Chapter 1
CHAPTER-I

INTRODUCTION

Mathematics, the key of all Sciences, includes Pure Mathematics, Applied Mathematics, Mathematical Statistics and Operational Research. In addition Mathematical Economics, Mathematical Sociology, Mathematical Psychology, Mathematical Model in Finance and Commerce, Mathematical Decision Science, Acuition Science, Computer Science, Mathematical Biology, Mathematical Medicine and many other disciplines use different vocabularies of Mathematics. All these use common mathematical approach, common algebraic and calculus methods, common differential and difference equations, common calculus of variations, common statistical techniques, common numerical and computer techniques. That is why all these fall under the category of Mathematical Sciences.

Applied Mechanics plays an important role to engineering and technology due to its applications for the rapid development of new innovations of human endeavour and have created a better world for mankind. However, this advancement has given place to a number of problems for society, namely, energy crisis, atmospheric pollution, socio-economic, moral, ethical and political chaos. Besides, commercial and financial transactions, construction of bridges, dams, weapons, communication, transport, health care and many other human pursuits involve applied mechanics.
Fluid Mechanics has been recognized as one of the most important branch of Applied Mathematics having its unique place due to its applications in Science and Technology in every field of human beings. Due to wide acceptability of fluid mechanics in the fast race of exciting developments during these days, the researches in this field are considered to be more and more important both from the theoretical and practical point of view indicating the fact that fluid mechanics is a subject of widespread interest and indispensable utility in almost all the fields of Engineering and Technology.

The researches on Fluid Mechanics are gaining indispensable importance during these days. Due to researches in various fields of knowledge; social, scientific and technological advancement has taken place. The advancement is accelerating the process of industrialization and urbanization in almost all countries of the world. In Fluid Mechanics the researches on flow of viscous fluids are being used in all fields of engineering, astrophysics, biomedical science, meteorology, physical chemistry, geophysics and allied fields. The development of aeronautical, chemical, mechanical and computer engineering during last few decades on the one hand and finding oil wells in oceans on the other hand have added stimuli to the study of viscous fluid flows.

So far as the phenomenon of free convection flow of viscous fluids is concerned, it arises in flow of fluids when temperature change causes density variations leading to buoyancy forces acting on the fluid elements. The study of free convection flow through porous medium has scientific and engineering applications. For example in the field of agricultural engineering to study the underground water resources; in petroleum technology to study the movements of natural gases, oil and water through the oil reservoirs; in
chemical engineering to study the filtration and purification processes and so on.

In addition to above, magnetohydrodynamics is an important branch of fluid mechanics. It is developed to study the astrophysical phenomena arising from the discovery of existence of large magnetic fields in some stars and to study the possibilities of magnetohydrodynamic (MHD) generators as a new source of energy. Besides, the flow of viscous fluids in presence of magnetic field under varied restrictions is of increasing importance due to its applications in laboratory researches, engineering, technology, medical sciences, industries and so on. Hydromagnetic free convection flows have also been studied due to their importance in atmospheric and oceanic circulations, nuclear reactors, power transformers etc.

Rotating flows are of peculiar interest due to their applications in mechanical engineering, chemical engineering, petroleum engineering, physiological sciences and astrophysics. Rotating MHD flows have got application in cosmology and geophysics. These flows have industrial application in MHD generators, gaseous core, nuclear reactors etc. The importance of rotating and electromagnetic force on the hydromagnetic flow and their application to cosmology and solar physics has drawn attention of many research workers to study these flows. Besides, MHD rotating flows with heat transfer through porous medium, porous cylinders, rectangular channels and the laminar plane channel flows are of immense importance during these days due to their wide application in so many fields. In fact mathematicians, researchers aero-dynamists, mechanical engineers, medical-doctors, bio-engineers, physiologists etc. have recognised the importance of these flows. In view of the above facts, the researcher has made investigations under the topic "A STUDY OF FREE CONVECTION"
FLOW OF VISCOUS FLUIDS IN ROTATING SYSTEM" approved by the R. D. C. (Mathematics) of C. J. S. M., University, Kanpur (Formerly, Kanpur University, Kanpur) at its meeting held on 13-02-2004 in reference to the application of the researcher submitted on 25-07-2003 to the office of research section for permission to supplicate for the "DOCTOR OF PHILOSOPHY" degree of the University in the subject MATHEMATICS.

In the present thesis, an attempt has been made to present a comprehensive account of investigations carried out by the researcher during the period of research work.

1. NEWTONIAN AND NON-NEWTONIAN FLUIDS

The classical theory of 'fluid flow' was developed from studies of an imaginary fluid known as 'ideal fluid' or 'perfect fluid'. The ideal fluid is assumed to be incompressible and without viscosity or elasticity. The shearing motion in such fluid is completely frictionless, so that shear stresses are absent. For ideal fluids the mathematical relations are available in abundance. However, their application to real fluid motions was too much limited. This gives rise to development of 'dynamic theory' for the simplest class of real fluids, namely those possessing a linear relationship between shear stress and rate of shear, commonly described as 'Newtonian Fluids'. Newtonian fluids satisfy a hypothesis called 'Newtonian hypothesis'.

