Chapter 2

The influence of dust particles on electromagnetic ion cyclotron waves in a bi-Lorentzian plasma

2.1 Introduction

Plasmas occurring naturally in space are generally non-Maxwellian; they are often characterised by an energetic tail distribution (Vasyliunas, 1968 and Williams et al. 1988). A useful distribution function to model such plasmas is the generalised Lorentzian or Kappa distribution. The well known Maxwellian and Kappa distributions differ substantially in the high energy tail region; the drop towards zero is much more abrupt for a Maxwellian distribution when compared to that of a Kappa distribution with a low spectral index κ. This difference is however less significant as κ increases. When compared to particles with a Maxwellian distribution, such high energy particles can enhance the growth of plasma waves, especially when the phase velocity of the wave is large compared to the thermal bulk velocity of the plasma. Such conditions commonly occur in space and other magnetospheric plasmas and Kappa distributions have been used to analyse and interpret spacecraft data in the Earth's magnetospheric plasma sheet (Vasyliunas, 1968 and Williams et al. 1988), the solar wind (Abraham - Shrauner et. al.,1979, Church and Thorne, 1983), Jupiter (Leubner, 1982) and Saturn (Armstrong et.al., 1981). In fact it has been found that, in practice, many space plasmas can be modelled more efficiently by a superposition of Kappa distributions than by Maxwellians.

In recent years there has been a growing interest in the study of dusty plasmas. These essentially consist of electrons and ions along with negatively charged dust grains. Planetary rings, asteroid zones, cometery comae and tails and the earth's lower magnetosphere are examples of space plasmas where such situations occur. For low frequency and long wavelength modes the dust grains can be described essentially as negative ions with charge to mass ratios that can vastly differ from what one is accustomed to in conventional plasmas.

The propagation and stability of low frequency waves in dusty plasmas has been studied recently by a number of authors and these have been reviewed briefly in Chapter 1. The charged dust grains will interact mainly electromagnetically. This interaction between the solar wind and comets has been used to interpret various kinds of cometary dust regions (Ip, 1984); the observed solar wind particles being fairly successfully modelled by the bi - Lorentzian or Kappa distribution function in many instances (Abraham Schrauner et.al.,1979).

As mentioned above, the characteristic frequencies of the dust particles are even smaller than the corresponding ion frequencies. We have therefore studied the influences of dust particles on the electromagnetic ion cyclotron (EMIC) waves in a plasma where the ions (protons) are modelled by a bi-Lorentzian or Kappa distribution. The dust particles, because they are heavier than the ions, have been treated as cold; the electrons too have been treated similarly for simplicity.

We have considered the stability of these waves in both low - and high - β plasmas. We find that in low - β plasmas the growth increases with temperature anisotropy of the ions, the charge number Z_D of the dust particles and density ratio n_D/n_i. However, in high - β plasmas the growth rate exhibits a mixed behaviour as regards the charge number Z_D and n_D/n_i. The growth rate is independent of the mass ratio m_i/m_D, for both the cases at least for the massive dust particles.

2.2 The general dispersion relation

We consider circularly polarised electromagnetic waves propagating parallel to the homogeneous magnetic field. The dispersion formula for the propagation of these waves is well known and is given by (Liemohn, 1974)

$$c^2k^2 = \omega^2 + \frac{\pi\omega}{k}\omega_p^2 \int_0^\infty dv_\perp \int_{-\infty}^{\infty} \frac{I_o\, dv_\parallel}{v_\parallel - v_c} \tag{2.1}$$

where

$$I_o(v_\perp, v_\parallel) = -v_\perp^2 \frac{\partial F}{\partial v_\perp} + \frac{k}{\omega} v_\perp^2 \left[v_\parallel \frac{\partial F}{\partial v_\perp} - v_\perp \frac{\partial F}{\partial v_\parallel} \right]$$

and

$$v_c = \frac{\omega \pm \Omega}{k}$$

ω_p and Ω which denote the plasma frequency and gyrofrequency respectively are defined as

$$\omega_{pj} = \left[\frac{4\pi n_j Z_j^2 q_j^2}{m_j}\right]^{\frac{1}{2}} \quad \text{and} \quad \Omega_j = \frac{q_j B_o}{m_j c}$$

where q_j is the charge and Z_j, the charge number, m_j is the mass and n_j the density of each species. F is the distribution function. The cyclotron resonance velocity v_c in the above equation is the value of $v_\parallel$ which Doppler shifts ω to Ω in the rest frame of the particle to allow the interaction. The positive and negative signs in v_c refer to the right handed and left handed circularly polarised modes respectively (Renuka and Viswanathan, 1980). We consider, in this Chapter, the left circularly polarised waves.

