Chapter - 5
CHAPTER-V

UNSTEADY HEAT AND MASS TRANSFER FLOW OF A VISCOUS FLUID PAST AN INFINITE POROUS PLATE WITH TIME DEPENDENT SUCTION VELOCITY AND HEAT FLUX IN ROTATING SYSTEM

ABSTRACT

Unsteady free convection flow of a viscous fluid past an infinite porous vertical plate embedded in homogeneous porous medium is studied taking time dependent suction velocity and constant heat flux. The suction velocity at the plate is oscillatory. Perturbation technique is applied to obtain approximate solutions for velocity, temperature and concentration fields. The expressions for skin-friction, rate of heat and mass transfer are also derived. The effects of various physical parameters on velocity, temperature and concentration fields are discussed with the help of figures and tables.

1. INTRODUCTION

In nature as well as in industries many transport processes exist in which the transfer of heat and mass occurs simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species.
Extensive research efforts have been made to the problems on heat and mass transfer in view of their application to astrophysics, geophysics and engineering. The phenomenon of heat and mass transfer is also encountered in chemical processing industries such as food processing and polymer products. In engineering applications, the concentration differences are created by injecting the foreign gases or by coating the surface with evaporating material, which evaporates due to the heat of the surface. In practice, hydrogen, helium, water vapor, oxygen, ammonia etc. are the foreign gases, which are injected in the air flowing past bodies.

Stokes (1851), in his celebrated memoir on the motion of pendulums, investigated the flow of an incompressible fluid near an infinite flat plate, which is impulsively started from rest into motion in its own plate with a constant velocity. Cole (1868) has introduced perturbation methods for flow of fluids, which are being frequently use by the researchers of applied mathematics and physics. Huang and Chen (1985), Jang and Ni (1989), Hunt and Tien (1990), Jang et. al. (1991), Bejan (1992), Ganapathy (1994), Angirasa et. al. (1997), Inaba et. al. (2002), Chamkha and Quadri (2003) and Singh and Singh (2004) have studied problems on heat and mass transfer of viscous incompressible fluids under different physical situations. Recently, Singh (2005) has presented a detailed analysis a problem on unsteady free convection and mass transfer flow of an incompressible viscous liquid through a porous medium past an infinite vertical porous plate subject to time dependent suction velocity normal to the plate using perturbation technique. More recently, Singh et. al. (2007) have
studied a problem on heat and mass transfer flow past a vertical plate embedded in a non homogeneous porous medium.

In the above stated investigations, the suction velocity is considered either constant or periodic. However, in engineering problems there are situations where the use of heat and mass transfer with heat flux in porous medium with periodic suction velocity and rotating system becomes inevitable. Hence the aim of the present study is to make a theoretical analysis on heat and mass transfer flow of a viscous fluid past an infinite porous plate with periodic suction velocity at the plate taking into account a constant heat flux. The results obtained have been discussed with the help of graphs and tables.

2. FORMULATION OF THE PROBLEM

Consider unsteady flow of an incompressible, viscous liquid by the presence of free convection and mass transfer flow through a porous medium past an infinite vertical porous plate. In Cartesian coordinate system \((x', y', z')\), the vertical plate is assumed in the plane \(z' = 0\) and \(z'\)-axis is taken normal to the plate pointing towards the flow medium. The plate is rotating uniformly with the liquid in a rigid state of rotation with a constant angular velocity \((0, 0, \Omega)\) about-\(z'\)-axis. In a stationary condition, the plate and the fluid are at the same temperature \(T_\infty\) with concentration level \(C_\infty\). The presence of both concentration difference and temperature difference between the surface of the plate and the fluid makes the fluid motion initiated primarily due to the fluid buoyancy force. Besides, oscillatory suction velocity \(w' = -w_0(1 + \varepsilon e^{i\Omega t})\) has been taken into
account at the plate \((w'_0 > 0\), the negative sign show that the suction is towards the plate). Therefore for the present configuration with usual Boussinesq approximation and constant heat flux, the governing equations are as follows:

**Momentum equation:**

\[
\frac{\partial u'}{\partial t'} - 2v'\Omega - w'_0 \left(1 + \xi e^{int}\right) \frac{\partial u'}{\partial z'} = g\beta (T' - T_\infty) + g\beta^* (C' - C_\infty) + \frac{9}{k} \frac{\partial^2 u'}{\partial z'^2} - \frac{g}{k} u' \tag{1}
\]

\[
\frac{\partial v'}{\partial t'} + 2u'\Omega - w'_0 \left(1 + \xi e^{int}\right) \frac{\partial v'}{\partial z'} = \frac{9}{k} \frac{\partial^2 v'}{\partial z'^2} - \frac{g}{k} v' \tag{2}
\]

