Chapter - 2
CHAPTER-II

MASS TRANSFER EFFECTS ON UNSTEADY FREE CONVECTION COUETTE FLOW OF RIVLIN-ERICKSEN FLUID

ABSTRACT

In this chapter, mass transfer effects on unsteady free convection Couette flow of an incompressible, electrically conducting Rivlin-Ericksen fluid between two vertical parallel plates under the influence of magnetic field is discussed. Solutions for velocity, temperature and concentration fields along with the expressions for skin-friction, heat and mass transfer are using Laplace transform technique followed by particular cases and deductions.

1. INTRODUCTION

In nature and industry numerous transport processes occur wherein the density differences are caused by temperature as well as chemical composition differences and gradients or by material or phase constitution. Rivlin and Ericksen (1955) have introduced constitutive equations for a class of viscoelastic fluid known as Rivlin-Ericksen fluid. Several authors including Sharma & Sharma (1991), Singh & Singh (1994, 95). Singh & Kumar (1995). Das (2001.
02) have studied flow of Rivlin-Ericksen fluid past an infinite plate. In the present paper we have presented an analysis on flow of an incompressible Rivlin-Ericksen fluid in a vertical channel.

2. FORMULATION OF THE PROBLEM

In cartesian coordinate system, we assume $x'$-axis along a wall of the channel in the direction of flow and the $y'$-axis normal to it. A magnetic field of uniform strength $B_0$ is applied in the direction normal to the flow region. Initially, at $t' \leq 0$ the channel walls are fixed and the walls as well as fluid are at the same temperature $T_0'$ and species concentration is uniformly distributed in the flow region such that it is everywhere $C'_0$. When $t' > 0$, one of the plate starts moving with velocity $U$. The temperature of the channel walls is instantaneously raised (or lowered) to $T_w$ and the concentration level is raised (or lowered) to $C'_w$ and thereafter maintained constant. In addition, our analysis is based on the assumptions of Singh (2000).

Therefore the equations, relevant to the problem, in non-dimensional form are:

\[
\frac{\partial u}{\partial t} = G_r T + G_m C + \left(1 + S \frac{\partial}{\partial t} \frac{\partial^2 u}{\partial y^2}\right) - Mu 
\]  \hspace{1cm} \text{...(2.1)}

\[
\frac{\partial T}{\partial t} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} 
\]  \hspace{1cm} \text{...(2.2)}

\[
\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} 
\]  \hspace{1cm} \text{...(2.3)}
The initial and boundary conditions in non-dimensional form are:

\[ t \leq 0; \quad u(y,t) = T(y,t) = C(y,t) = 0 \quad \text{for} \quad 0 \leq y \leq 1 \]

\[ t > 0; \quad u(y,t) = T(y,t) = C(y,t) = 1 \quad \text{at} \quad y = 0 \]

\[ u(y,t) = T(y,t) = C(y,t) = 0 \quad \text{at} \quad y = 1 \quad \ldots(2.4) \]

The non-dimensional quantities introduced to obtain above equations are:

\[ y = \frac{y'}{h}, \quad t = \frac{t'\delta}{h^2}, \quad u = \frac{u'}{U}, \quad T = \frac{T' - T_0'}{T_w' - T_0'}, \quad C = \frac{C' - C_0'}{C_w' - C_0'} \]

\[ G_r = \frac{g\beta(t'_w - T_0') h^2}{\delta U} \quad \text{(Grashof number)}, \quad P_r = \frac{\mu C_p}{K} \quad \text{(Prandtl number)}, \]

\[ S = \frac{S'}{h} \quad \text{(Viscoelastic parameter)}, \quad M = \frac{B_0^2 h^2 \delta p}{9} \quad \text{(Magnetic parameter)}, \]

\[ G_m = \frac{g\beta (C_w' - C_0') h^2}{\delta U} \quad \text{(Modified Grashof number)} \]

and \[ S_c = \frac{9}{D} \quad \text{(Schmidt number)} \]

The other symbols have their usual meaning.