As stated above the bi-Lorentzian or Kappa distribution has often been successful in describing the observed ion populations in the solar wind. We are therefore interested, in this Chapter, on the influences that dust particles have on electromagnetic ion cyclotron waves propagating parallel to the magnetic field in a plasma where the hot ions are modelled by the Kappa distribution. The dust particles are assumed to be cold and the electrons too, for simplicity. The bi-Lorentzian, which reduces to the anisotropic Maxwellian distribution when the spectral index κ tends to infinity, is given by

$$F = \frac{1}{\pi^{3/2}} \frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})} \frac{1}{\theta_\perp^2 \theta_\parallel} \left[1 + \frac{v_\perp^2}{\kappa\theta_\perp^2} + \frac{v_\parallel^2}{\kappa\theta_\parallel^2}\right]^{-(\kappa+1)} \tag{2.2}$$

In (2), $\theta_\perp$ and $\theta_\parallel$ are related to the mass m and the temperatures $T_\perp$ and $T_\parallel$ respectively parallel and perpendicular to the magnetic field by

$$\theta_\perp^2 = \left[\frac{\kappa - \frac{3}{2}}{\kappa} \frac{2k_B T_\perp}{m}\right] \quad and \quad \theta_\parallel^2 = \left[\frac{\kappa - \frac{3}{2}}{\kappa} \frac{2k_B T_\parallel}{m}\right] \tag{2.3}$$

with k_B being the Boltzmann constant. Substituting the derivatives of (2) into the dispersion formula (1) for left circularly polarised EMIC waves and carrying out the $dv_\perp$ and $dv_\parallel$ integrations, we get the dispersion relation for electromagnetic ion cyclotron (EMIC) waves propagating parallel to the external magnetic field as

$$D(\omega,k) = \omega^2 - c^2k^2 + \omega_{pi}^2\left\{\frac{1}{k\theta_\parallel}\left[\left(1+\frac{\zeta_\parallel^2}{\kappa}\right)Z_\kappa^*(\zeta_\parallel) + \frac{2\kappa-1}{2\kappa^2}\zeta_\parallel\right]\left[\omega - A(\omega-\Omega_i)\right] - A\right\}_i - \sum_{e,D}\omega_{pj}^2\frac{\omega}{\omega-\Omega_j} = 0 \qquad (2.4)$$

In (4) the subscript j indicates the species (j='i' for ions, 'e' for electrons and 'D' for dust). The anisotropy factor A for the ions is defined as

$$A = 1 - \frac{\theta_\perp^2}{\theta_\parallel^2}$$

The modified plasma dispersion function $Z_\kappa^*(\zeta)$ arises from the $dv_\parallel$ integration and is defined as (Summers and Thorne,1991)

$$Z_\kappa^*(\zeta_\parallel) = \frac{1}{\sqrt{\pi}}\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\int_{-\infty}^{\infty}\frac{ds}{(s-\zeta_\parallel)(1+\frac{s^2}{\kappa})^{\kappa+1}} \qquad (2.5)$$

where the argument $\zeta_\parallel$ is defined as

$$\zeta_\parallel = \frac{\omega - \Omega_i}{k_\parallel\theta_\parallel} \qquad (2.6)$$

The definitions of the other terms in (4) are standard. As a check on (4), we note that as $\kappa \rightarrow \infty$ it reduces to the corresponding relation in an anisotropic Maxwellian plasma (Gomberoff and Vega, 1989).

2.3 Low -β case

2.3.1 Dispersion relation

In this section we derive the dispersion relation for EMIC waves when the ions have a low parallel temperature. Under such circumstances $\zeta_{\|} > 1$ and we need the asymptotic expansion of $Z^*_\kappa(\zeta_{\|})$, which is given by (Summers and Thorne, 1991)

$$Z^*_\kappa(\zeta_{\|}) = -\frac{(2\kappa-1)}{2\kappa}\frac{1}{\zeta_{\|}}\left[1+\frac{\kappa}{2\kappa-1}\frac{1}{\zeta_{\|}^2}+...\right]+\frac{i\sqrt{\pi}\kappa!}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\frac{1}{\left[1+\frac{\zeta_{\|}^2}{\kappa}\right]^{\kappa+1}} \tag{2.7}$$