It was observed by Newton that in a simple rectilinear motion of fluid, the shearing stress between two neighbouring fluid layers is proportional to the relative velocity between the two layers and inversely proportional to the distance between the two layers. Thus, if the two neighbouring fluid layers are moving with velocities $u$ and $(u + \delta u)$ and are at a distance $\delta y$, then the shearing stress

$$\tau \propto \frac{\delta u}{\delta y} \text{ or } \tau = \mu \frac{\delta u}{\delta y}$$
This hypothesis is called ‘Newtonian's hypothesis’. The fluid satisfying this hypothesis is called \textbf{Newtonian fluid}. The constant of proportionality $\mu$ is called its \textbf{coefficient of viscosity} and the ratio $\frac{\mu}{\rho}$, is termed as \textbf{kinematic coefficient of viscosity}, where $\rho$ is the density of the fluid. Newtonian hypothesis has been used in explaining many physical phenomena in various branches of Fluid Mechanics. This tempts us to remark that most of fluids, at least in ordinary situations, behave like Newtonian fluids.

In recent years, specially with the emergence of polymers, it has been found that there are fluids which show a distinct deviation from Newtonian hypothesis. Such fluids are called \textbf{Non-Newtonian Fluids}. Examples of such fluids are solutions and melts of high polymers, suspensions of solids in liquids, emulsions and materials possessing both viscous and elastic properties and so on. The non-Newtonian fluids have been studied to solve the problems arising in industries dealing with plastic and synthetic fibers, petroleum, pharmaceuticals, cement, foods, paper-pulp, paints, rubber, soaps, detergents, cosmetics, biological fluids, solid rocket propellants, fermentation processes, oil-field operations, ore processing, printing etc.

Non-Newtonian fluid flows find applications in the flow of blood, in the development of nuclear reactors using Thorium and Uranium slurries etc. Further, the injection of non-Newtonian fluids into the boundary layer on immersed bodies such as: ships, hulls and submarines for reduced drag; addition of Magnesium or Boron particles to jet engine fuels for greater thrust; the propulsion of rockets by electrostatic acceleration of electrically charged colloidal particles etc. There has arisen a strong need to develop new theories to explain the behaviour of different types of non-Newtonian fluids.
2. POROSITY AND PERMEABILITY

Due to its natural occurrence and application in various branches of engineering, the study of flow through porous medium has drawn attention of a number of researchers. A porous material is defined as a solid containing holes or voids, either connected or non-connected, dispersed within it in either regular or random manner provided that such holes occur frequently within the solid. A great variety of natural and artificial materials such as; a bucket of sand, a piece of limestone, a brick, a tuft of cotton or a loaf of bread are porous in nature. The porosity of a porous material is defined as the fraction of the total volume of the material occupied by voids. Two classes of porosity have been defined, namely, absolute porosity and effective porosity. Absolute porosity is the fractional void space with respect to total volume regardless of pore concentration while effective porosity is the fraction of the total volume constituted by interconnecting pores. A fluid can flow only through connected spaces and therefore the study of effective porosity is meaningful. Porosity of a material changes with pressure; therefore porosity of clay decreases with increasing depth below with earth's surface.

The permeability is defined as that property of the porous material, which characterizes the case with which a fluid may be made to flow through the material by an applied pressure gradient. It is the fluid conductivity of the porous material. Darcy [1856] introduced the theory characterizing the fluid conductivity of a porous material. He stated that if horizontal flow of an incompressible fluid takes place through a porous material of length $L$ in the direction of fluid, and cross-sectional area $A$, then the permeability $k$ of the material is defined as

$$k = \frac{\mu \Omega}{A(\Delta P/L)}.$$
where $\Omega$ is the fluid flow rate in volume per unit time, $\mu$ is the viscosity of the fluid and $\Delta P$ is the applied pressure difference across the length $L$. The dimension of $k$ is defined as the square of the length. The unit widely used for the permeability $k$ is the Darcy.

3. INCOMPRESSIBLE AND COMPRESSIBLE FLUIDS

Incompressibility means that the density of the fluid is same at all the points and remains constant as time passes. The analysis of the flow of a fluid becomes simplified if we consider the fluid to be non-viscous and incompressible. This assumption is quite good for liquids and is valid in certain cases of flow of gases. Viscosity of a fluid is related to the internal friction when a layer of fluid slips over another layer, the mechanical energy is lost against such viscous forces. It has been observed that the total mass of fluid going into the tube through any cross-section should, therefore, be equal to the total mass coming out of the same tube from any other cross-section in the same time. This leads to the equation of continuity, a fundamental equation, to be satisfied by the flow of fluid.

In incompressible theory one important factor is that the velocity of sound is too much large and may be treated as infinity. Therefore any disturbance given at a point is transmitted every-where in the fluid instantaneously. This is contrary to the observations. Thus the phenomena connected with the propagation of disturbances cannot be dealt with incompressible theory. Secondly, there is a marked difference in the flow behaviour, when the fluid velocity is less than the velocity of sound and when it is greater then the velocity of sound. With the development of space sciences, the importance of the study of flows, where the fluid velocity is greater than the velocity of sound is increasing day by day.
To investigate any compressible fluid flow we have six equations; one mass conservation equation, three momentum equations, one energy equation and one equation of state, besides the boundary conditions. From six equations we can determine six variables namely, three velocity component, pressure, density and temperature. However, in practice, the situation is not so simple because the equations are highly non-linear. An important simplification in incompressible theory is that the mass conservation equation and the momentum equation do not depend on the temperature and the pressure is a dynamic variable. Thus we can solve the mass conservation and momentum equations without bothering about the temperature. After we have known the velocity and density distributions, we can obtain the temperature distribution from the energy equation. In the case of compressible flows all these equations have to be solved simultaneously. This increases the complexity of the problem; therefore to solve such problems we need to make some simplifying approximations.