3. SOLUTION OF THE PROBLEM

Using Laplace transform technique, the solutions of equations (2.1)-(2.3), under the conditions (2.4), for \( P_r \neq 1, S_c \neq 1 \) [following the notations of Singh (2000)] are:

\[ T = \sum_{r=0}^{\infty} F_r(X_r, Y_r, P_r, 0.0, t) \quad \ldots(3.1) \]
\[ C = \sum_{r=0}^{\infty} F_1(X_r, Y_r, S_c, 0.0, t) \]  \ldots (3.2)

\[ u = \sum_{r=0}^{\infty} \left[ (1 + A_1 + A_4) F_1(X_r, Y_r, K, M, t) - (A_1 + A_4) F_2(X_r, Y_r, P_r, S_c, 0.0, t) \right] e^{-M_1 t} + \left[ (A_2 + A_3) F_1(X_r, Y_r, K, M, t) - (A_2 + A_3) F_2(X_r, Y_r, P_r, S_c, 0.0, t) \right] e^{-M_2 t} + \left[ A_4 F_1(X_r, Y_r, K, M, t) - A_6 F_2(X_r, Y_r, S_c, 0.0, t) \right] e^{-M_3 t} \]  \ldots (3.3)

Where \[ F_1(Z_1, Z_2, Z_3, Z_4, Z_5) = F(Z_1, Z_3, Z_4, Z_5) - F(Z_2, Z_3, Z_4, Z_5) \]
\[ F_2(Z_1, Z_2, Z_3, Z_4, Z_5, Z_6) = F_1(Z_1, Z_2, Z_3, Z_4, Z_5) + F_1(Z_1, Z_2, Z_4, Z_5, Z_6) \]
\[ F(Z_1, Z_2, Z_3, Z_4) = \frac{1}{2} \left[ \exp\left( - \sqrt{Z_1 Z_2} \right) \operatorname{erfc}\left( \frac{Z_1 \sqrt{Z_2}}{2 \sqrt{Z_4}} - \sqrt{\frac{Z_3 Z_4}{2 \sqrt{Z_4}}} \right) + \exp\left( \sqrt{Z_1 Z_2} \right) \operatorname{erfc}\left( \frac{Z_1 \sqrt{Z_2}}{2 \sqrt{Z_4}} + \sqrt{\frac{Z_3 Z_4}{2 \sqrt{Z_4}}} \right) \right] \]

where \[ K = 1 - SM, \quad K_1 = \frac{G_r}{S(P_r - 1)}, \quad K_2 = \frac{G_m}{S(S_c - 1)}, \quad M_1 = \frac{1}{S}, \]
\[ M_2 = \frac{M}{P_r - 1}, \quad M_3 = \frac{M}{S_c - 1}, \quad A_1 = \frac{-K_1}{M_1 M_2}, \]
\[ A_2 = \frac{K_1}{M_1 (M_1 + M_2)}, \quad A_3 = \frac{K_1}{M_2 (M_1 + M_2)}, \quad A_4 = \frac{-K_2}{M_1 M_3}, \]
\[ A_5 = \frac{K_2}{M_1 (M_1 + M_3)}, \quad A_6 = \frac{K_2}{M_3 (M_1 + M_3)}, \quad X_r = 2r + y \]

and \[ Y_r = 2r + 2 - y \]
4. SKIN-FRICTION, RATE OF HEAT AND MASS TRANSFER

The skin-friction at the plate \( y = 1 \) is:

\[
\tau = \left( \frac{\partial u}{\partial y} \right)_{y=1} = \sum_{r=0}^{\infty} \left[ f_1(D_r, C_r, K, M, t) \right. \\
\left. - \left( A_4 + A_9 e^{-M_{4t}} - A_6 e^{M_{4t}} \right) f_3(D_r, C_r, S_c, 0.0, t) \right] 
\]

\( \ldots(4.1) \)