We are interested in the parallel propagation of IC waves, which are low frequency waves. Thus we have neglected ω^2 in comparison to c^2k^2 and also assumed that $\omega << \Omega_e$. Thus substituting the asymptotic expansion (7) into (4) and using the charge neutrality condition

$$n_i = n_e + Z_D n_D \tag{2.8}$$

and equating the real and imaginary parts, we get

$$\begin{aligned} Re\ D(k,\omega) = \frac{c^2k^2}{\omega_p^2} - \frac{1}{k\theta_{\|}}\Big[\omega - A(\omega-\Omega_i)\Big]\Big[-\Big(\frac{2\kappa-1}{2\kappa}\Big)\frac{1}{\zeta_{\|}} \\ \Big[1+\frac{\kappa}{2\kappa-1}\frac{1}{\zeta_{\|}^2}\Big(1+\frac{3\kappa}{2\kappa-3}\frac{1}{\zeta_{\|}^2}\Big)\Big]\Big[1+\frac{\zeta_{\|}^2}{\kappa}\Big] \\ +\frac{\kappa-\frac{1}{2}}{\kappa^2}\zeta_{\|}\Big] + A - \frac{\omega_{pe}^2}{\omega_{pp}^2}\frac{\omega}{\Omega_e} + \frac{\omega_{pD}^2}{\omega_{pp}^2}\frac{\omega}{\omega-\Omega_D} = 0 \end{aligned} \tag{2.9}$$

Using (8) for simplification, we arrive at the dispersion relation as

$$ReD(k,\omega) = \frac{c^2k^2}{\omega_p} - \frac{x^2}{1-x} + \left[\frac{x}{x-1} - A\right]\frac{\beta_\parallel}{2}\frac{c^2k^2}{\omega_p^2}\frac{1}{(x-1)^2} - Z_D\frac{n_D}{n_i}x\frac{x}{x+Z_D\frac{m_i}{m_D}} = 0$$

which on re-arrangement leads to

$$\frac{c^2k^2}{\omega_{pi}^2} = \frac{x^2\left[\frac{1}{(1-x)} + Z_D\frac{n_D}{n_i}\frac{1}{(x+Z_D\frac{m_i}{m_D})}\right]}{1-\frac{1}{2}\frac{\beta_{\parallel i}}{(1-x)^2}\left[\frac{x}{(1-x)}+A\right]} \quad (2.10)$$

where $x = \omega_r/\Omega_i$, $(\omega = \omega_r + i\omega_i)$ and $\beta_{\parallel i} = \frac{8\pi n_i k_B T_{\parallel i}}{B_o^2}$. and

$$ImD(k,\omega) = -\frac{1}{k\theta_{\parallel i}}\left[A(\omega-\Omega_i) - \omega\right]\left[\frac{\kappa!\sqrt{\pi}}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\frac{1}{\left(1+\frac{\zeta_\parallel^2}{\kappa}\right)^\kappa}\right] \quad (2.11)$$

The expression for the growth/damping rate can be derived using the imaginary part (11) and the derivative of (10). It is

$$\gamma = -\frac{\omega_i}{\Omega_i} = -\frac{\frac{\Omega_i}{k\theta_{\parallel i}}\left[A(1-x)+x\right]\left[\frac{\kappa!\sqrt{\pi}}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\left(1+\frac{\zeta_\parallel^2}{\kappa}\right)^\kappa}\right]}{k^2\left(\frac{k_BT_\parallel}{\Omega^2 m}\right)_i\left[\frac{2A(1-x)+(1+2x)}{(1-x)^4}\right] + \left[\frac{x(2-x)}{(1-x)^2} + Z_D\frac{n_D}{n_i}\frac{x\left(x+2Z_D\frac{m_i}{m_D}\right)}{\left(x+Z_D\frac{m_i}{m_D}\right)^2}\right]} \quad (2.12)$$

2.3.2 Discussion

We now discuss certain interesting features of (10) and (12). We first consider (10). When the dust particles are massive and not highly charged, that is, when $Z_D\frac{m_i}{m_D} << x$, it can be simplified into

$$\left[1 - Z_D\frac{n_D}{n_i}\right](\Delta x)^4 - \left[\frac{c^2k^2}{\omega_{pi}^2} + 1 + (1 - Z_D\frac{n_D}{n_i})\right](\Delta x)^3 + (\Delta x)^2$$
$$-\frac{c^2k^2}{\omega_{pi}^2}\frac{\beta_{\|i}}{2}\left[1 - A\right]\Delta x + \frac{c^2k^2}{\omega_{pi}^2}\frac{\beta_{\|i}}{2} = 0 \qquad (2.13)$$

