7.1 INTRODUCTION

The classical transportation problem is a subclass of linear programming problem in which a fixed cost is incurred for every origin. Most of the factors which can be applied to input data, and related parameters, such as available supply and forecast demand are often imprecise or fuzzy because most of the information is incomplete. And the decision makers must handle conflicting goals that govern the use of the constrained resources with organizations simultaneously. Many distribution problems are in practice which can only be modelized as fuzzy capacitated transportation problems.

In the present chapter we proposed a fuzzy linear programming method for solving the transportation decision problem with fuzzy constraints due to available supply, forecast demand and damage units and find a minimization total production and transportation cost corresponding to total delivery time, and when the transportation problems admits of an objective function value which is lower than the optimal objective function value by transporting larger quantities of goods.
The transportation problem is an efficient in terms of how to minimize the transportation cost and production cost. Since transportation problem is an extension of the classical transportation problem in which a fixed cost is available for every origin. Since the application of the transportation model is not limited to transporting commodities between sources to destinations. This chapter presents two model in the area of production and transporting problems which was originally formulated by Hirsch and Dantzig(1954) Sandtrock(1988) gave a simplex algorithm for solving the fuzzy cost capacitated transportation problem. Basu et. al.(1994) gave an algorithm for finding optimal solution of solid fixed charge transportation problems, the fixed charge transportation problems have been studied by Arora et al(2004). Generalized, the decision maker conflict these goals frequently in the framework of fuzzy aspiration levels. Li and Lai(2000) Abd El-Wahed(2006) modelized the conventional linear programming method and existing solution algorithms which can not be solve all transportation problem decision in uncertain demands or environments.

Zimmermann(1976) first introduced fuzzy set theory into the ordinary linear programming and multi-objective linear programming problems with fuzzy goals and fuzzy constraints. As well as Zimmermann's fuzzy linear programming has developed into different fuzzy optimization techniques to solve transportation problems.

Chanas et. al(1996) presented an fuzzy linear programming model for solving these type of transportation problem which is going due to
uncertainty as uncertain demand of market with crisp cost coefficients and fuzzy supply and demand. Moreover Thirwani(1998) presented a model for many distribution problems in which we have to modelized the fuzzy charge transportation problems as rails, roads and trucks which consists of a fixed cost and a variable cost.

The second class of transportation problems, where the objective function to be optimized is a ratio of two linear functions, optimization of a ratio of criteria which gives more insight in these types of situations Dinkelbach (1967) solved linear fractional programming problem. Swarup (1966) also gave a method to solve a linear fractional transportation problem, another types of transportation problem consists capacitated transportation problem, we modelized a non-linear capacitated transportation problem admits of a total objective function value which is lower than the optimum and is attainable by transporting maximum quantities of the goods where the production and transportation cost will be minimize. Li and Lai (2006) presented a fuzzy compromise programming solution method for obtaining a non-dominated compromise solution, for fuzzy transportation planning decision, problems with multiple fuzzy constraints. In this chapter we develops an interactive fuzzy linear programming techniques for solving multi-objectives transportation problems with fuzzy constraints, available supply, uncertain demand with damage transporting units and minimize total production and transportation cost.
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7.2 NOTATIONS

The following notations have been taken for the proposed model to be discussed in this chapter.

\( I = \text{index set of warehouses} \ (i = 1, 2, 3 \ldots m) \)

\( J = \text{index set of destinations} \ (j = 1, 2, 3 \ldots n) \)

\( x_{ij} = \text{the amount transported from the } i^{\text{th}} \text{ warehouse to the } j^{\text{th}} \text{ destination.} \)

\( g = \text{index for objectives for all} \ (g = 1, 2, 3 \ldots k) \)

\( Q_{ij} = \text{units transported from source } i \text{ to destination } j \)

\( z_1 = \text{total production and transportation costs in rupees (i.e. objective functions)} \)

\( z_2 = \text{total delivery time in hours} \)

\( c_{ij} = \text{transportation cost per unit delivered from source } i \text{ to destination } j \)

\( P_{ij} = \text{production cost per unit delivered from source } i \text{ to destination } j \)

\( f_i = \text{fixed charge associated with } i^{\text{th}} \text{ warehouse} \)

\( t_{ij} = \text{transportation time per unit delivered from source } i \text{ to destination } j. \)

\( \tilde{D}_j = \text{total uncertain/forecast demand of each destination } j \)

\( \tilde{S}_i = \text{total available supply for each source } i \text{ in units} \)
\( d_{ij} \) = damage cost per unit due to transportation of goods.

\( q_{ij} \) = units transported to destination after transportation due to damage of goods.