**Energy equation:**

\[
\frac{\partial T'}{\partial t'} - w'_0 \left(1 + \xi e^{int}\right) \frac{\partial T'}{\partial z'} = \frac{K_T}{\rho C_p} \frac{\partial^2 T'}{\partial z'^2} \tag{3}
\]

**Diffusion equation:**

\[
\frac{\partial C'}{\partial t'} - w'_0 \left(1 + \xi e^{int}\right) \frac{\partial C'}{\partial z'} = D_M \frac{\partial^2 C'}{\partial z'^2} \tag{4}
\]

The boundary conditions are

\(t' > 0\): \(u' = v' = w'_0 \left(1 + \xi e^{int}\right),\quad \frac{\partial T'}{\partial t'} = \frac{q}{K_T'}\)

\[C' = C'_w + \xi \left(C'_w - C'_\infty\right) e^{int}\quad \text{at } z' = 0\]

\(u' \rightarrow 0,\; v' \rightarrow 0,\; T' \rightarrow T_\infty,\; C' \rightarrow C_\infty\quad \text{as } z' \rightarrow \infty \tag{5}\)

where \(T'\) is the temperature of the fluid, \(C'\) is the concentration of species, \(u',\; v'\) are the velocities of the fluid in the \(x'\) and \(y'\) directions respectively, \(T_\infty\) is the temperature of the liquid far away from the plate, \(C_\infty\) is the concentration of species far away from the plate, \(K_T'\) is the thermal conductivity, \(C_p\) is the specific heat at constant pressure. \(D_M\) is the thermal diffusivity and the other symbols have their usual meaning.
We introduce the following non-dimensional variables:

\[ u = \frac{u'}{w_0}, \quad v = \frac{v'}{w_0}, \quad z = \frac{z'}{w_0}, \quad K_T = \frac{K_T w_0^2}{g}, \quad t = \frac{t' w_0^2}{g}, \quad n = \frac{n' \Omega}{w_0^2}. \]

\[ k = \frac{k' g^2}{w_0^2}, \quad T = \frac{(T' - T_\infty) K_T'}{T'_w - T'_\infty} \quad \text{and} \quad C = \frac{C' - C_\infty}{C'_w - C'_\infty}. \]

Introducing above stated non-dimensional quantities in (1) - (4), we obtain:

\[ \frac{\partial u}{\partial t} - 2Ev - (1 + \varepsilon e^{\text{int}}) \frac{\partial u}{\partial z} = G_r T + G_m C + \frac{\partial^2 u}{\partial z^2} - \frac{1}{k} u \]  
(6)

\[ \frac{\partial v}{\partial t} + 2Eu - (1 + \varepsilon e^{\text{int}}) \frac{\partial v}{\partial z} = \frac{\partial^2 v}{\partial z^2} - \frac{1}{k} v \]  
(7)

\[ \frac{\partial T}{\partial t} - (1 + \varepsilon e^{\text{int}}) \frac{\partial T}{\partial z} = \frac{1}{P_r} \frac{\partial^2 T}{\partial z^2} \]  
(8)

\[ \frac{\partial C}{\partial t} - (1 + \varepsilon e^{\text{int}}) \frac{\partial C}{\partial z} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2} \]  
(9)

where \( S_c = \frac{g}{D_M} \) (Schmidt number), \( G_r = \frac{g \beta \Psi g^2}{K_T w_0^3} \) (Grashof number),

\[ G_m = \frac{g \beta \Psi (C_w - C_\infty)}{w_0^3} \] (Modified Grashof number).

\[ P_r = \frac{\mu C_p}{K_T} \] (Prandtl number)

and \( E = \frac{g \Omega}{w_0^2} \) (Rotation parameter).