Incompressible viscous fluid flows have been studied in detail by so many authors. It is worthwhile to discuss a simple case for flow of viscous incompressible fluid between two parallel plates.

Let us consider that the two infinite plates are situated at \( y = 0 \) and \( y = d \), and the flow is along \( x \)-axis, which is taken to be parallel to the plates. Since the plates are taken to be of infinite length, therefore we shall assume that the flow is only along \( x \)-axis and depends on \( y \)-axis. Moreover, we take the flow to be steady and therefore the flow variables do not depend on time.

We assume the velocity vector as \( \mathbf{v}(u(y,0,0)) \). Then the mass-conservation equation i.e. equation of continuity is satisfied identically and the momentum equations become:
The equation (2) gives that the pressure does not depend on \( y \).

Moreover, if we differentiate (1) with respect to \( x \), we find that \( \frac{d^2 p}{dx^2} = 0 \). This gives that \( \frac{dp}{dx} = -P \) is a constant. Thus in equation (1) we can take total derivative instated of partial derivative. This gives:

\[
\frac{d^2 u}{dy^2} = -\frac{1}{\mu} P
\]  

Its solution is

\[
u = Ay + B - \frac{1}{2\mu} Py^2
\]  

where \( A \) and \( B \) are constants of integration and are to be determined from boundary conditions. The shearing stress is then given by:

\[
\tau = \mu \frac{du}{dy} = \mu A - Py
\]  

Next let us take up the temperature distribution and so we consider the energy equation. Since we have taken the velocity along \( x \)-axis and all the variables depending only on \( y \), then energy equation reduce to:

\[
k \frac{d^2 T}{dy^2} + \mu \left( \frac{du}{dy} \right)^2 = 0
\]  

Substituting the value of \( u \) from (4), we get:

\[
\frac{d^2 T}{dy^2} = -\frac{1}{12\kappa\mu} \left[ 6\mu^2 A^2 y^2 - 4\mu A Py^3 + P^2 y^4 \right] + Cy + D
\]  

where \( C \) and \( D \) are integration constants.
Let the plates \( y = 0 \) and \( y = d \) be maintained at constant temperatures \( T_1 \) and \( T_2 \) respectively. Then the boundary conditions on temperature are:

\[
\begin{align*}
T &= T_1 \quad \text{at} \quad y = 0 \\
T &= T_2 \quad \text{at} \quad y = d
\end{align*}
\] (9)

Using the boundary conditions (9), we have:

\[
T = \frac{1}{12\kappa\mu} \left[ 6\mu^2 A^2 (d - y) - 4\mu AP \left( d^2 - y^2 \right) + P^2 \left( d^3 - y^3 \right) \right] + \left( T_2 - T_1 \right) \frac{y}{d} + T_1
\]

(10)

On the other hand if the plate \( y = 0 \) is thermally insulated and the plate \( y = d \) is maintained at a constant temperature \( T_2 \), then the boundary conditions are:

\[
\frac{dT}{dy} = 0 \quad \text{at} \quad y = 0 \\
T = T_2 \quad \text{at} \quad y = d
\]

(11)

using the boundary conditions (11), we have:

\[
T = \frac{1}{12\kappa\mu} \left[ 6\mu^2 A^2 (d^2 - y^2) - 4\mu AP \left( d^3 - y^3 \right) + P^2 \left( d^4 - y^4 \right) \right] + T_2
\]

(12)

4. FREE AND FORCED CONVECTION FLOW

It is the general observation that when a hot body is kept in contact with a cold body, the cold body warms up and the hot body cools down. In this process the internal energy of the hot body decreases and the internal energy of the cold body increases. Thus, energy is transferred from the hot body to the cold body when they are placed in contact. We notice that no mechanical work is done during this transfer of energy (neglecting any change in volume of the body). This is due to fact that there are no displacements involved. The transfer of energy from a hot body to a cold body is a non-mechanical process. The energy that is transferred from one body to the other, without any
mechanical work involved, is called ‘heat’. Thus, heat is a form of energy. It is energy in transit whenever temperature differences exist. Once it is transferred, it becomes the \textbf{internal energy} of the receiving body. Thus it is obvious that the word heat is meaningful only as long as the energy is being transferred.

The heat is energy in transit and its unit in SI is ‘\textbf{Joule}’. However, another unit of heat ‘\textbf{Calorie}’ is in wide use. The calorie is now defined in terms of Joule as $1 \text{Cal} = 4.186 \text{Joule}$. We also use the unit kilocalorie, which is equal to 1000 calorie as the name indicates. The amount of heat needed to raise the temperature of one gram of water by $1^\circ C$ depends slightly on the actual temperature of water and the pressure. That is why, the range $14.5^\circ C$ to $15.5^\circ C$ and the pressure one atmosphere was specified in the definition. If no transfer of heat takes place between two bodies, these bodies are said to be in thermal equilibrium when they are placed in contact.