where $\Delta x = 1 - x$. This dispersion relation is greatly modified under various limiting conditions. For example when A = 1 (that is, when the hot ions are extremely temperature anisotropic with $T_\perp \to 0$) we find that the coefficient of Δx drops out of (13). When $Z_D\frac{n_D}{n_i} = 1$ we find that (13) becomes a cubic equation with the coefficient of $(\Delta x)^3$ greatly modified. Again, in the long wavelength limit, when $k \to 0$, (13) reduces to a quadratic in Δx with one solution being $\Delta x = 1(\omega = 0)$, the other being $\Delta x = \frac{1}{1-Z_D\frac{n_D}{n_i}}$. The latter gives $\omega = \frac{-Z_D\frac{n_D}{n_i}}{1-Z_D\frac{n_D}{n_i}}\Omega_i$ i.e. the wave exists at a multiple of the ion gyrofrequency only for $Z_D\frac{n_D}{n_i} > 1$. It again reduces to a different cubic equation when the hot ions are again highly temperature anisotropic but now with $T_{\|i} \to 0$.

We next consider the expression for the growth/damping rate, namely (12). We find that $\gamma = 0$ when in the numerator $A(1-x) + x = 0$ or when $\frac{T_\perp}{T_\|} = \frac{1}{(1-x)}$; that is, for a given normalised frequency x, there exists a critical temperature ratio $\frac{T_\perp}{T_\|}$ for which the growth $\gamma = 0$. Also in the long wavelength limit, when $k \to 0$, γ is sensitively dependent on the parameters of the dust particles. Interestingly, if in the denominator of (12), $2A(1-x) + (1+2x) = 0$ we get $A(1-x) = -(1+2x)/2$. Substituting this into the numerator of (12), we find that the wave is now unstable for $x < 1$; the growth being again sensitively dependent on the parameters of the dust.

2.4 High $\beta_{\|}$ case

2.4.1 Dispersion relation

In this section we complement our study of section 3 by considering the other limit of the argument of the modified plasma dispersion function namely, $\zeta_{\|} < 1$. The small argument expression for it is given by (Summers and Thorne, 1991)

$$Z_{\kappa}^{*}(\zeta_{\|}) = -\frac{(2\kappa-1)(2\kappa+1)}{2\kappa^2}\zeta_{\|}\left[1-\frac{2\kappa+3}{3\kappa}\zeta_{\|}^2+...\right]+\frac{i\sqrt{\pi}\kappa!}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\frac{1}{\left[1+\frac{\zeta_{\|}^2}{\kappa}\right]^{(\kappa+1)}} \quad (2.14)$$

Substituting (14) for $Z_{\kappa}^{*}(\zeta_{\|})$ into (4) and proceeding in a similar manner, the real and imaginary parts are given by

$$ReD(k,\omega) = \frac{1}{k\theta_{\|i}}\Big[\omega - A(\omega-\Omega_i)\Big]\Big[-\frac{(2\kappa-1)(2\kappa+1)}{2\kappa^2}\zeta_{\|}\Big[1-\frac{2\kappa+3}{3\kappa}\zeta_{\|i} \left[1+\frac{\zeta_{\|i}}{\kappa}\right]+\frac{\kappa-\frac{1}{2}}{\kappa^2}\zeta_{\|i}\Big]+A-\frac{\omega_{pe}^2}{\omega_{pp}^2}\frac{\omega}{\Omega_e}+\frac{\omega_{pD}^2}{\omega_{pp}^2}\frac{\omega}{(\omega-\Omega_D)} = 0 \quad (2.15)$$

and

$$ImD(k,\omega) = -\frac{1}{k\theta_{\|i}}\Big[\omega - A(\omega-\Omega_i)\Big]\Big[\frac{\kappa!\sqrt{\pi}}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\frac{1}{\left(1+\frac{\zeta^2}{\kappa}\right)^{\kappa}}\Big] \quad (2.16)$$

The real part of the dispersion relation can be written in the form of a quadratic equation and is given by

$$\left(\frac{c^2k^2}{\omega_{pi}^2}\right)^2 + \left[A + \left(1 - Z_D\frac{n_D}{n_i}\right)x + Z_D^2\frac{m_i}{m_D}\frac{n_D}{n_i}\frac{x}{x + Z_D\frac{m_i}{m_D}}\right]\left(\frac{c^2k^2}{\omega_{pi}^2}\right)$$
$$-\left[A - \frac{x}{x-1}\right]\left[\frac{2\kappa-1}{2\kappa-3}\frac{2}{\beta_{\parallel i}}(x-1)^2\right] = 0 \qquad (2.17)$$

