CHAPTER-6

FUZZY THEORETIC APPROACH FOR TRANSPORTATION PROBLEMS BASED ON FUZZY OPTIMIZATION

6.1 INTRODUCTION

Most of the researchers in the fuzzy set theory are based on fuzzy optimization techniques and non linear programming problems with fuzzy constraints for which we use parametric based solution methods. Many topic in transportation planning and decision making can be characterized as subjective, ill-defined, ambiguous, and vague. Decision-making processes for transportation investment, traveler's choice of routes, modes and driver's behaviour are taking illustration that are not entirely based on the clear cut decision criteria, often, the difficulty of dealing with these decision problems are caused by the analyst's attempt to view the results in the frame of binary logic; in the same manner, to seek a result in one of two worlds, yes or no, or wrong or right, and nothing between with no uncertain term. As the scope of transportation problems, we purpose a multi-objective approach to solve parametric programming approach, and a multi-objective pareto-based evolutionary algorithm is used. Due to uncertainty and scope of transportation analysis we deal with the types of uncertainty that are different from the traditional form which has been taken by probability theory.
In this chapter we study multi-objective evolutionary algorithms and linear programming with fuzzy coefficients in constraints for transportation problems based on fuzzy optimization techniques. Operations on fuzzy sets are the formal mechanism to operate on fuzzy sets to define new concepts. The theory facilitates modelling of situations that are observed as approximate or vague; most often a fuzzy set represents a concept associated with natural phenomena. There are many areas in which fuzzy set theory is useful in analyzing qualitative or quantitative or descriptive information and modelling a system whose properties are known or expressed only in natural language. Now we will taken a new method for solving linear programming problems with coefficients in constraints. Which is shown and can be reduced to a linear semi-infinite programming problem. Liu, Baoding(2001) proposed fuzzy programming with fuzzy decisions and fuzzy simulation based genetic algorithm. Li, Fan(2001) formulated a study on solution to constrained optimization problems in fuzzy expert systems. Gao, Pei Wang(2002) formulated a new simplex algorithm for fuzzy linear programming. Al-Saleem, Ameer(2004) considered exact and approximation algorithms for scheduling unrelated machines under fuzzy environment.

Since introduced by Zadeh, more than a quarter century ago, fuzzy set theory has gained an increasing level of acceptance in science and engineering and providing a systematic framework for dealing with the
vagueness and imprecision inherent to human thought process. Because the subjects in transportation problems and planning cannot be separated from human perception and decision processes, transportation problems are a good domain for fuzzy theory application. In fact a suitable number of attempts have already been made to apply fuzzy set theory to transportation problems. This part present the nature of the transportation problems and identify the need for a new paradigm to deal with complex problems that we face currently.

6.2 KNOWLEDGE BASE TRANSPORTATION PROBLEMS

Such types of situations in which transportation analysis take place may be characterized in many different ways, in the following we characterize them in the context of four basic elements of system analysis, which are input, model or knowledge base, output and goals and objectives. Information used for making projection and decisions in transportation is usually imprecise. Accuracy of input is often inconsistent among different data sets and the lack of data is often supplemented by interpolation and extrapolation based on the available data. Data pertaining to perception and feeling are difficult to handle because the boundaries are vague and unclear. Traditionally such data are treated as a single value or a rigid interval. Example: Imprecise and hard to measure quantities, sight distance, reaction time, value of time,
capacity of roadways, adjustment factors in highway capacity calculation is treated.

The knowledge about causalities or relations is generally not very clear in the case of human and societal affairs, and it is expressed often in languages rather than in a precise equation and formula; although, often, such human phenomena have been modeled in a rigid mathematical formula in social science.

Further, the domain to which a particular knowledge base applies is not clear. Traditional rule-based models, such as expert systems, have a clearly defined domain in which each rule applies. If either input or the knowledge-base is vague then the output inevitably becomes vague. Such an output is the reflection of the propagation of uncertainty. In reality, however, the uncertainty in the output is often presented with a mask of certainty. During the analysis process, uncertainty must propagate but the initial uncertainty in the input and that in the knowledge base, testing of uncertainty has been dealt with by the sensitivity analysis, in which the range of input values is used to generate a range of possible output.

The goals of the decision or control problem are often not clearly defined in transportation; usually they are stated qualitatively, such as to reduce congestion and improve air quality. Therefore, whether the outcome of an action satisfies the goals or not is not clearly determined. Further, transportation plans intend to achieve may objectives, but the
priorities and weights among them are not certain. The purpose of the
database is to store data for each specific task of the expert system. The
data may be obtained through a dialog between the expert system and the
user, such data are parameters of the problem or other relevant facts and
other data may be obtained by the inference of the expert system.