If $\Delta Q$ amount of heat crosses through any cross-section in time $\Delta t$, $\frac{\Delta Q}{\Delta t}$ is called the ‘\textbf{heat current}’. It is found that in steady state the heat current is proportional to the area of cross-section $A$, proportional to the temperature difference $(T_1 - T_2)$ between the ends and inversely proportional to the length $x$. Thus

$$ \frac{\Delta Q}{\Delta t} = \kappa \frac{A(T_1 - T_2)}{x} $$

where $\kappa$ is a constant for the material of the slab and is called the \textbf{thermal conductivity} of the material.

If the area of cross-section is not uniform or if the steady state conditions are not attached, the equation can only be applied to a thin layer of material perpendicular to the heat flow. If $A$ be the area of cross-section at a place, $dx$ be a small thickness along the direction of heat flow, and $dT$ be the
temperature difference across the layer of thickness $dx$, the heat current through this cross-section is

$$\frac{\Delta Q}{\Delta t} = -\kappa A \frac{dT}{dx}$$

The quantity $\frac{dT}{dx}$ is called the 'temperature gradient'. The minus sign indicates that $\frac{dT}{dx}$ is negative along the direction of the heat flow.

All bodies in thermal equilibrium are assigned equal temperature. A hotter body is assigned higher temperature than a colder body. Thus, the temperatures of two bodies decide the direction of heat flow when the two bodies are put in contact. Heat flows from the body at higher temperature to the body at lower temperature and ultimately both the bodies become at thermal equilibrium.

In convection, heat is transferred from one place to the other by the actual motion of heated material. For example, in a hot air blower, air is heated by a heating element and is blown by a fan. The air carries the heat wherever it goes. When water is kept in a vessel and heated on a stove, the water at the bottom gets heat due to conduction through the vessel's bottom. The density of the water at the bottom decreases and consequently it rises. Thus, the heat is carried from the bottom to the top by the actual movement of the water. If the heated material is forced to move, say by a blower or a pump, the process of heat transfer is called 'forced convection'. If the material moves due to difference in density, it is called 'natural convection' or 'free convection'.

Inside the human body, the main mechanism for heat transfer is forced convection. Heart serves as the pump and blood as the circulating fluid. Heat is lost to the atmosphere through all the three processes viz. conduction,
convection and radiation. The rate of loss depends on the clothing, the tiredness and perspiration, atmospheric temperature, air current, humidity and several other factors. The system, however, transports the just required amount of heat and hence maintains a remarkably constant body temperature.

Convective flows with temperature and concentration differences have also been studied. In these flows, the transfer of heat and mass takes place simultaneously. In practice, ammonia, water vapor, oxygen, hydrogen, helium etc. are main gases, which are added in particulate air or other liquids embedded with dust particles. In mathematical problems, the concentration difference takes place through the 'diffusion equation'.

In some flow problems, the fluid motion is influenced by variations in density and viscosity within the fluid, are characterized by the term 'stratified flow'. Many researches have been done towards analysis of stratified flow models. Yih [1959] studied the effect of density variations on the fluid flow. Ahmadi & Manvi [1971] have studied viscous MHD flow. The mechanics of stratified flows might logically be considered in three phases namely (i) a study of pattern of flow in well defined layers, (ii) an investigation of the stability of layered flow and (iii) a treatment of the advanced stage of mixing wall beyond the point of instability. The researches on rotating flows with suction / injection, rotating flows in porous medium and MHD flows in rotating system, are of widespread interest to Applied Mathematicians, Aerodynamicists, Medical Doctors and Bio-Engineers. Besides, rotating MHD flows have got application in industrial application such as in MHD generators, nuclear reactors and so on.

The importance of the effect of rotating and electro-magnetic force on the hydro-magnetic flow and their application to solar physics has drawn attention of many research workers. In order to discover the above effects on
the hydro-magnetic flow phenomena and to examine the structures of the associated boundary layers, the initial value investigation of the Stokes and the Eckman problems in the presence of a magnetic field are of considerable interest in geophysical fluid dynamics. Besides, MHD rotating flows through porous medium, porous walls, porous cylinders, rectangular channels and the laminar plane channel flows are of immense importance during these days, particularly in presence of suction / injection.

5. DUSTY VISCOUS FLOWS

The motion of dusty fluids has also been studied by a number of research workers in the past few years. Flow of dusty fluids through porous media is of considerable importance in a wide range of disciplines in science and technology namely in soil mechanics, groundwater hydrology, petroleum engineering, water purifications, industrial filtrations, ceramic engineering and powder metallurgy. Besides, oscillatory dusty flow of fluids also occurs frequently and is important from the point of view of their applications. Ahmadi & Manvi [1971] have given general equation of flow of fluid through porous media. In addition, the study of free convection / mass transfer of dusty fluid flows in fluid saturated porous media has applications in geophysical and geothermal engineering. These includes moisture transport in thermal insulation, pore water convection near salt dams, movements of contaminants in ground water and waste dissolution and releases in underground nuclear waste disposal. Other important application of dust particles in boundary layer include fluidization (flow through packed beds), soil salvation by natural winds, dust entertainment in clouds during explosion, pulp, paper technology and so on. Following, Saffman [1962], dusty viscous laminar flows have also been discussed by making the following assumptions.