\( a_{ij} \) = machine-hour/unit

\( M_{i \text{max}} \) = maximum machine capacity available for each source \( i \)

\( W_{j \text{max}} \) = maximum warehouse space for each destination \( j \)

\( B \) = total budget in rupees

\( b_{ij} \) = warehouse space per unit delivered from source \( i \) to destination \( j \)

### 7.3 MATHEMATICAL MODEL OF FUZZY MULTI-OBJECTIVE TRANSPORTATION PROBLEMS

Let us consider a logistics center for determining the transportation decision of a homogeneous commodities from \( m \) sources to \( n \) destinations. Since each source has an available supply of the units which is to be distributed to different destinations and each destination has a uncertain demand of the commodity to be received from various sources, our aim is to minimize the total production and transportation costs in the fixed of budget

Minimize total production and transportation costs

Min. \( z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} (p_{ij} + c_{ij})Q_{ij} \) \( \ldots \)(7.1)

and the minimize total delivery time
Min. \( z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij}Q_{ij} \) \( \ldots(7.2) \)

where, \( \simeq \) is refers to the fuzzification of the aspiration levels.

For the constraints on total available supply for each source \( i \) to destinations \( j \)

\[ \sum_{j=1}^{n} Q_{ij} \leq \tilde{S}_i \text{ for all value of } i \] \( \ldots(7.3) \)

when the demand of the goods in real life applications are uncertain and forecast for each destination \( j \) due to damage items with transportation

\[ \sum_{i=1}^{m} q_{ij} \geq \tilde{D}_j \text{ for all value of } j \] \( \ldots(7.4) \)

where \( Q_{ij} = a_{ij} + d_{ij} \) as well as \( q_{ij} = Q_{ij} - d_{ij} \)

fuzzy constraints on total budget

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} (p_{ij} + c_{ij})Q_{ij} \leq B \] \( \ldots(7.5) \)

fuzzy constraints on machine capacities for each source \( i \) to destinations \( j \)

\[ \sum_{j=1}^{n} a_{ij}Q_{ij} \leq M_{i \text{ max}} \text{ for all } i \] \( \ldots(7.6) \)

fuzzy constraints on warehouse space for each destination \( j \)

\[ \sum_{i=1}^{m} b_{ij}q_{ij} \leq W_{j \text{ max}} \text{ for all } j \] \( \ldots(7.7) \)
with non negative constraints on decision variables

\[ Q_{ij} \geq 0 \text{ for all } i & j \]

if we have \( f_g(z_g) \) be the membership function to the flexibility of uncertain demand which represent fuzzy numbers for solving the multi-objective transportation programming decision problem in a fuzzy sense and corresponding to fuzzy objective functions

\[
f_g(z_g) = \begin{cases} 
1 & , z_g \leq z_g^L \\
\frac{z_g^U - z_g}{z_g^U - z_g^L} , & z_g^L \leq z_g < z_g^U \\
0 & , z_g \geq z_g^U 
\end{cases} \quad \ldots (7.8)
\]

where, \( z_g \) be the objective functions and \( z_g^U, z_g^L \) be the upper bounds and lower bounds of the membership function for total available supply

\[
f_i(q_{ij} + d_{ij}) = \begin{cases} 
1 & , q_{ij} + d_{ij} \leq s_i^I \\
\frac{s_i^U - (q_{ij} + d_{ij})}{s_i^U - s_i^L} , & s_i^L < q_{ij} + d_{ij} < s_i^U \quad , i = 1, 2, 3, \ldots m \ldots (7.9) \\
0 & , q_{ij} + d_{ij} \geq s_i^U 
\end{cases}
\]

where, \( (q_{ij} + d_{ij}) = \sum_{j=1}^{n} Q_{ij} \forall i = 1, 2, 3 \ldots m \) and corresponding linear membership functions for all of the fuzzy objective functions and
the fuzzy inequality constraints to the upper and lower bounds \( z_g^U \) and \( z_g^L \) respectively.