Assuming \( W = u + iv \), the equations (6) and (7) give:

\[ \frac{\partial W}{\partial t} + 2iEW - (1 + \varepsilon e^{\text{int}}) \frac{\partial W}{\partial z} = G_r T + G_m C + \frac{\partial^2 W}{\partial z^2} - \frac{1}{k} W \]  
(10)

The boundary conditions (5) are transformed to:
3. SOLUTION OF THE PROBLEM

To solve the above equations, we assume \( \varepsilon << 1 \):

\[
W(z, t) = W_1(z) + \varepsilon W_2(y) e^{int},
\]

\[
T(z, t) = T_1(z) + \varepsilon T_2(y) e^{int},
\]

\[
C(z, t) = C_1(z) + \varepsilon C_2(y) e^{int}
\]  

\begin{equation}
W \to 0, \quad T \to 0, \quad C \to 0 \quad \text{as} \quad z \to \infty \tag{11}
\end{equation}

On comparing harmonic and non-harmonic terms in (8)-(10), we obtain:

\[
\frac{d^2 T_1}{dz^2} + P_r \frac{dT_1}{dz} = 0 \tag{13}
\]

\[
\frac{d^2 T_2}{dz^2} + P_r \frac{dT_2}{dz} - \text{inP}_r T_2 = -P_r \frac{dT_1}{dz} \tag{14}
\]

\[
\frac{d^2 C_1}{dz^2} + S_c \frac{dC_1}{dz} = 0 \tag{15}
\]

\[
\frac{d^2 C_2}{dz^2} + S_c \frac{dC_2}{dz} - \text{inS}_c C_2 = -S_c \frac{dC_1}{dz} \tag{16}
\]

\[
\frac{d^2 W_1}{dz^2} + \frac{dW_1}{dz} - \left(2iE + k^{-1}\right) W_1 = -G_r T_1 - G_m C_1 \tag{17}
\]

\[
\frac{d^2 W_2}{dz^2} + \frac{dW_2}{dz} - \left(i(2E + n) + k^{-1}\right) W_2 = -G_r T_2 - G_m C_2 - \frac{dW_1}{dz} \tag{18}
\]

Introducing (12) in boundary conditions (11), we obtain:

\[
W_1 = 1, W_2 = 1, \quad \frac{\partial T_1}{\partial z} = -1, \quad \frac{\partial T_2}{\partial z} = 0, \quad C_1 = 1, C_2 = 1 \quad \text{at} \quad z = 0
\]

\[
W_1 \to 0, W_2 \to 0, \quad T_1 \to 0, T_2 \to 0, \quad C_1 \to 0, C_2 \to 0 \quad \text{as} \quad z \to \infty \tag{19}
\]
The solutions of (13)-(18) satisfying boundary conditions (19) are:

\[ T_1(z) = \frac{1}{P} e^{-P_r V} \]  

(20)

\[ T_2(z) = e^{-a_1 z} (A_1 \cos b_1 z + B_1 \sin b_1 z) + \frac{1}{n} e^{-P_r z} \]  

(21)

\[ C_1(z) = e^{-S_c y} \]  

(22)

\[ C_2(z) = e^{-a_2 z} \left( \cos b_2 z - \frac{S_c}{n} \sin b_2 z \right) \]  

(23)

\[ u_1(z) = P_1 e^{-P_r z} + P_2 e^{-S_c z} + e^{-a_3 z} (P_3 \cos b_3 z - Q_3 \sin b_3 z) \]  

(24)

\[ u_2(z) = e^{-a_1 z} (R_0 \cos b_1 z + Q_0 \sin b_1 z) + e^{-a_2 z} (P_0 \cos b_2 z + Q_0 \sin b_2 z) + e^{-a_3 z} (P_1 \cos b_3 z + Q_1 \sin b_3 z) + e^{-a_4 z} (P_2 \cos b_4 z + Q_2 \sin b_4 z) \]  

(25)

\[ v_1(z) = Q_1 e^{-P_r z} + Q_2 e^{-S_c z} + e^{-a_3 z} (Q_3 \cos b_3 z + P_3 \sin b_3 z) \]  

(26)

\[ v_2(z) = e^{-a_1 z} (Q_0 \cos b_1 z - R_0 \sin b_1 z) + e^{-a_2 z} (Q_0 \cos b_2 z - R_0 \sin b_2 z) + e^{-a_3 z} (Q_1 \cos b_3 z - R_1 \sin b_3 z) + e^{-a_4 z} (Q_2 \cos b_4 z - R_2 \sin b_4 z) \]  

(27)

Introducing (20)-(27) into (12) the velocity, temperature and concentration fields are given by:

\[ T(z,t) = 1 - e^{-P_r z} - e \left[ X_1 \cos nt - Y_1 \sin nt + i \left( Y_1 \cos nt + X_1 \sin nt \right) \right] \]  