(i) The temperature is uniform within the particles.
Chemical reaction, mass transfer and radiation between the particles and fluid are not considered.

The interaction between the particles themselves has not been considered.

The buoyancy force has been neglected.

The flow is fully developed, *i.e.*, the velocity developed in the channel is independent of the axial coordinate of the channel.

The number density of the dust particles is constant throughout the motion.

The displacement current in MHD flow is neglected as the flow velocity is very small in comparison to the velocity of the light (*v* ≪ *c*).

The induced magnetic field has been neglected.

There is no external applied electric field.

The magnetic field is considered fixed relative to plate/plates/channels.

The fluid is electrically neutral, *i.e.*, there is no surplus electric charge distribution present in the fluid.

Fluid properties (density, viscosity) are invariable in the fluid.

Saffman's equations of motion for dusty viscous fluid flows reveal that the effect of dust enters through two parameters *F* and *τ*, where *F* depends on dust concentration and *τ* depends upon the size of dust particles.
In the study of flow of dusty fluids, the volume of the dust particles is generally assumed to be negligible. Saffinan [1962] studied stability of laminar flow of a dusty gas neglecting the volume fraction of dust particles. However, there are situations where this assumption is not justified such as the case of high fluid densities or the case of high particle mass fraction.

6. FLOWS THROUGH POROUS MEDIA

As a result of natural occurrence and due to applications in engineering and technology the flow of viscous fluids through a porous media has been a subject of intensive studies during recent years. In nature such flows occur in cases of water movement in the earth and sand structures such as dams, flow to wells from water bearing formation intrusion of sea water to the coastal area, the flow of river water through the porous banks and thus land erosion. The production of petroleum and natural gases, well drilling and lodging require many predictions based on the results of fluid flow through porous medium. The flow of blood through lungs and arteries are also examples of flow through porous medium. In many cases mentioned above the fluid flows through two regions. The region where there is no porous medium and fluid flows freely is called ‘free flow region’. The other region is called ‘porous region’ and in this region fluid flows through the pores of a permeable solid. Flows through these two regions are matched by suitable ‘matching conditions’ at the interface to obtain the solution of the problem.

Darcy's law holds good for laminar flow at law Reynolds number, which is the ratio of inertia and viscous forces. It has been demonstrated experimentally that for the flow through sand Darcy's law is valid if Reynolds number is less then unity. Brinkman [1947] has proposed that, if permeable medium is a swarm of homogeneous particles which are small spheres and are kept in position by external forces, as in a bed of closely packed particles
which support each other by contact, then the flow of viscous fluid in such a medium is governed by the following equation:

$$\Delta p = \mu \nabla V + \mu \nabla^2 V + F$$

where $F$ is the body force and other symbols have their usual meaning.

When the fluid flows in the presence of a porous material, the free flow outside such a body obeys Navier-Stokes equation. ‘Darcy’s law’ or ‘modified Darcy’s law’ holds for the flow inside the porous material. To make the problem determinate, it is assumed that there is a continuity of the mass flow at the interface and from this the continuity of the ‘normal velocity’ follows. Second assumption, that generally used, is the continuity of the pressure. For small permeability, the surface of a solid body could be regarded as a solid surface containing tiny holes and that ‘no slip condition’ on the tangent velocity could be applied. For bodies with a large permeability, no slip condition for the tangential velocity is sometimes invalid. On such a surface Beaver & Joseph [1967] proposed a physically plausible but empirical boundary conditions for plane boundaries. They have experimentally verified the above condition in the case of a flow through the channel in which one of the walls consists of a porous surface. The above condition is called ‘BJ-boundary condition’. Beaver & Joseph [1967] have also discussed the rectilinear flow of a viscous fluid through a channel formed by an impermeable wall ($y = h$) and a permeable lower wall ($y = 0$). The plane $y = 0$ defines a nominal surface for the permeable material. A uniform pressure gradient is maintained in the longitudinal direction in both the channel and the permeable material. The flow through the porous material is assumed to be governed by Darcy’s law and that in the free flow region by the Navier-Stokes equation. Both the flows are matched at the interface by the BJ-boundary condition.
7. MAGNETOHYDRODYNAMIC (MHD) FLOWS

The motion of an electrically conducting fluid, like mercury, under a magnetic field, in general, gives rise to induced electric currents on which mechanical forces are exerted by the magnetic field. On the other hand the induced electric currents also produce induced magnetic field and thus the original magnetic field is also changed. Thus there is a two-way interaction between the flow field and the magnetic field. The magnetic field exerts force on the fluid by producing induced currents. The induced currents change the original magnetic field. Therefore the hydromagnetic flows, i.e., the flows of electrically conducting fluids in the presence of magnetic field are more complex than the ordinary hydrodynamic flows. Mathematically also, the hydromagnetic equations have three non-linear terms while the hydrodynamics has only one. The number of governing equations also increases. The study of hydromagnetic flows is called hydromagnetics. It is also called magnetohydrodynamics (MHD). The later name has come obviously as generalization of hydrodynamics.

The study of hydromagnetics goes back to Faraday who predicted induced currents in the ocean due to the magnetic field of earth. In the present time, Hartmann [1937] was first to discuss both experimentally and theoretically the hydromagnetic flow between two parallel plates. Since then the literature on hydromagnetics has increases many-fold.