Information and data that have vague boundaries include the following:

(i) Notion of desire, goal and target i.e. desired cost, desired
time-saving, desired arrival time.

(ii) Notion of satisfaction and acceptability level i.e. satisfactory
level of achievement, acceptable air pollution level, acceptable cost,
acceptable delay, acceptable error and willingness to pay.

(iii) Perceptions and quantities based on memory i.e. time spent
for an activity, distance traveled, prices paid.

(iv) Description of perceived condition or quality i.e. traffic
congestion, comfort level, acceptable safety level.

(v) Performance as a combination of attributes that are
interacting one another i.e. loss of highway, aesthetic quality and concept
of livability.

If a typical transportation problem deals with sources where a
supply of some commodity is available and destinations where the
commodity is demanded. The classic statement of the transportation
problem uses a matrix with the rows representing sources and columns
representing destinations. The algorithms for solving the problem are based on this matrix representation. The costs of shipping from sources to destinations are indicated by the entries in the matrix. If shipment is impossible between a given source and destination a large cost is entered. This discourages the solution from using such types of cells supplies and demands are shown along the margins of the matrix. Let us consider an example the classic transportation problem has total supply equal to total demand. The network model of the transportation problem follows; sources are identified as the nodes on the left and destinations on the right. The network has a special form important in fuzzy set theory.

Variations of the classical transportation problem are easily handled by modifications of the network model. If we links finite capacity, the arc upper bounds can be made finite. If supplies represent raw materials that are transformed into products at the sources and the demands are in units of product, the gain factors can be used to represent transformation efficiency at each source. If some minimal flow is required in certain links, then lower bounds can be set to non-zero values.

6.3 FUZZY EXPERT SYSTEM

In many situations, the knowledge pertaining to the problem domain which is usually represented by a set of fuzzy production rules, which connect antecedents with consequences, premises with conclusions or condition with actions. It is applicable where one has the data about
any operation but due to unawareness of the functioning of the system, Decision maker can not make this data usable. If becomes difficult for makers to take the decisions about the chronological order of the system. The expert system is not only a mapping of questions to answers. Rather on expert system use some sense of intelligibility. If we provide some data about the parameters of the concern problem, then expert system gives us judgement about the parameters. The purpose of the database is to store data for each specific task of the expert system. The data may be obtained through a dialog between the expert system. So we can say that an expert systems is knowledge engineering, which elicit the human expert knowledge into a suitable form.

Expert knowledge base consists of some facts, relations, and judgements etc. But all these degree of imprecision and uncertainty. Since an expert system can never be intelligent like human expert system and it is cleared in the form of crisp values, therefore to reduce a gap between this real life situation and the expert system we would like to assign a fuzzy number to the linguistic terms answered by the expert system. The inference engine may also use knowledge regarding the fuzzy production rules in the knowledge base. This type of knowledge, whose appropriate name is metaknowledge. The knowledge acquisition module, which is included only in some expert systems. The linguistic terms are assigned fuzzy set to fuzzify the results for covering the uncertain condition.
6.4 VOID IN THE CLASSICAL THEORY APPROACH

Traditionally, probability theory has been the only analysis the uncertain situations. It is used to obtain the degree of a particular event occurs. In this approach, the probability distribution function represents the information or in frequencies that each of possible event occurs, and this distribution becomes the evidence for testing the hypothesis. Many problems of transportation have been dealt with by probability theory. An example is the application of queuing theory, in which the number of service channels is determined in order to limit the probability that the arriving units are delayed more than a certain tolerable time, or the
probability that the length of the queue becomes more than a tolerable value.

Given the nature of the transportation problem as described in the previous chapters the important issue is how to formalize the uncertainty associated with the linguistic expression in the mathematical structure so that the observation and knowledge can be processed systematically and put in the form of a mathematical model. Such a scheme would allow the computer to simulate the situation and also replicate the human control and inference processes. Particularly useful would be the ability to model a choice of path in a crowded pedestrian environment, diagnosing the cause of the environmental damage, and automating these processes. For discover and describe underlying relationships and to apply them for prediction, diagnosis and control has always been at the core of our profession. The relationships are normally formalized in the form of $y = f(x)$. However, in human control and inference process, relationships are usually expressed in language in the form of fuzzy network models. “If the vehicle in front decelerates fast, then decelerate fast” or “if traffic is congested, then leave home early”. In these situations, the issue is not simply whether the relationship between $x$ and $y$ does exist or not exist.