whose solution is given by

$$\frac{c^2k^2}{\omega_{pi}^2} = -\frac{1}{2}\left[A + x\left(1 - Z_D\frac{n_D}{n_i}\right) + Z_D^2\frac{n_D}{n_i}\frac{m_i}{m_D}\frac{x}{x + Z_D\frac{m_i}{m_D}}\right] \pm \frac{1}{2}\left\{\left[A + x\left(1 - Z_D\frac{n_D}{n_i}\right)\right.\right.$$
$$\left.\left. + Z_D^2\frac{n_D}{n_i}\frac{m_i}{m_D}\frac{x}{x + Z_D\frac{m_i}{m_D}}\right]^2 + \frac{8}{\beta_{\parallel i}}(1-x)^2\frac{2\kappa-1}{2\kappa-3}\left(A + \frac{x}{1-x}\right)\right\}^{1/2} \qquad (2.18)$$

As a check on (18) we note that for the spectral index $\kappa \to \infty$ and the density of dust, $n_D = 0$, it reduces to the corresponding expression in Gomberoff and Vega (1989).

Finally, the expression for the growth /damping rate is given by

$$\gamma = \frac{\sqrt{\pi}\Omega_i}{k\theta_{\parallel i}} \frac{\left[\frac{\kappa!}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\right]\frac{1}{\left(1+\frac{\zeta_{\parallel i}^2}{\kappa}\right)^{\kappa}}\left[A(1-x)+x\right]}{\left[1 - Z_D\frac{n_D}{n_i} - Z_D^3\frac{n_D}{n_i}\left(\frac{m_i}{m_D}\right)^2\frac{1}{(x+Z_D\frac{m_i}{m_D})^2}\right] + \frac{2\kappa-1}{2\kappa-3}\frac{2}{\beta_{\parallel i}}\frac{\omega_{pi}^2}{c^2k^2}\left[2A(1-x)+(2x-1)\right]} \qquad (2.19)$$

2.4.2 Discussion:

In this sub-section we discuss relations (18) and (19). We first consider (18). Comparing it with (10), the dispersion relation under the approximation $\zeta_{\parallel i} > 1$, we find that (18) is dependent on the spectral index κ of the hot component. However, more interestingly, we find that in addition to the cut-off at the ion gyrofrequency there is a new cut-off at the gyrofrequency of the dust particle when it is positively charged (that is, when $x = Z_D \frac{m_i}{m_D}$ or $\omega = \Omega_D$). Since, unlike in a conventional plasma, both Z_D and m_D can take different values this could even give rise to a stop-band in the wave propagation characteristics rather than simple cut-off.

Next consider (19) , the expression for γ. As in section 3.2 we find that $\gamma = 0$, when $\frac{T_\perp}{T_\parallel} = \frac{1}{1-x}$. However, in contrast to the case where $\beta_{\parallel i} < 1$, it is in the short wavelength limit that γ is sensitively dependent on the parameters of the dust particles. However, in contrast to the previous case, this growth or damping is very sensitive to Z_D, the charge number of the dust. Finally, if in the denominator of (19) $2A(1 - x) + (2x - 1) = 0$, we find the wave to be damped for all frequencies provided $Z_D \frac{n_D}{n_i} \geq 1$; a condition that can easily be satisfied considering the large amount of charge that a dust particle can carry.

2.5 Results:

We consider in this section, the computation of the expressions for the growth/damping rate of sections 3 and 4, for typical parameters of the solar wind. The solar wind parameters were chosen since it is a system where low- and high-β situations occur under quiet and turbulent conditions of the solar wind. However, the results that we obtain are not directly applicable to the solar wind since the electrons have been treated as cold; modelling them kinetically results in a masking of the interesting effects due to ions and dust by the electrons.

We first consider typical solar wind parameters for quiet conditions. The parameters are (Abraham Schrauner et.al.1979):

$T_{\parallel i} = 9 * 10^{4o}K, B_o = 6 * 10^{-5}$Gauss and $n_i = 2.8cm^{-3}$. With these parameters $\beta_{\parallel i} = 0.2428$; the spectral index κ was kept a constant at 2.

Figure 1 is a plot of the growth/damping rate versus the normalised frequency x as a function of $\frac{T_\perp}{T_\parallel}$. A comparison of the curves show that the wave is fully damped for $\frac{T_\perp}{T_\parallel}$=1 (A=0.0); however both the range of the instability and the growth rate increases with $\frac{T_\perp}{T_\parallel}$. Thus temperature anisotropy is the source of instability in an electron - ion plasma.