In gist, Magnetohydrodynamic (MHD) flows indicate the motion of electrically conducting fluids in the presence of magnetic field. Due to the effect of magnetic field an electric current is created in the fluid, which modifies the fluid motion. The property of electric current implies that there are two electric charges in the motion in the fluid. The motion of charges generates a magnetic field, which exerts a force on a charged particle of fluid.
moving within the fluid. Basic laws of electricity and magnetism govern such forces. The ultimate dynamical behaviour of the fluid is due to the combined effect of these forces. The study of MHD flows through porous media is of great importance in many engineering and technological fields. Problems on MHD flows involving porous media and permeable boundaries in the field of agricultural engineering, oil engineering and petroleum industry are of paramount importance for the well being of the society. The treatment by magneto-therapy is of common use in these days in problems of blood flow in arteries and in human joints.

The method of MHD power generation is gaining vital importance all over the world for economical generation of electrically by unconventional methods. At present the plants developed are small and experimental type. More developed plants will be required before MHD is regarded as a commercial proposition. In principle, MHD power generation is identical to the conventional generators, which are in operation and based on Faraday's law that the relative motion between the electrically conducting medium and a suitable magnetic field produces electric currents within the medium. For MHD power generation combustion plasma at elevated temperature is expended at a high velocity through a magnetic field. The gaseous conducting plasma replaces the metallic conductor of the conventional power generators. Besides, due to fast developments of microelectronics on microprocessor-based technology, magnetic flow meters have used MHD flow technology. At present there is fast development of industries, therefore the unconventional methods of energy generation play an important role. The application of electromagnetic field in various fluid flows has found an important place in recent researches. The study of MHD flow is of prime importance for applied mathematicians as well as to mechanical engineers, oil engineers, bio-engineers, aero-dynamicists, physiologists and medical doctors. The dynamics
of MHD flows is concerned with the interaction of electrically conducting fluids and electromagnetic fields. Such interactions occur in nature as well as in the man-made devices. In the laboratory, many new devices have been made which utilize the MHD interaction directly, such as propulsion units and power generators or which involve fluid electromagnetic field interactions such as electron beam dynamics, electrical discharges and many others.

The basic equations known as 'Maxwell equations' for electromagnetic forces are as follows:

\[ \nabla \times \vec{E} = \frac{4\pi \vec{q}}{\varepsilon} \]  
(1)

\[ \nabla \cdot \vec{H} = 0 \]  
(2)

\[ \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \]  
(3)

and \[ \nabla \times \vec{H} = 4\pi \vec{J} \]  
(4)

The above equations are employed in conjunction with Ohm's law:

\[ \vec{J} = \sigma (\vec{E} + \mu_e \vec{q} \times \vec{H}) = -\mu \frac{\partial \vec{H}}{\partial t} \]  
(5)

where \( \vec{E} \) is the electric field intensity, \( \varepsilon \) is the dielectric constant of the medium, \( \vec{H} \) is the magnetic field intensity, \( \mu_e \) is the magnetic permeability, \( \vec{J} \) is the conjunction current density vector, \( \vec{q} \) is the charge per unit volume and \( \sigma \) is the electrical conductivity of the medium.

If the medium is porous, basic equations for the flow of conducting viscous incompressible fluids are as follows:

\[ \text{div}(\vec{v}) = 0 \]  
(Equation of continuity)  
(6)

and \[ \rho \frac{\partial \vec{v}}{\partial t} = -\nabla p + \mu \nabla^2 \vec{v} - \frac{\mu_k}{k} \vec{v} + \vec{J} \times \vec{E} \]  
(7)
where $\vec{V}$ represents the velocity vector, $p$ the pressure, $\rho$ the density, $\mu$ the viscosity, $k$ the permeability of the medium, $\vec{B}$ the magnetic field and $\vec{J}$ the coniunction current density vector given by Ohm's law.

The most important parameter in the theory of magnetohydrodynamics is the ‘Hartmann number’, which is defined as:

$$\text{Hartmann number } (M) = \mu_e H_0 L \left( \frac{\sigma}{\mu} \right)^{1/2}$$

where $\mu_e$ is the magnetic permeability, $H_0$ is the magnetic intensity, $L$ is the length, $\sigma$ is the electrical conductivity and $\mu$ is the coefficient of viscosity of the fluid.

Magnetic Reynolds number ($R_m$) is a measure of the ratio of magnetic convection to magnetic diffusion. If $R_m << 1$, it can be shown that the induced magnetic field is small compared to the applied magnetic field. Symbolically, $R_m = V_0 L \sigma \mu_e$, where $\sigma$ is the electrical conductivity.

### 8. FLOWS IN ROTATING SYSTEM

At present considerable attention has been given to the study of hydromagnetic convective flow of viscous fluid in rotating system in connection with the theories of fluid motion, two phase flows, stratified flows, flow of immiscible fluids, flow through porous media, flow with suction/injection, flow in presence of heat source/heat flux, flow past a porous/hot/accelerated vertical plate, flow through channels of different shape with varied restrictions and so on. New ideas are being added to the literature to possible applications in geophysics, engineering problems, geothermal energy, steam stimulation of oil field, food drying and heat pipes.