(28)

\[ C(z,t) = e^{-S_c z} + e \left[ X_2 \cos nt + Y_2 \sin nt - i \left( Y_2 \cos nt - X_2 \sin nt \right) \right] \]  

(29)
\[ u(z,t) = u_1 + \varepsilon (u_2 \cos nt - v_2 \sin nt) \]  
\[ v(z,t) = v_1 + \varepsilon (u_2 \sin nt + v_2 \cos nt) \]

where \[ X_1 = e^{-\alpha_1 z} (A_1 \cos b_1 z + B_1 \sin b_1 z) - e^{-p_{r} z}, \]
\[ Y_1 = e^{-\alpha_1 z} (B_1 \cos b_1 z - A_1 \sin b_1 z), \]
\[ X_2 = e^{-\alpha_2 z} \left( \cos b_2 z - \frac{S_c}{n} \sin b_2 z \right), \]
\[ Y_2 = e^{-\alpha_2 z} \left( \sin b_2 z + \frac{S_c}{n} \cos b_2 z \right) - \frac{S_c}{n} e^{-S_c z} \]

Hence the transient velocity, temperature and concentration distribution at \( nt = \pi/2 \), are given by:

\[ T \left( y, \frac{\pi}{2n} \right) = T_1 + \varepsilon Y_1 \]  
(32)

\[ C \left( y, \frac{\pi}{2n} \right) = C_1 - \varepsilon Y_2 \]  
(33)

\[ u \left( y, \frac{\pi}{2n} \right) = u_1 - \varepsilon v_2 \]  
(34)

\[ v \left( y, \frac{\pi}{2n} \right) = v_1 - \varepsilon u_2 \]  
(35)

The constants are not included in the text to save the space.

4. SKIN-FRICTION AND MASS TRANSFER

The transient skin-friction \( (\tau_p) \) due to primary velocity, transient skin-friction \( (\tau_s) \) due to secondary velocity and rate of mass transfer \( (S_h) \) in terms of Sherwood number at the plate, are:

\[ \tau_p = \left( \frac{\partial u}{\partial z} \right)_{z=0} = L_1 + \varepsilon [A_3 \cos nt + B_3 \sin nt] \]  
(36)

\[ \tau_s = \left( \frac{\partial v}{\partial z} \right)_{z=0} = L_2 + \varepsilon [A_3 \sin nt - B_3 \cos nt] \]  
(37)

\[ S_h = \left( \frac{\partial u}{\partial z} \right)_{z=0} = -S_c + \varepsilon [B_2 \cos nt + A_2 \sin nt] \]  
(38)
TABLE-1
Effects of various parameters on skin-frictions $\tau_p$ and $\tau_s$
($nt = \pi/2$ and $\epsilon = 0.02$)

<table>
<thead>
<tr>
<th>$Gr$</th>
<th>$Gm$</th>
<th>$Pr$</th>
<th>$Sc$</th>
<th>$E$</th>
<th>$k$</th>
<th>$\tau_p$</th>
<th>$\tau_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.0</td>
<td>18.0</td>
<td>0.71</td>
<td>0.30</td>
<td>1.0</td>
<td>20.0</td>
<td>4.2475</td>
<td>-3.8764</td>
</tr>
<tr>
<td>18.0</td>
<td>18.0</td>
<td>0.71</td>
<td>0.30</td>
<td>1.0</td>
<td>20.0</td>
<td>6.7845</td>
<td>-4.4872</td>
</tr>
<tr>
<td>15.0</td>
<td>22.0</td>
<td>0.71</td>
<td>0.30</td>
<td>1.0</td>
<td>20.0</td>
<td>7.0847</td>
<td>-4.8476</td>
</tr>
<tr>
<td>15.0</td>
<td>18.0</td>
<td>7.00</td>
<td>0.30</td>
<td>1.0</td>
<td>20.0</td>
<td>3.2864</td>
<td>-2.4837</td>
</tr>
<tr>
<td>15.0</td>
<td>18.0</td>
<td>0.71</td>
<td>0.66</td>
<td>1.0</td>
<td>20.0</td>
<td>4.0837</td>
<td>-3.5673</td>
</tr>
<tr>
<td>15.0</td>
<td>18.0</td>
<td>0.71</td>
<td>0.30</td>
<td>2.0</td>
<td>20.0</td>
<td>2.7548</td>
<td>-4.0567</td>
</tr>
<tr>
<td>15.0</td>
<td>18.0</td>
<td>0.71</td>
<td>0.30</td>
<td>1.0</td>
<td>50.0</td>
<td>4.2586</td>
<td>-3.9048</td>
</tr>
</tbody>
</table>