But the strength of association between particular $x$ and $y$ values is important. Often in a description of a relationship are somewhat related
or very much related. This is usually the kind of description of relationships that are involved in the transportation analysis. If we take an example i.e. formalize a police officer's manual traffic control rules, which are based on his experience, at an intersection in a mathematical form, we may be able to develop a traffic control that emulates the police officer's control. Similarly if we can put the driving rules in mathematical form, we may be able to automate some of the driving tasks. Further, a complex transportation system's workings, the relationships between supply and demand that are expressed in natural language, are put into a more formal expression, then predicting and diagnosing the situations may be systematized.

The analysis of the transportation problems and planning needs a mechanism that allows for a formalized treatment of vagueness in human expression and human-based information. Fuzzy set theory is a paradigm that expresses linguistic integrity in the analysis process, but also creates new avenues for control and inference.

6.5 FUZZY INFERENCE AND OPTIMIZATION FOR OPTIMAL VALUE

For most daily activities, human behaviour and decision is structured on the language-based inference system, for example, when one sees a long queue of vehicles on the highway, one would infer a traffic incident ahead. If the queue is very long, then one would infer a
very large incident. If it is a short queue, one infers a relatively small incident.

In this case, one perceives the queue length ahead, and also one has the general knowledge regarding the relationship between the queue length and the severity of traffic incident. Then, one builds a knowledge-base rule, which associates the queue length and the severity of accident. Then if one has the information on the current queue length, one can infer the severity of the incident.

The most fundamental structure of an inference is the following type:

Premise 1 : \( x \in A \)

Premise 2 : If \( x \in A \) then \( y \in B \)

Consequent : \( y \in B \)

In this form of logic, if the two premises are true, then the consequent is always true. This logical structure has been the foundation of reasoning from the time of Aristotle, and it is called modus ponens.

This structure is generalized into a generalized modus ponens, in which \( A' \), \( A \) and \( B \) are expressed in fuzzy terms:

Premise 1 : \( x \in A' \) (fuzzy input condition)

Premise 2 : If \( x \in A \) then \( y \in B \) (fuzzy relation)

Consequent : \( y \in B' \) (fuzzy output)
While $A'$ and $A$ are not same fuzzy set, but they are drawn from the same universal set. The degree of match between $A$ and $A'$ defines the degree of validity of the consequent “$y$ is $B$”. Premise 1 is a fuzzy set of input (or data), and premise 2 is a fuzzy relation expressed by the form shown in chapter 1. Defuzzification is a step that defuzzifies the consequent. A situation may arise such that one needs to reduce the fuzzy outcome into a single value representation; after all a decision is binary. For this, we need a process to convert the fuzzy output into a single value.

![Figure 6.2 (fuzzy system input and output process)](image)

Several approaches are proposed for this step. The most popular approach is take the center of gravity of the shape of the membership function of $B'(z)$ with respect to $z$. This types of modelling is known as the fuzzy system. In a fuzzy system, input is fuzzy and the behaviour of the system is known only fuzzy. The output becomes inevitably fuzzy.
The process is useful for many situation of transportation analysis, in which the causalities are represented by the linguistic rule basis, and the observed data is fuzzy. An example is the human choice process. Is shown in the above figure (6.2).

The traditional optimization processes finds the values of the decision variables so as to minimize or maximize the objective function, within the domain of values of variables that are defined by a set of constraints. Both the objective and the constraints are a function of the decision variables. The formulation requires the strict satisfaction of the constraints; thus sometimes, the optimum value may not be found. In the fuzzy optimization the process is to find the values of the decision variable so as achieve the maximum “satisfaction” or “compatibility” with the objective and the constraints. The degree of satisfaction with the objective and constraints is measured by the membership grade between 0 and 1. The approach is based on the idea that the optimum solution is the best compromise between the objective and constraints. In the classical optimization approach, the solution may become infeasible in real life, however, through a compromise, a solution is usually found. This is the result of fuzzifying the boundaries of the objective and the constraints; that is compromising. Such types of fuzzification of boundaries is taken by compromise solution.