Figure 2 depicts γ versus x as a function of Z_D (=1000, 5000 and 10000) for $\frac{T_\perp}{T_\parallel} = 2.5$, $\frac{m_i}{m_D} = 1 * 10^{-7}$ and $\frac{n_d}{n_i} = 1 * 10^{-4}$. We find that the growth rate increases with Z_D. The growth rate, in all the cases, starts from low values, reaches a maximum at about $x = 0.45$ and thereafter decreases.

Figure 3 again depicts γ versus x but now as a function of $\frac{n_D}{n_i}$ (=$1 * 10^{-4}, 1 * 10^{-5}$ and $1 * 10^{-6}$); the other parameters for the figure being $Z_D = 5000$, $\frac{m_i}{m_D} = 1 * 10^{-7}$ and $\frac{T_\perp}{T_\parallel} = 2.5$. We find that the growth rate is small for low values of $\frac{n_D}{n_i}$; and it tends to increase dramatically when $\frac{n_D}{n_i}$ is greater than a critical value. The dust particles thus enhance the instability of the ion cyclotron waves.

Another aspect studied was the variation of γ with respect to $\frac{m_i}{m_D}$ ($= 1*10^{-6}, 1*10^{-7}$ and $1 * 10^{-8}$). No variation in γ was found; thus the instability is insensitive to the mass of the dust, atleast for the very massive dust particles.

We next consider the results for the relations of section 4, for high $\beta_{\parallel i}$ plasmas. While for the quiet solar wind $\beta_{\parallel i}$ is less than 1, it is often greater than 1 during turbulent conditions. For example, during one such disturbed condition the observed parameters were (Buti and Lakhina, 1973)

$\frac{T_{\perp i}}{T_{\perp e}} = 0.12$, $\frac{T_{\parallel i}}{T_{\perp i}} = 4.9$ and $\beta_{\perp e} = 3.4$ giving $\beta_{\parallel i} = 1.999$

The other parameters namely $T_{\parallel i}$ and κ were kept the same since they do not play a very significant role in our calculations; though there is a slight variation in $T_{\parallel i}$ for the low - and high - β cases.

Figure 4 depicts γ versus x, but as a function of Z_D (5000 and 10000); the other parameters being $\frac{n_D}{n_i} = 1 * 10^{-3}$, $\frac{m_i}{m_D} = 1 * 10^{-7}$ and $\frac{T_{\perp}}{T_{\parallel}} = 2.5$. We find the mode to be unstable over a part of the frequency range for both Z_D; the growth rate however decreases with increasing Z_D the region of instability.

Figure 5 is intended to demonstrate the sensitivity of γ to the density of dust particles and depicts γ versus x as a function of $\frac{n_D}{n_i}(= 1 * 10^{-3}, 1 * 10^{-2}$ and $1 * 10^{-1})$; the other parameters being $\frac{m_i}{m_D} = 1 * 10^{-7}, Z_D = 5000$ and $\frac{T_{\perp}}{T_{\parallel}} = 2.5$. The wave is unstable upto a critical frequency for all the three densities studied. For the low value of $\frac{n_D}{n_i}(= 1 * 10^{-3})$ the wave is strongly unstable below this frequency and strongly damped above this frequency. The curves tend to flatten as $\frac{n_D}{n_i}$ increases; thus with increasing dust densities the EMIC wave tends to propagate almost freely.

Similar to the low-β case, no variation in γ was found as a function of $\frac{m_i}{m_D}(= 1*10^{-6}$ to $1*10^{-8})$.

Finally, the instability was also studied as a function of the spectral index κ of the hot ion species for both values of $\beta_{\|i}$. We find a small reduction in the growth rate with increasing κ.

2.6 Conclusions

We have, in this Chapter, studied the influence of dust particles on the EMIC instability propagating parallel to the magnetic field in a plasma where the hot - ions are described by a bi-Lorentzian or Kappa distribution; the other components of the system being electrons and negatively charged dust particles. The electrons have been treated as cold since they mask interesting effects due to ions and dust particles when modelled kinetically. Expressions for the dispersion relations and growth rates were derived for both the low - and - high $\beta_{\|i}$ cases. We find that only the charge and density of the dust particles have a strong influence on the growth rate of the ion cyclotron waves.