Taking a membership function for total uncertain demand of \( j \)th fuzzy inequality constraints corresponding to the \( j \)th destinations

\[
f_j(q_{ij} + d_{ij}) = \begin{cases} 
1 & q_{ij} + d_{ij} \geq D_j^U \\
(q_{ij} + d_{ij}) - D_j^L & D_j^L < q_{ij} + d_{ij} < D_j^U \\
0 & q_{ij} + d_{ij} \leq D_j^L
\end{cases} \quad j = 1, 2, 3..., n 
\]

...(7.10)

where \( q_{ij} + d_{ij} = \sum_{i=1}^{m} Q_{ij}, \forall j = 1, 2, 3... n \)

Since, decision making generally gives a planning problem with multiple fuzzy objectives when making a transportation problem decision. The main proposed of fuzzy linear programming approach employing the linear membership functions to represent fuzzy numbers it is an development of fuzzy linear programming method for solving the transportation planning decisions with fuzzy constraints due to available supply, uncertain demand and damage cost for deteriorating items.
7.4 OPTIMIZED VALUE OF OBJECTIVE FUNCTION

The optimum value of transportation problem consists of a total objective function value which is lower than the optimum and attainable by transporting larger quantities of the goods over the exact transporting routes which is to be optimized as

\[
\text{Min. } z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} (x_{ij}^L + x_{ij}^U) + \sum_{i=1}^{m} f_i \quad \ldots(7.11)
\]

subject to constraints

\[
\begin{align*}
\sum_{j \in J} x_{ij} &\leq a_i \quad \forall \ i \in I \\
\sum_{i \in I} x_{ij} &= b_j \quad \forall \ j \in J
\end{align*}
\]

\[
\hspace{1cm} \ldots(7.12)
\]

\[
x_{ij} \geq 0 \quad \forall \ i \in I, \ j \in J
\]

with \( f_i = \sum_{l=1}^{r} \delta_{il} f_{il} : i = 1, 2, 3 \ldots m \quad \ldots(7.13) \)

be the fixed charged \( i^{th} \) warehouse to \( j^{th} \) destination

where, \( \delta_{il} = \begin{cases} 
1, \text{ if } \sum_{j=1}^{n} x_{il} > A_{il} : i \in I \ l = 1, 2, \ldots r \\
0, \text{ otherwise}
\end{cases} \)

If \( g_i \) be the fixed space cost associated with \( i^{th} \) warehouse and than minimization amount of each route for \( e_{ij} \) which is variable profit per unit amount transported from the \( i^{th} \) warehouse to \( j^{th} \) destination which gives the relation
Min. \[ z_2 = \left( \sum_{i \in I} \sum_{j \in J} c_{ij} (x_{ij}^L + x_{ij}^U) + \sum_{i \in I} f_i \right) \left( \sum_{i \in I} \sum_{j \in J} e_{ij} (x_{ij}^L + x_{ij}^U) + \sum_{i \in I} g_i \right) \] \tag{7.14}

subject to constraints
\[ \sum_{j \in J} x_{ij} \leq a_i \quad \forall \ i \in I \]
and \[ \sum_{i \in I} x_{ij} = b_j \quad \forall \ j \in J \] \tag{7.15}

where \( L_{ij} \leq x_{ij} \leq U_{ij}; \quad \forall \ i, j \in I, J \) respectively

\[ \sum_{i \in I} \sum_{j \in J} e_{ij} (x_{ij}^L + x_{ij}^U) \] assume that for positive value \( i.e., > 0 \) for every feasible solution and all \( U_{ij} \) are finite where \( L_{ij}, U_{ij} \) be the minimum and maximum quantities of the goods that can be transported along \( (i, j)^{th} \) route and minimization of objective function \( z_2 \) (7.14) has a unique solution.

when an optimal basic feasible solution of objective function \( z_2 \) gives the \( z^0 \) of the objective function then it may be observed to \( a_i \) and \( b_j \) which can be formulated by an objective function

Min. \[ z_2 = \sum_{i \in I} \sum_{j \in J} c_{ij} (x_{ij}^L + x_{ij}^U) + \sum_{i \in I} f_i \sum_{i \in I} \sum_{j \in J} e_{ij} (x_{ij}^L + x_{ij}^U) + \sum_{i \in I} g_i' \] \tag{7.16}

subject to constraints
\[ \sum_{j \in J} x_{ij} \geq a_i \quad \forall \ i \in I \]
and \[ \sum_{i \in I} x_{ij} \geq b_j \quad \forall \ j \in I \] \tag{7.17}

where \( g_i' \) be the total space cost of all warehouses.
7.5 MINIMUM-COST CAPACITATED TRANSPORTATION PROBLEM

The minimum-cost capacitated transportation flow problem is based on the following assumptions:

(i) A non-negative unit flow cost is associated with each arc.

(ii) Arcs may have positive lower capacity limits.

(iii) Any node in the transportation network may act as a source.

This model consists the flows in the different arcs that minimize the total cost while satisfying the flow restrictions on the arcs and the supply and demands amounts at the nodes. We firstly present the capacitated network flow model and its equivalent linear programming formulation. The linear programming formulation is the basis for the development of a special capacitated simplex algorithm for solving the network transportation model.