TABLE-2
Rate of mass transfer
($nt = \pi/2$ and $\epsilon = 0.02$)

<table>
<thead>
<tr>
<th>S. No.</th>
<th>$Sc$</th>
<th>$Sh$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.22</td>
<td>-0.2367</td>
</tr>
<tr>
<td>2</td>
<td>0.30</td>
<td>-0.3183</td>
</tr>
<tr>
<td>3</td>
<td>0.60</td>
<td>-0.6207</td>
</tr>
<tr>
<td>4</td>
<td>0.66</td>
<td>-0.6748</td>
</tr>
<tr>
<td>5</td>
<td>0.78</td>
<td>-0.7962</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>-1.1874</td>
</tr>
</tbody>
</table>

5. DISCUSSION AND CONCLUSIONS

In order to get physical insight into the problem, the effects of Grashof number ($Gr$), modified Grashof number ($Gm$), Prandtl number ($Pr$), Schmidt number ($Sc$), rotation parameter ($E$) and permeability parameter ($k$) on primary velocity ($u$) and secondary velocity ($v$) at $nt = \pi/2$ and $\epsilon = 0.02$ are numerically observed and are shown in Figure-1 and Figure-2 while the effects of Prandtl number ($Pr$) on temperature field at $nt = \pi/2$ and $\epsilon = 0.02$ and the effects of
Schmidt number \((S_c)\) at \(n = \pi/2\) and \(\varepsilon = 0.02\) are shown in Figure-3 and figure-4 respectively. The effects of above stated parameters on skin-friction \((\tau_p)\) due to primary velocity and skin-friction \((\tau_s)\) due to secondary velocity at the plate at \(n = \pi/2\) and \(\varepsilon = 0.02\) are observed numerically and are represented in Table-1 while the effects of Schmidt number \((S_c)\) on rate of mass transfer are presented numerically in Table-2. To be realistic, the value of Schmidt number \((S_c)\) are chosen 0.22, 0.30, 0.60, 0.66, 0.78 and 1.0 for hydrogen, helium, water-vapor, oxygen, ammonia and methanol respectively, which represent diffusing chemical species of most common interest in air. The values of the Prandtl number \((Pr)\) are chosen to be \(Pr = 0.71\) and \(Pr = 7.00\), which corresponds to air and water at \(20^\circ C\) respectively. The remaining parameters are chosen arbitrarily but do retain applications in energy systems. The conclusions of the study are as follows:

(i) An increase in \(G_r, G_m\) or \(S_c\) increases primary velocity while an increase in \(Pr, E, k\) decreases the primary velocity.

(ii) The primary velocity increases in the vicinity of the plate and after attaining a maximum value it decreases as \(z\) increases.

(iii) An increase in \(Pr, E, k\) increases secondary velocity while an increase in \(G_r, G_m\) or \(S_c\) decreases the secondary velocity.

(iv) The secondary velocity decreases in the vicinity of the plate and after attaining a minimum value it increases as \(z\) increases.

(v) An increase in \(Pr, S_c\) or \(E\) decreases the skin-friction \((\tau_p)\) due to primary velocity while an increase in \(G_r, G_m\) or \(k\) increases the skin-friction \((\tau_p)\) due to primary velocity.
(vi) An increase in $G_r, G_m, E$ or $k$ decreases the skin-friction ($\tau_s$) due to secondary velocity while an increase in $S_c$ or $P_r$ increases the skin-friction ($\tau_s$) due to secondary velocity.

(vii) An increase in $S_c$ decreases the rate of mass.

