimal. It gives a better optimal compromise solution when we takes number of objectives and constraints. This feature makes the fuzzy approach more practical than other approaches solving the multi-objective transportation problem.
In this chapter the transportation problem considers not only the supply and demand but also the conveyance capacity as the fuzzy numbers for satisfying the transportation requirements and gives a method to find the membership function of the fuzzy total transportation cost when the unit transportation costs, demand, supply and conveyance capacities are convex fuzzy numbers. Since, transportation models approach an important role in supply chain management and business administration for reducing cost and improving service. The idea of solving the transportation problem is based on Zadeh's extension principle to transform the fuzzy multi-objective transportation problem into a pair of mathematical programme.