CHAPTER-7

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Coding applications have grown rapidly in the past several years. This area of applied mathematics includes the study and discovery of various coding schemes that are used to correct the errors that are introduced during data transmission. There are various types of errors and a number of techniques for correction of these errors under Hamming metric for both binary and non-binary cases. Hamming distance in non-binary cases is just not capable of considering errors, which are limited in other ways. Thus from both mathematical as well as from practical considerations other than Hamming metric is to be employed. All such distances over GF (q), q > 3, are available through SK-partitions. To correct only those patterns that need to be corrected rather than have the wasted capacity of correcting non-errors by default, we have considered new types of error pattern using under SK-metric, and have studied a largely extended wider class of codes.

The lower bounds on redundancy or upper bounds on word length for specified limited patterns of errors, lays down the ideal situation for existence of such codes. The choices provided by use of SK-partitions and SK-distances provide a choice of metrics, including that for Hamming distance, matching the channel characteristics. It seems quite possible to extend this to constructions of codes correcting limited patterns of errors.

Mathematicians and Coding theorists have remained quite enamored with search of prefect codes. Under Hamming distance, there are only two rather trivial such classes. Search of perfect codes under new error patterns has been undertaken. Obtained bounds on the number of parity check digits for correcting errors on $t$ positions of SK-weight -1, SK-weight two or less over $\mathbb{Z}_q$, $(q \geq 7)$ a prime respectively. We have investigated the existence of perfect codes correcting error in $t$ positions of SK-weight-1 over $\mathbb{Z}_7$, and perfect codes correcting errors of SK-weight -2 or less over $\mathbb{Z}_{13}$. While search has been quite rewarding, the search for specific values of $q$ is bound to enrich the class of what may be termed ‘special perfect codes’.
Channel is a physical medium through which information is transmitted. Probabilistic being the nature of noise, with all the care in designing codes correcting designed errors, there is a chance of errors. Thus proper characterization of channel for correction criteria, in our case SK-metric, is required for reliability, that is probability of error. We have defined a \( q \)-nary symmetric channel which is compatible with the SK - partition. This channel is a generalization of widely used concept of Binary-Symmetric Channels. We have denoted this \( q \)-nary symmetric channel for SK-partition \( \wp \) by ‘\( q \wp SC \)’.

We have results on probabilities of errors for a code designed for different classes of errors.

With generalization of additive errors and of symmetric channel for the new setting under SK-studies, there are several other possibilities that arise for further studies.

Another area of challenge for coding theorists is that of developing codes with larger number of errors. BCH and some other designed codes turn out to be inefficient for increasing number of error. Small codes with desired error correction capabilities can be easily developed; developing large codes presents a real practical problem. A practical way to obtain large codes is to obtain them by suitable composition of those of shorter lengths. Our strategy is to build codes with larger size or longer length from code of smaller size or shorter length. We introduced a new kind of product of polynomials defined over a field. We have called it ‘Ordered Power Product’ (OPP). OPP has wider implication in abstract algebra, that we have indicated. Using the new type of product of polynomials, for the first time in the literature, we have defined a new product of two cyclic codes and devise a method of getting a cyclic code from the ‘ordered power product’ of two cyclic codes. This amounts to developing new product type of codes, which are more efficient.

Our studies and ideas provide lead to several areas for continued research. In brief we envisage these as follows:

- Consider other bounds in particular Low rate bound and Linear programming bound, for codes correcting limited patterns of random errors using SK- metric and construction of codes.

- In continuation to ideas in Chapter 4, explore Coding theorems and error bounds for ‘\( q \wp SC \)’ channel.
- Generalization of Reed-Solomon, BCH and other important codes for $q$-nary channels in terms of distances arising from SK-Partitions.
- Search for more perfect and quasi perfect codes under generalized distances.
- Exploring the possibility of partitioned product cyclic codes and their further studies.
- Study of Unequal error protection code under SK-Metric