\[
D(x^*) = \text{Max.Min.}\{\mu(x), \phi(x)\} \text{ for all } x \quad \text{ ...(6.1)}
\]
where, \( x \) is the decision variable; and \( x^* \) is the optimal solution, \( \mu_1(x) \) and \( \mu_2(x) \) are respectively, the membership function of the objective and the constraint \( D(x^*) \) represents the maximum degree of satisfaction of both are objective and constraint. Generally, we now see that there is no difference between objective and constraints as follows:

Figure 6.3
(fuzzy optimization concept for no feasible and feasible solution)

Because the intersection can be formed by many objectives and constraints, this formulation solves the multi-objective and multi
constraint programs, in which the solution is found in the intersection of the objectives and constraints, by maximizing the value of the membership grade of intersecting membership functions. Thus generalized expression of the optimal solution is

\[ D(x^*) = \text{Max.Min.}\{\mu_1(x), \mu_2(x), \phi_1(x), \phi_2(x), \phi_3(x)\} \quad \text{...(6.2)} \]

Hence, the fuzzy optimization techniques are applicable for such types of problems in which both the objective and constraints are not clearly defined and the concept of fuzzy optimization has been applied to the traditional optimization algorithm, such as fuzzy linear programming transportation problem and in the case of fuzzy linear programming transportation problem, supply and demand are known only in fuzzy numbers and it determines the amount to be shipped between each demand-supply node pair \((x_{ij})\), mathematically it is solved according to the following condition:

Max. \(f(x)\)

Subject to: \(G(x_{ij}) \leq f(x)\) and \(D_j(x_{ij}) \leq f(x)\)

\[ S_i(x_{ij}) \leq f(x) \]

whenever, \(\sum_{j=1}^{n} x_{ij} \approx S_i, \sum_{i=1}^{m} x_{ij} \approx D_j\) \quad \text{...(6.3)}
where, $G(x_{ij})$ represents the goal, such as the minimal total cost or time of transportation $S_i(x_{ij})$ and $D_j(x_{ij})$ represents the approximate total supply and demand at $i$ & $j$.

6.6 SOLUTION OF FUZZY PROGRAMMING PROBLEM BY EVOLUTIONARY ALGORITHMS

Let us consider a constrained optimization problem is expressed as:

\[
\text{Min. } f(x) \\
\text{Subject to constraints } \quad g_j(x) \leq 0, \quad j=1, 2, 3, \ldots n \quad \ldots(6.4)
\]

where $f(x)$ be any objective function and $g_j(x) \leq 0, \quad j=1, 2, 3, \ldots n$ are constraints defined on the $n$ tuples of real numbers $(R^n)$, let $X$ be a subset of $R^n$.

The inequality (6.4) might be solved for the values $x_1, x_2, \ldots, x_{n-1}, x_n$ which is belonging to the solution space and satisfy the objective function. Since different types of optimization problems can be categorized based on the characteristics of $X$. Therefore if $f(x)$ and $g_j(x)$, for $j=1, 2, 3, \ldots n$ are linear functions and describes a linear optimization problems. Evolutionary algorithms are stochastic operation algorithms which often can find actual solutions rather than the classical optimization techniques, mainly for optimal solution for a non linear programming problem is difficult to obtain.
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If in different types of uncertainty that can appear in such types of cases which has vagueness in nature due to lack of information or lack of imprecision so we can modeled as a fuzzy optimization. Consider a non-linear programming problem with fuzzy constraints. So for mathematically

\[
\begin{align*}
\text{Min. } & f(x) \\
\text{s.t. } & g_j'(x) \leq b_j, \ j=1, \ 2, \ 3, \ ....n \\
x_i \in [L_i, \ U_i], \ i=1, \ 2, \ 3..... \ m \ 	ext{and} \ l_i \geq 0 \quad \ldots (6.5)
\end{align*}
\]

where, \( x=(x_1, x_2, x_3, \ldots, x_n) \in \mathbb{R}^n \) be a \( n \) dimensional real valued parameter and \([L_i, \ U_i] \subset \mathbb{R} \) (where \( i=1, \ 2, \ 3...m \ equire )), \( b_j \in \mathbb{R}, f(x), g_j'(x) \) are continuous derivative arbitrary functions and for solving these fuzzy non-linear programming problems are observed fuzziness of the problem.