The rate of heat transfer \((N_u)\) at the wall \( y=1 \) is:

\[
N_u = -\left( \frac{\partial T}{\partial y} \right)_{y=1} = \sum_{r=0}^{\infty} f_1(D_r, C_r, P_r, 0.0, t) 
\]

\( \ldots(4.2) \)

The rate of mass transfer \((S_h)\) at the wall \( y=1 \) is:

\[
S_h = -\left( \frac{\partial c}{\partial y} \right)_{y=1} = \sum_{r=0}^{\infty} f_1(D_r, C_r, S_c, 0.0, t) 
\]

\( \ldots(4.3) \)

Where \( A_7 = 1 + A_1 + A_7, \quad A_8 = A_2 + A_5, \quad C_r = 2r, \quad D_r = 2r + 2. \)

\[
f_1(Z_1, Z_2, Z_3, Z_4, Z_5) = f(Z_1, Z_3, Z_4, Z_5) + f(Z_2, Z_3, Z_4, Z_5),
\]

\[
f(Z_1, Z_2, Z_3, Z_4) = \frac{1}{2} \sqrt{Z_2 Z_3} \left[ \exp\left( -Z_1 \sqrt{Z_2 Z_3} \right) \text{erfc}\left( \frac{Z_1 \sqrt{Z_2}}{2 \sqrt{Z_4}} - \sqrt{Z_3 Z_4} \right) \right. \\
\left. + \exp\left( Z_1 \sqrt{Z_2 Z_3} \right) \text{erfc}\left( \frac{Z_1 \sqrt{Z_2}}{2 \sqrt{Z_4}} + \sqrt{Z_3 Z_4} \right) \right]
\]

\[
+ \sqrt{\frac{Z_2}{\pi Z_4}} \exp\left( -\frac{Z_1^2 Z_2^2}{4Z_4} - Z_3 Z_4 \right)
\]

5. PARTICULAR CASES

Case 1: When \( P_r \neq 1 \) and \( S_c = 1 \), the results shown in (3.1)-(3.3) and (4.1)-(4.3) reduce to:
\[ T_i = \sum_{r=0}^{\infty} F_1(X_r, Y_r, P_r, 0, 0, t) \]  
\[ C_i = \sum_{r=0}^{\infty} F_1(X_r, Y_r, 10, 0, 0, t) \]  
\[ u_i = \sum_{r=0}^{\infty} \left[ (A_1 + A_2 e^{-M_1 t} + A_3 e^{M_2 t}) f_1(X_r, Y_r, K, M, t) \right. \]
\[ -\left( A_1 + A_2 e^{-M_1 t} + A_3 e^{M_2 t} \right) f_1(X_r, Y_r, P, 0, 0, t) \]
\[ -\left( A_9 + A_{10} e^{-M_1 t} \right) f_1(X_r, Y_r, 1.0, 0, 0, t) \]  
\[ \tau_i = \sum_{r=0}^{\infty} \left[ (A_1 + A_2 e^{-M_1 t} + A_3 e^{M_2 t}) f_1(D_r, C_r, K, M, t) \right. \]
\[ -\left( A_1 + A_2 e^{-M_1 t} + A_3 e^{M_2 t} \right) f_1(D_r, C_r, P, 0, 0, t) \]
\[ -\left( A_9 + A_{10} e^{-M_1 t} \right) f_1(D_r, C_r, 1.0, 0, 0, t) \]  
\[ N_{u_i} = \sum_{r=0}^{\infty} f_1(D_r, C_r, P_r, 0, 0, t) \]  
\[ S_{h_i} = \sum_{r=0}^{\infty} f_1(D_r, C_r, 1.0, 0, 0, t) \]  

Where \( A_9 = -\frac{K_3}{M_1}, \) \( A_{10} = \frac{K_3}{M_1}, \) \( A_1 = 1 + A_1 + A_9, \) \( A_2 = A_2 + A_{10} \) and \( K_3 = \frac{M_1 G_m}{M} \)

The subscript I denotes corresponding results for case I.