Network Representation

Consider a capacitated network $G = (N, A)$, where $N$ is the set of nodes, and $A$ is the set of arcs which is as follows:

$x_{ij}$ = amount of flow from node $i$ to node $j$

$U_{ij}(L_{ij})$ = upper and lower capacity of arc $(i, j)$

$c_{ij}$ = unit transportation cost from node $i$ to node $j$ as well as source $i$ to destination $j$
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\[ f_i = \text{net flow at node } i \]

The labels \( f_i \) assumes a positive (negative) value when a net supply (demand) is associated with node \( i \)

![Diagram](image)

Figure 7.1 (Capacitated arc with external transportation flow)

**Capacitated Network Model for Linear Programming**

The formulation of the capacitated network model as a linear programming provides the foundation for the development of the capacitated simplex algorithm, which we will capacitated given as

Minimize \[ z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}, \quad \forall \ i, j \]

subject to constraints \[ \sum_{(j,k \in A)} x_{jk} - \sum_{(i,j \in A)} x_{ij} = f_j, \quad (j = 1, 2, 3, \ldots, n) \]

\[ L_{ij} \leq x_{ij} \leq U_{ij} \quad \ldots(7.18) \]

the equation for node \( j \) measures the net flow \( f_j \) in node \( j \) as—

outgoing flow from node \( j \) – incoming flow into node \( j = f_i \)

Node \( j \) acts as a source if \( f_i > 0 \) and \( f_j < 0 \) so we can always remove the lower bound \( L_{ij} \) from the constraints by using the substitution

\[ x_{ij} = x'_{ij} + L_{ij} \quad \ldots(7.19) \]
The flow variable, $x_{ij}$ has a new upper limit of $U_{ij} - L_{ij}$. Hence the net flow at node $i$ becomes $f_i - L_{ij}$, and that at node $j$ is $f_j + L_{ij}$, defined as

$$f_i, f_j, f_i - L_{ij}, f_j + L_{ij}$$

Figure 7.2 (Removal of the lower bound in arcs)

### 7.6 NUMERICAL ILLUSTRATION

Anapurna company supplies corn from three silos (warehouse) to three poultry farms. The supply amounts at the three silos are 100, 200 and 50 thousands tons and the demand at the three farms is 150, 80 and 120 thousand tons. Anapurna company mostly uses railroads to transport the corn to the farms, with the exception of three routes where trucks are used.

Consider the available route between the silos and the farms. The silos are represented by nodes 1, 2 and 3 whose supply amounts are [100], [200] and [50] respectively. The farms are represented by nodes 4, 5 and 6 whose demand amounts are [−150], [−80] and [−120]. The routes allow transshipment between the silos. Arcs (1, 4), (3, 4) and (4, 6) are truck routes with minimum and maximum capacities, for example, the
capacity of route (1, 4) is between 50 and 80 thousand tons. All other routes use trainloads, whose maximum capacity is practically unlimited.

The transportation cost per ton are indicated on the respective arcs.

Figure 7.3 (transportation network)

Before and after the lower bounds are substituted and the main constraints of the linear program relate the input-output transportation at each node, which gives the following linear programme.

<table>
<thead>
<tr>
<th>Table</th>
</tr>
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<tbody>
<tr>
<td>Minimize</td>
</tr>
<tr>
<td>Node 1</td>
</tr>
<tr>
<td>Node 2</td>
</tr>
<tr>
<td>Node 3</td>
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<td>Node 4</td>
</tr>
<tr>
<td>Node 5</td>
</tr>
<tr>
<td>Node 6</td>
</tr>
<tr>
<td>Lower bounds</td>
</tr>
<tr>
<td>Upper bounds</td>
</tr>
</tbody>
</table>
This arrangement of the constraints coefficients with variables $x_{ij}$ has exactly one positive (+1) in row $i$ and one negative (−1) in row $j$. The delay of the coefficients are 0, this structure is typical of network flow models.

$$\Rightarrow \text{the variables with lower bounds are } x_{14} = x'_{14} + 50$$

$$x_{34} = x'_{34} + 70 \text{ and } x_{46} = x'_{46} + 100$$

### 7.7 CONCLUSION

The presented chapter proposed a method attempts to minimize the total production and transportation costs when we transported any number of items which can be produced by a company due to damage cost of worst items and the total delivery time with reference to available supply and machine capacities at each source corresponding to uncertain or fluctuating the demands and the formation of the transportation problem according to Bellman-Zadeh's concept of decision making in a fuzzy environment enables a more natural consideration in the determination of the supply values of the deliverers and of the demand values of the receivers and we discussed in this chapter a fixed charged capacitated transportation problem where the objective function is the sum of two linear fractional functions consisting of variables costs and times.