Let us consider the linear membership function related to each fuzzy constraint

\[
\mu_j(x) = \begin{cases} 
0 & \text{if } g_j'(x) \geq b_j + d_j \\
\frac{b_j + d_j - g_j'(x)}{d_j} & \text{if } b_j \leq g_j'(x) \leq b_j + d_j \\
1 & \text{if } g_j'(x) \leq b_j 
\end{cases} \quad \ldots (6.6)
\]

for each constraint’s value \( b_j + d_j \) where \( j=1, \ 2, \ 3...n \), then
Min. \( f(x) \)

\[
\text{s.t. } g_j(x) \leq b_j + d_j(1 - \alpha); \quad j = 1, 2, 3, \ldots, n
\]
\[
x \in [L_i, U_i], \quad i = 1, 2, 3, \ldots, m, \quad l_i \geq 0 \quad \ldots (6.7)
\]

where \( \alpha \in [0, 1] \)

if we have a fuzzy set corresponding to membership function

\[
\mu_s(x) = \begin{cases} \sup_{x \in S(\alpha)} \alpha & \text{if } x \in U_\alpha S(\alpha) \\ 0 & \text{otherwise} \end{cases}
\]

in which \( S(\alpha) = \left\{ x \in \mathbb{R}^n : \frac{R^n}{z(x)} \right\} = \min_{x \in X(\alpha)} f(x') \)

corresponding to, \( X(\alpha) = \left\{ x \in \mathbb{R}^n : \frac{R^n}{g_j(x)} \leq b_j + d_j(1 - \alpha) \right\} \)

\[
x_i \in [L_i, U_i], \quad i = 1, 2, 3, \ldots, m
\]
\[
\quad j = 1, 2, 3, \ldots, n
\]
\[
\forall \ i \ & j \text{ and } \alpha \in [0, 1]
\]

An evolutionary parametric based approach to solve fuzzy transportation problems have been given in inequality (6.7). Therefore we shall try to find an approximate solution of the problem. In this way we are proposing an evolutionary approach finding non optimal.

Let a set \( Q = \{\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_q\} \) with \( \alpha_p = [0, 1] \)

where \( p = 1, 2, 3, \ldots, q \)

and \( \alpha_p < \alpha_{p+1} \) for \( p = 1, 2, 3, \ldots, q-1 \)
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and a fuzzy relation solution \( \bar{x}=(\bar{x}_1,\bar{x}_2,\bar{x}_3,...,\bar{x}_n) \) could be constructed, then we transformed the inequality (6.7) into a two-objective non-linear programming problem in which the parameter \( \alpha \) is treated as a new decision variable in the constraints and a second objective is minimize, then a new problem is treated as follows:

\[
\text{Min. } f(x), \alpha \\
\text{s.t. } g_j(x) \leq b_j + d_j(1 - \alpha), \ j = 1, 2, 3, \ldots n \\
x_i \in [L_i, U_i], \quad i = 1, 2, 3, \ldots m \text{ and } l_i \geq 0 
\]

where, \( \alpha \in [0, 1] \)

6.7 FUZZY LINEAR PROGRAMMING IN SUPPLY PRODUCTION PROBLEM

Since in mathematical programming problems we will take a membership function in a supply production problem with continuous variables and parameter of this programming are modeled by non-linear membership function such as s-curve membership function. So development of methods of solution were directed towards single objective mathematical programs such as the simplex method for linear programming. In applying mathematical programming decision-makers realized that there are real-life problems though that considers multiple objectives. Many problem in operations research as well as transportation problems, decision science, mainly been studied from optimizing points of view. As the decision making is much influenced by the disturbances, optimization approach is not always the best. It is because under such influences, many problems are ill-structured. Therefore, a satisfaction
approach may be much better than an optimization one. For this type of problems, it is acceptable that the aspiration level on the treated problem is resolved on the base of current knowledge possessed by a decision maker which should be considered to solve a problem for vagueness in the fuzzy system denoted by fuzzy numbers.

In the real world decision-making theory it uses the optimization techniques. Which is connected with a single criteria and decision processes with multiple criteria deal with decision of decision maker's. Different types of membership functions were used in fuzzy linear programming problem and its application such a linear membership function, and a tangent type of membership function an interval linear membership function.

As a tangent type of membership function, and hyperbolic membership function are non-linear function, a fuzzy mathematical programming defined with a non-linear membership function results in a non-linear programming. Now modified of curve membership function and named. s-curve membership function is employed to overcome such deficits which is a linear membership function. While S-curve membership function is more flexible enough to describe the vagueness in the fuzzy parameters for the supply production problems.

Using fuzzy linear programming in supply production planning and their applications to decision making are carried out and s-curve membership function of fuzzy linear programming based on a vagueness
in the fuzzy parameters such as resource variables given by a decision maker is analyzed.