Because of wide applicability of hydromagnetic free convective flow of viscous fluids in rotating system, the researcher propose to under take the present study. He proposes to carry his investigations on free convective flow
of viscous fluids in the presence of transverse magnetic field in rotating system. The effects of transverse magnetic fields and other important parameters on the free convective flow of an electrically conducting viscous fluid will be discussed. The scientific study, obviously, will be a valuable contribution to the knowledge and will be important for the advancement of the literature in diverse fields of engineering and technology.

The investigations on various models on flow of incompressible viscous fluids have been made due to fast development of industries. The problems on flow through porous ducts, porous medium, plane channels, rectangular channels, circular tubes and many other models of channel flows in rotating system are of immense importance during these days. The study of oscillating flows is found useful in the paper industry, physiological mechanics, petroleum industry, surface water hydrology and several technological as well as engineering fields. The problems of free convection heat transfer have been studied due to their numerous applications in geophysical and engineering problems. Dusty flows driven by combined buoyancy effects of thermal diffusion and diffusion of chemical species are encountered in many geophysical, astrophysical and engineering applications including power technology, performance of solid fuels in rocket nozzles, air craft icing etc.

The researchers working in the field of incompressible viscous fluid flows have paid their attention on problems associated with channel flows, flow through rectangular channels, flow through porous walls, flow through porous medium, flow through tubes, flow through different shapes of conic sections, flow past an infinite plate, flow with suction / injection, flow of blood in arteries, dusty flows, heat transfer, heat and mass transfer etc. Some important authors associated with the field of study of are: Crank & Nicolson
& Banerjee [1996], Rathod & Hosurker [1996], Ahmad & Sarma [1997],
Sharma et. al. [1997], Singh & Singh [1997], Hossain et. al. [1998], Iyengar
et. al. [1998], Kumar & Sharma [1998], Gupta et. al. [1998], Alam & Sattar
[1999], Gupta [1999], Paul et. al. [1999], Singh et. al. [1999], Gupta & Pathak
[1999], Seth & Ghose [1999], Acharya et. al. [2000], Alam & Sattar [2000],
Das et. al. [2000], Shah & Verma [2000], Singh [2000], Singh [2000], Singh
& Singh [2000], Singh & Singh [2001, 01], Gupta & Johari [2001], Helmy
[2001], Mishra [2001], Panda et. al. [2001], Sreekanth et. al. [2001], Takhar
et. al. [2001, 01], Jha [2001], El-Hakiem [2001], Tashtoush [2001], Kumar et.
al. [2002], Jha et. al. [2002], Singh et. al. [2002, 2002], Sonth et. al. [2002],
Habib et. al. [2002], Inaba et. al. [2002], Chamkha & Quadri [2002],
Mahapatra & Gupta [2002], Salah El-Din [2002], Sahoo et. al. [2002], Takhar
& Nath [2003], Takhar et. al. [2003], Huang & Li [2003], Chamkha & Quadri
[2003, 03], Kumar & Hundal [2003], Kumar et. al. [2003], Mobedi & Yuncu
[2003], Al-Subaie & Chamkha [2003], Ganesan & Palani [2003], Bhagarava
et. al. [2003], Singh [2003, 2003], Singh [2003], Singh et. al. [2003], Sood et.
al. [2003], Chamkha et. al. [2004], Kumari & Nath [2004, 04], Siu & Lee
[2004], Singh et. al. [2004, 04], Talukdar et. al. [2004], Dias et. al. [2004],
Chen [2004], Agrawal et. al. [2004], Al-Sanea [2004], Sun & Wichman
[2004], Che et. al. [2005], Ezzat & Jakaria [2005], Hussain & Rees [2005],
Lamsaadodi et. al. [2005], Mai et. al. [2005], Singh [2005], Singh et. al. [2005],
Singh at. al. [2006].

To give the general idea of the investigations given in the thesis, it
appears appropriate to give the essence of the problems considered for
investigations. Thus abstract of all the chapters is being given here. It may be
added here that all the problems taken for consideration have been discussed
taking numerical values of general interest and followed by established
researchers of the field.
Chapter-I

The heading of the first chapter is "INTRODUCTION". In this chapter, Newtonian and non-Newtonian fluids, porosity and permeability, incompressible and compressible fluids, free and forced convection flows, dusty viscous flows, flows through porous media, magnetohydrodynamic (MHD) flows, flows through rotating system, etc. have been described. Thereafter, chapter-wise introduction of the problems has been given to give a general idea about the investigations embodied in the thesis.

Chapter-II

The second chapter of the thesis deals with the effects of mass transfer on unsteady MHD free convection flow and heat transfer past an infinite porous vertical plate in rotating system under the heading "EFFECTS OF MASS TRANSFER ON UNSTEADY MHD FREE CONVECTION FLOW PAST AN INFINITE POROUS VERTICAL PLATE IN ROTATING SYSTEM". The plate is subjected to a time dependent suction in a homogeneous porous medium. The flow is considered under the influence of uniform magnetic field applied perpendicular to the flow region. The effects of magnetic parameter, rotation parameter, heat source parameter, permeability parameter, thermal diffusion parameter, Grashof number, Prandtl number, modified Grashof number and Schmidt number on primary velocity and secondary velocity are observed and discussed with the help of graphs. The effects of these parameters on skin-friction are discussed with the help of tables.
Chapter-III

In the present chapter the effects of heat and mass transfer, rotation, constant suction and time dependent porous medium on MHD flow of an incompressible, electrically conducting, viscous fluid past an infinite porous vertical plate under the influence of uniform magnetic field applied perpendicular to the flow region is discussed under the heading "EFFECTS OF MASS TRANSFER AND ROTATION ON FREE CONVECTION MHD FLOW PAST A POROUS PLATE IN A NON-HOMOGENEOUS POROUS MEDIUM". The results of the study are discussed with the help of graphs and tables.