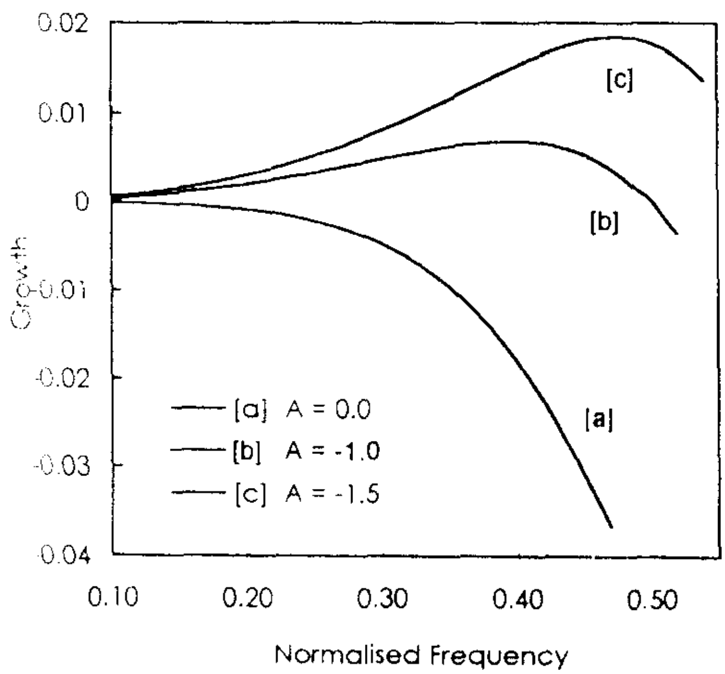

Figure 1 : Plot of the normalised frequency versus growth rate as a function of temperature anisotropy A in an electron - ion plasma

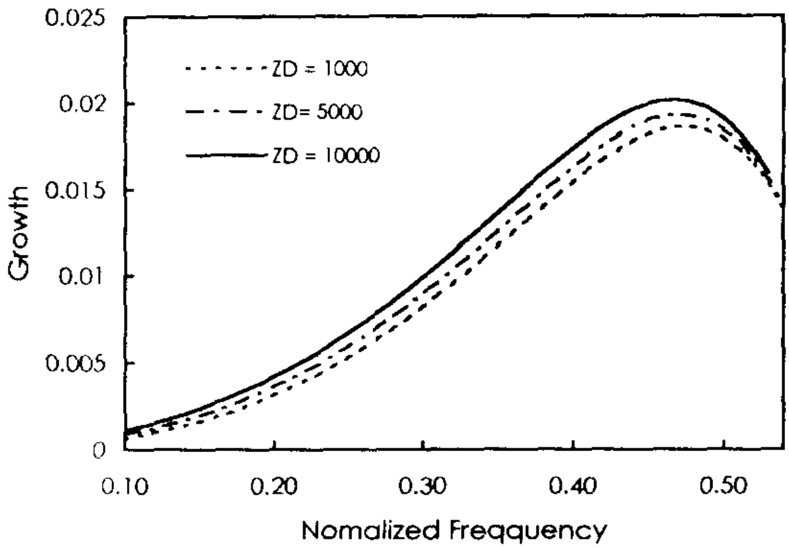

Figure 2 : Plot of the normalised frequency versus growth rate in a dusty plasma as a function of Z_D, the charge number of the dust. The other parameters are A=-1.5, $m_i/m_D = 1.0X10^{-7}$ and $n_D/n_i = 1.0X10^{-4}$

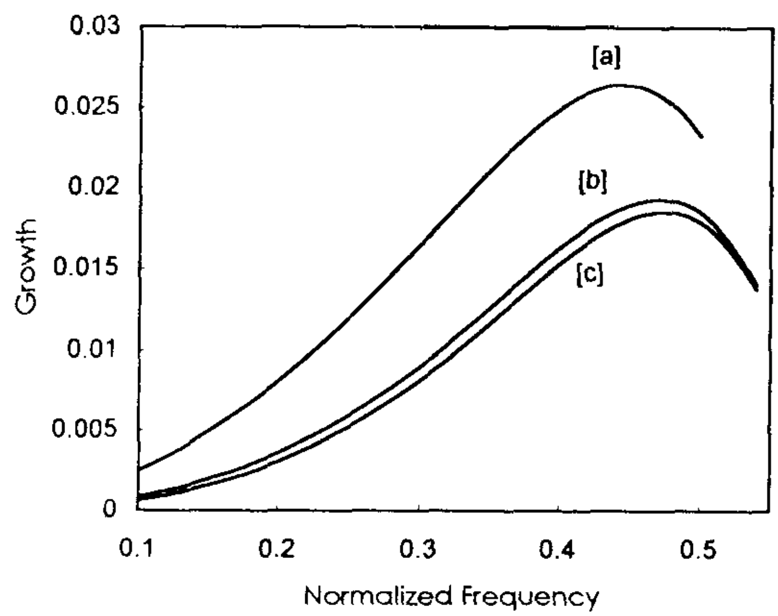

Figure 3: Plot of the normalised frequency versus growth rate in a dusty plasma as a function of n_D/n_i, the dust density (= $1.0X10^{-4}$, curve [a]; $1.0X10^{-5}$, curve [b] and $1.0X10^{-6}$, curve [c]). The other parameters are A = -1.5, $m_i/m_D = 1.0 \times 10^{-7}$ and Z_D=5000