APPENDIX

\[
\begin{align*}
    a_1 &= \frac{P_r}{2} + \sqrt[2]{\frac{P_r}{2} + 16n^2 + P_r}^{1/2}, \\
    b_1 &= \frac{\sqrt{P_r}}{2\sqrt{2}} \left[ \frac{\sqrt{P_r^2 + 16n^2 - P_r}}{1} \right]^{1/2}, \\
    a_2 &= \frac{S_c}{2} + \sqrt[2]{\frac{S_c}{2} + 16n^2 + S_c}^{1/2}, \\
    b_2 &= \frac{\sqrt{S_c}}{2\sqrt{2}} \left[ \frac{\sqrt{S_c^2 + 16n^2 - S_c}}{1} \right]^{1/2}, \\
    a_3 &= \frac{1}{2} + \frac{1}{2\sqrt{2}} \left[ \frac{\sqrt{1 + 4k^{-1}}^2 + 64E^2 + (1 + 4k^{-1})}{1} \right]^{1/2}, \\
    b_3 &= \frac{1}{2\sqrt{2}} \left[ \frac{\sqrt{(1 + 4k^{-1})^2 + 64E^2 - (1 + 4k^{-1})}}{1} \right]^{1/2}, \\
    a_4 &= \frac{1}{2} + \frac{1}{2\sqrt{2}} \left[ \frac{\sqrt{(1 + 4k^{-1})^2 + 16(2E + n)^2 + (1 + 4k^{-1})}}{1} \right]^{1/2}, \\
    b_4 &= \frac{1}{2\sqrt{2}} \left[ \frac{\sqrt{(1 + 4k^{-1})^2 + 16(2E + n)^2 - (1 + 4k^{-1})}}{1} \right]^{1/2}, \\
    A_1 &= \frac{b_1 P_r}{n(a_1^2 + b_1^2)}, \\
    B_1 &= \frac{a_1 P_r}{n(a_1^2 + b_1^2)}, \\
    P_1 &= \frac{G_r \left( P_r^2 - P_r - k^{-1} \right)}{P_r \left[ \left( P_r^2 - P_r - k^{-1} \right) + 4E^2 \right]}, \\
    Q_1 &= \frac{2EG_r}{P_r \left[ \left( P_r^2 - P_r - k^{-1} \right) + 4E^2 \right]}, \\
    P_2 &= \frac{G_m \left( S_c^2 - S_c - k^{-1} \right)}{(S_c^2 - S_c - k^{-1}) + 4E^2}, \\
    Q_2 &= \frac{2EG_m}{\left( S_c^2 - S_c - k^{-1} \right) + 4E^2}, \\
    P_3 &= 1 - P_1 - P_2, \\
    Q_3 &= 1 - Q_1 - Q_2.
\end{align*}
\]
\[ P_4 = G_r A_1, \]
\[ Q_4 = G_r B_1, \]
\[ P_5 = -G_m, \]
\[ Q_5 = \frac{G_m S_c}{n}, \]
\[ P_6 = a_3 P_3 - b_3 Q_3, \]
\[ Q_6 = b_3 P_3 - a_3 Q_3, \]
\[ P_7 = P_1 P_r, \]
\[ Q_7 = Q_1 P_r - \frac{G_r}{n}, \]
\[ P_8 = P_2 S_c, \]
\[ Q_8 = S_c \left( \frac{Q_2 - \frac{G_m}{n}}{n} \right), \]
\[ P_9 = \frac{c_1 P_4 + d_1 Q_4}{c_1^2 + d_1^2}, \]
\[ Q_9 = \frac{c_1 Q_4 - d_1 P_4}{c_1^2 + d_1^2}, \]
\[ P_{10} = \frac{c_2 P_3 + d_2 Q_5}{c_2^2 + d_2^2}, \]
\[ Q_{10} = \frac{c_2 Q_5 - d_2 P_3}{c_2^2 + d_2^2}, \]
\[ P_{11} = \frac{c_3 P_6 + d_3 Q_6}{c_3^2 + d_3^2}, \]
\[ Q_{11} = \frac{c_3 Q_6 - d_3 P_6}{c_3^2 + d_3^2}, \]
\[ P_{12} = 1 - P_9 - P_{10} - P_{11} - P_{13} - P_{14}, \]
\[ Q_{12} = -Q_9 - Q_{10} - Q_{11} - Q_{13} - Q_{14}, \]
\[ P_{13} = \frac{(P_r^2 - P_r - k^{-1}) P_7 - (2E + n) Q_7}{(P_r^2 - P_r - k^{-1})^2 + (2E + n)^2}, \]
\[ Q_{13} = \frac{(P_r^2 - P_r - k^{-1}) Q_7 + (2E + n) P_7}{(P_r^2 - P_r - k^{-1})^2 + (2E + n)^2}, \]
\[ P_{14} = \frac{(S_c^2 - S_c - k^{-1}) P_8 - (2E + n) Q_8}{(S_c^2 - S_c - k^{-1})^2 + (2E + n)^2}, \]
\[ Q_{14} = \frac{(S_c^2 - S_c - k^{-1}) Q_8 + (2E + n) P_8}{(S_c^2 - S_c - k^{-1})^2 + (2E + n)^2}. \]