Chapter-IV

In the present chapter effects of mass transfer and rotation on the unsteady free convection flow of an incompressible, homogeneous, electrically conducting, viscous fluid past an infinite porous vertical plate bounded by time dependent porous medium is studied under the heading "EFFECTS OF MASS TRANSFER AND ROTATION ON FREE CONVECTION FLOW PAST A PLATE WITH TIME DEPENDENT SUCTION IN NON-HOMOGENEOUS POROUS MEDIUM". It is considered that the plate is subjected to time dependent suction velocity and the flow is under the influence of transversely applied uniform magnetic field. The effects of important parameters on primary and secondary velocity and corresponding skin-frictions are discussed with the help of graph and tables.

Chapter-V

The effects of mass transfer on unsteady MHD free convection flow past an infinite porous vertical plate in rotating system are studied considering radiation and oscillatory heat flux under the heading "EFFECTS OF MASS TRANSFER ON UNSTEADY MHD FREE CONVECTION FLOW WITH RADIATION AND OSCILLATORY HEAT FLUX IN ROTATING SYSTEM". The plate is subjected to a
time dependent suction in a homogeneous porous medium and the flow is considered under the influence of uniform magnetic field applied perpendicular to the flow region. The effects of magnetic parameter, rotation parameter, Schmidt number, radiation parameter and heat flux parameter on primary and secondary velocity are observed and discussed through graphs. The effects of these parameters on skin-friction and rate of mass transfer at the plate are derived, discussed and their numerical values for different parameters are presented through tables.

Chapter-VI

In the present investigation the unsteady free convection flow due to heat and mass transfer bounded by an infinite vertical porous plate in rotating system under the influence of a uniform transverse magnetic field of constant strength is studied under the heading “HYDROMAGNETIC FREE CONVECTION WITH COMBINED HEAT AND MASS TRANSFER IN ROTATING SYSTEM”. The fluid and the plate are in a state of solid body rotation with constant angular velocity. The temperature and concentration at the plate are assumed oscillatory with time about a constant non-zero mean. The problem is solved by using a regular expansion technique for small value of frequency parameter. Approximate solutions for the primary and secondary velocity, temperature and concentration fields are obtained. Expressions for skin-friction at the plate due to primary velocity and secondary velocity, rate of heat and mass transfer are also derived. Numerical calculations are carried out for different values of parameters occurring into the equations of the problem under study and shown through tables and graphs. The chapter contains four figures and four tables.
Chapter-VII

Hydromagnetic free convection laminar flow of an incompressible viscous, conducting fluid through a porous medium past an isothermal porous vertical plate in rotating system with suction/injection at the plate is studied considering jump in temperature under the heading “HYDROMAGNETIC FREE CONVECTION FLOW OF A VISCOUS LIQUID IN ROTATING SYSTEM PAST AN INFINITE PLATE WITH JUMP IN TEMPERATURE”. Approximate solutions for primary and secondary velocity of the liquid and temperature field are obtained using perturbation technique. Expressions for skin-frictions due to primary and secondary velocities and heat transfer are also derived. The results obtained are numerically discussed with the help of three tables and three graphs.

Chapter-VIII

This chapter deals with the free convection flow of an electrically conducting, incompressible viscous fluid through a non-homogeneous porous medium in the presence of heat sink, oscillatory suction velocity and under the influence of transversely uniform magnetic field. Both the liquid and the plate are considered in a state of rigid body rotation with a constant angular velocity under the heading “FREE CONVECTION MHD FLOW THROUGH NON-HOMOGENEOUS POROUS MEDIUM WITH SUCTION VELOCITY AND HEAT SINK IN ROTATING SYSTEM”. Regular perturbation technique is applied to obtain approximate solutions for primary velocity, secondary velocity and temperature distributions. Expressions for the skin-friction at the plate due to primary velocity and secondary velocity and rate of heat transfer are also derived. The effects of various parameters on velocities, temperature distribution, skin-friction and rate of heat transfer are numerically discussed with the help of three graphs and three tables.
Chapter-IX

An analysis indicating the effect of Hall current on the unsteady boundary layer flow in an incompressible, electrically conducting, viscous liquid with solid, non-conducting, spherical and uniformly distributed dust particles bounded by infinite, impermeable, oscillating plate under the influence of transversely applied uniform magnetic field has been presented under the heading "HYDROMAGNETIC CONVECTIVE FLOW PAST IMPERMEABLE PLATE IN ROTATING SYSTEM: DUSTY FLOW WITH HALL CURRENT". The system rotates as a rigid body rotation. Perturbation technique is used to obtain the solutions for primary velocity and secondary velocity of liquid, dust particles and temperature field. Expressions for shear stress at the plate due to primary and secondary velocity of the liquid and dust particles are derived. In addition rate of heat transfer is also obtained. The results obtained are numerically evaluated and plotted graphically followed by a discussion. This chapter contains three figures and three tables.