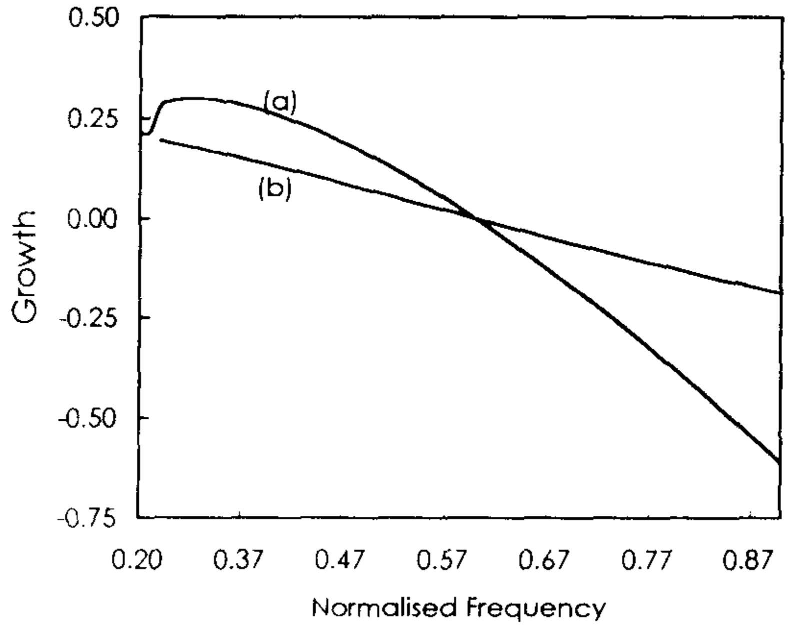

Figure 4 : Plot of the normalised frequency versus growth rate as a function of Z_D=5000 (curve (a)) and Z_D = 10000 (curve (b)). The other parameters are $mi/m_D = 1.0 \times 10^{-7}$, $n_D/ni = 1.0 \times 10^{-3}$ and A = -1.5.

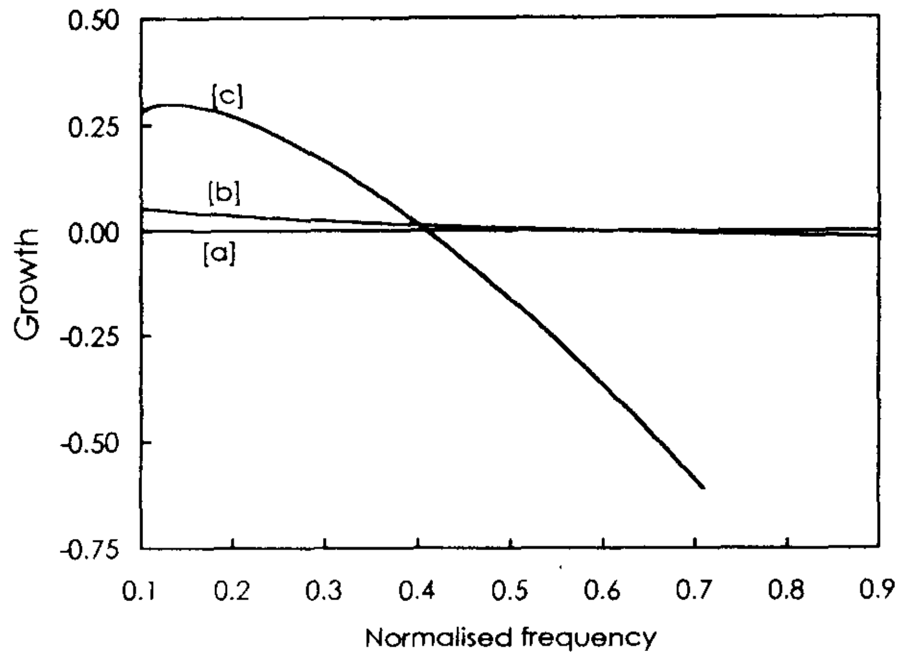

Figure 5 : Plot of the normalised frequency versus growth rate as a function of n_D/ni (=1.0 X 10^{-1} (curve [a]), 1.0X10^{-2} (curve [b]) and 1.0X10^{-3} (curve [c])). Z_D = 5000 while mi/m_D and A have the same values as in Figure 4.

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