\[ c_1 = a_1^2 - b_1^2 - a_1 - k^{-1}, \]
\[ d_1 = 2a_1 b_1 - b_1 - 2E - n, \]
\[ c_2 = a_2^2 - b_2^2 - a_2 - k^{-1}, \]
\[ d_2 = 2a_2 b_2 - b_2 - 2E - n, \]
\[ c_3 = a_3^2 - b_3^2 - a_3 - k^{-1}, \]
\[ d_3 = 2a_3 b_3 - b_3 - 2E - n, \]
\[ A_2 = b_2 - \frac{a_2 S_c}{n} + \frac{S_c^2}{n}, \]
\[ B_2 = -a_2 - \frac{b_2 S_c}{n} , \]
\[ A_3 = b_1 q_9 - a_1 p_9 + b_2 q_{10} - a_2 p_{10} + b_3 q_{11} - a_3 p_{11} + b_4 q_{12} - a_4 p_{12} - p_r p_1 - s_c p_{14} \]
\[ B_3 = b_1 p_9 + a_1 q_9 + b_2 p_{10} + a_2 q_{10} + b_3 p_{11} + a_3 q_{11} + b_4 p_{12} + a_4 q_{12} + p_r q_{13} + s_c q_{14} \]
\[ L_1 = -p_r p_1 - s_c p_2 - a_3 p_3 - b_3 q_3 \quad \text{and} \quad L_2 = -p_r q_1 - s_c q_2 + a_3 q_3 - b_3 p_3. \]
Figure 1: Effects of $Gr$, $Gm$, $Pr$, $Sc$, $E$ and $k$
on primary velocity field ($nt = \pi/2$, $\varepsilon = 0.02$)

<table>
<thead>
<tr>
<th>Curves</th>
<th>$Gr$</th>
<th>$Gm$</th>
<th>$Pr$</th>
<th>$Sc$</th>
<th>$E$</th>
<th>$k$</th>
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</thead>
<tbody>
<tr>
<td>I</td>
<td>15</td>
<td>18</td>
<td>0.71</td>
<td>0.30</td>
<td>1.0</td>
<td>20</td>
</tr>
<tr>
<td>II</td>
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<td>18</td>
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<td>0.30</td>
<td>1.0</td>
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<tr>
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<td>15</td>
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<td>IV</td>
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<td>18</td>
<td>7.00</td>
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<td>20</td>
</tr>
<tr>
<td>V</td>
<td>15</td>
<td>18</td>
<td>0.71</td>
<td>0.66</td>
<td>1.0</td>
<td>20</td>
</tr>
<tr>
<td>VI</td>
<td>15</td>
<td>18</td>
<td>0.71</td>
<td>0.30</td>
<td>2.0</td>
<td>20</td>
</tr>
<tr>
<td>VII</td>
<td>15</td>
<td>18</td>
<td>0.71</td>
<td>0.30</td>
<td>1.0</td>
<td>50</td>
</tr>
</tbody>
</table>

Figure 2: Effects of $Gr$, $Gm$, $Pr$, $Sc$, $E$ and $k$
on secondary velocity field ($nt = \pi/2$, $\varepsilon = 0.02$)

<table>
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<th>Curves</th>
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<th>$Gm$</th>
<th>$Pr$</th>
<th>$Sc$</th>
<th>$E$</th>
<th>$k$</th>
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<td>18</td>
<td>0.71</td>
<td>0.30</td>
<td>1.0</td>
<td>50</td>
</tr>
</tbody>
</table>
Figure 3: Effect of Pr on temperature field

\( nt = \pi/2, \varepsilon = 0.02 \)

Curve \( P_r \)
- I: 0.71
- II: 7.00
- III: 11.4
- IV: 0.025

Figure 4: Effect of Sc on concentration field

\( nt = \pi/2, \varepsilon = 0.02 \)

Curves \( S_c \)
- I: 0.22
- II: 0.30
- III: 0.60
- IV: 0.66
- V: 0.78
- VI: 1.00