The modified s-curve membership function is a particular case of the logistic function with specific values of variables $X$, $Y$ and $\alpha$. These values are given in membership grade function which is as follows:

$$
\mu(x) = \begin{cases} 
1 & x < x^a \\
0.999 & x = x^a \\
\frac{X}{1 + Ye^{\alpha x}} & x^a < x < x^b \\
0.001 & x = x^b \\
0 & x > x^b 
\end{cases} 
...(6.9)
$$

Where $\mu$ is the degree of membership function, and $\alpha$ determine the shape of membership function $\mu(x)$, where $\alpha > 0$. The larger parameter $\alpha$ get, the less their vagueness becomes. It is necessary that parameter $\alpha$, which determine the membership functions which is defined as $0.001 \leq \mu(x) \leq 0.999$.

This range is selected because in supply production the revenue and harmful pollution need not be always 100% of the requirement. At the same time the total revenue and total harmful pollution will not be 0%. Therefore there is a range between $x^a$ and $x^b$ with $0.001 \leq \mu(x) \leq 0.999$. Taking $x$ axis as $x^a = 0$ and $x^b = 1$ in order to find the values of $X$, $Y$ and $\alpha$, inequality (6.9) as

$$
X = 0.999(1 + Y) 
...(6.10)
$$
and \[ \frac{X}{1 + Ce^\alpha} = 0.001 \] ...(6.11)

So by substituting the value of \(X\) from inequality (6.10) into (6.11), we get

\[ \frac{0.999(1 + C)}{1 + Ce^\alpha} = 0.001 \] ...(6.12)

Therefore,

\[ \alpha = \log \frac{1}{0.001} \left( \frac{0.998}{C} + 0.999 \right) \] ...(6.13)

Since \(X\) and \(\alpha\) depend on \(Y\) so we let one more condition to get the values for \(X\), \(Y\) and \(\alpha\).

Taking when \(x_0 = \frac{x^a + x^b}{2}\), \(\mu(x_0) = 0.5\)

\[ \Rightarrow \frac{X}{1 + Ce^{\alpha/2}} = 0.5 \] ...(6.14)

and \[ \alpha = 2\log \left( \frac{2X - 1}{Y} \right) \] ...(6.15)

Now substitute the value of inequality (6.13) and (6.14) into the inequality (6.15) we get

\[ 2\log \left( \frac{2(0.999)(1 + Y - 1)}{Y} \right) = \log \frac{1}{0.001} \left( \frac{0.998}{Y} + 0.999 \right) \] ...(6.16)

Inequality (6.16) becomes \((0.998 + 1.998Y)^2 = Y(998 + 999Y)\) ...(6.17)

Now solving the inequality (6.17) we have

\[ Y = \frac{-994.011992 \pm \sqrt{988059.8402 + 396412776}}{1990.015992} \] ...(6.18)

Since \(Y\) be a +ve quantity which is 0.001001001 and \(X = 1\), \(\alpha = 13.81350956\). And with the modification of s-curve membership
function has the tangent hyperbolic function and also a trapezoidal and triangular membership functions are an approximation from a logistic function.

Figure 6.4 (Modification of s-curve membership function)

Therefore, the \( s \)-function is considered much appropriate to denote the vague goal level which a decision maker considers for the solution of the production planning problem by fuzzy optimization techniques. Furthermore, it is possible that the modified \( s \)-curve membership function changes its figure according to the parameters values. Then a decision
maker is able to apply his approach to a fuzzy supply production planning using these parameters and it is a triangular or trapezoidal membership functions which shows a lower level and a upper level at their membership values 0 and 1 and with concerning a non-linear membership function such as a modified s-curve function a lower level and upper level may be approximated with membership values 0.001 and 0.999 which is lies between closed interval [0, 1] with fuzzy membership grade.

6.8 CONCLUSION

In the present chapter a fuzzy theoretical approach for transportation problem is developed and analyzed that when the data and the relationships are approximate or expressed in language, and the approximate output is sufficient. With the use of proper set operators, the set operations can preserve uncertainty in the computation process and reaches the output whose uncertainty is inconsistent. Fuzzy set theory offers a real mathematical approach that complements the traditional approach in transportation planning or traffic problems by preserving uncertainty in the analysis process which is plentiful in this type of field. For many transportation problems, the traditional deterministic assumptions or probabilistic approach may satisfy.