**Case II**: When \( P_r = 1 \) and \( S_c \neq 1 \), the results shown in (3.1)-(3.3) and (4.1)-(4.3) reduce to:

\[ T_{\parallel} = \sum_{r=0}^{\infty} F_1(X_r, Y_r, 1.0, 0, 0, t) \]  
\[ C_{\parallel} = \sum_{r=0}^{\infty} F_1(X_r, Y_r, S_c, 0.0, t) \]
\[ u_{II} = \sum_{r=0}^{\infty} \left( A_{15} + A_{16}e^{-M_r} + A_{17}e^{M_r} \right) f_1(X_r, Y_r, K, M, t) \]
\[ - \left( A_{19} + A_{18}e^{-M_r} \right) f_1(X_r, Y_r, S_c, 0.0, 0.0, t) \]
\[ - \left( A_{19} + A_{18}e^{-M_r} \right) f_1(X_r, Y_r, 1.0, 0.0, 0.0, t) \] 
\[ \ldots (5.9) \]

\[ \tau_{II} = \sum_{r=0}^{\infty} \left( A_{15} + A_{16}e^{-M_r} + A_{17}e^{M_r} \right) f_1(D_r, C_r, K, M, t) \]
\[ - \left( A_{19} + A_{18}e^{-M_r} \right) f_1(D_r, C_r, S_c, 0.0, 0.0, t) \]
\[ - \left( A_{19} + A_{18}e^{-M_r} \right) f_1(D_r, C_r, 1.0, 0.0, 0.0, t) \] 
\[ \ldots (5.10) \]

\[ N_{u_{II}} = \sum_{r=0}^{\infty} f_1(D_r, C_r, 1.0, 0.0, 0.0, t) \] 
\[ \ldots (5.11) \]

\[ S_{h_{II}} = \sum_{r=0}^{\infty} f_1(D_r, C_r, S_c, 0.0, 0.0, t) \] 
\[ \ldots (5.12) \]

where \[ A_{13} = \frac{-K_4}{M_1}, \quad A_{14} = \frac{K_4}{M_1}, \quad A_{15} = 1 + A_4 + A_{13}, \]
\[ A_{16} = A_5 + A_{14} \quad \text{and} \quad K_4 = \frac{M_1 G_r}{M} \]

The subscript II denote corresponding results for case II.

**Case III**: When \( P_r = 1 \) and \( S_c = 1 \) the results shown in (3.1)-(3.3) and (4.1)-(4.3) reduce to:

\[ T_{III} = C_{III} = \sum_{r=0}^{\infty} f_1(X_r, Y_r, 1.0, 0.0, 0.0, t) \] 
\[ \ldots (5.13) \]

\[ u_{III} = \sum_{r=0}^{\infty} \left[ A_{17} + A_{18}e^{-M_r} \right] f_1(X_r, Y_r, K, M, t) \]
\[ - \left[ A_{19} + A_{18}e^{-M_r} \right] f_1(X_r, Y_r, 1.0, 0.0, 0.0, t) \] 
\[ \ldots (5.14) \]

\[ \tau_{III} = \sum_{r=0}^{\infty} \left[ A_{17} + A_{18}e^{-M_r} \right] f_1(D_r, C_r, 1.0, 0.0, 0.0, t) \]
\[-\left( A_{19} + A_{18}e^{-Mf} \right)f_1(D_r, C_r, 1.0, 0.0, t) \] ...

\[ N_{h_{III}} = S_{h_{III}} = \sum_{r=0}^{\infty} f_1(D_r, C_r, 1.0, 0.0, t) \] ...

where \( A_{17} = 1 + A_9 + A_{13}, \quad A_{18} = A_{10} + A_{14} \quad \text{and} \quad A_{19} = A_9 + A_{13} \)

The subscript III denote corresponding results for case III.

**6. DEDUCTIONS**

(i) When \( M = 0 \) the results obtained are exactly same as obtained by Singh et. al. (2003) except notations.

(ii) When \( M = 0 \) and \( S = 0 \) the results obtained are exactly same as obtained by Takhar and Jha (1998) except notations.