MODELLING AND ANALYSIS OF THE SPREAD OF CARRIER DEPENDENT INFECTIOUS DISEASES: EFFECTS OF INFRASTRUCTURES, DEPENDING WHOLLY ON HUMAN POPULATION DENSITY

4.1 INTRODUCTION

In the case of carrier dependent infectious diseases such as tuberculosis, diarrhea, cholera, typhoid fever, the spread depends not only on the carrier population density but also on human population density related factors such as infrastructure. In a habitat, infrastructure plays a very important role in the spread of the carrier dependent infectious diseases as it provides a good space for growth and survival of carriers. No attention has been paid to study the effects of infrastructure although several models have been proposed and analyzed to study the effect of environment on the spread of infectious diseases. In chapter 3, we have proposed a nonlinear model to study the effect of isolation on the spread of carrier dependent infectious diseases. In this chapter we study the effect of infrastructures on the spread of carrier dependent infectious diseases. It may be mentioned that in the modelling process, it is assumed that the growth rate and carrying capacity of the carrier population infrastructures were indirectly considered by assuming the growth rate of carrying capacity of the carrier population density are increasing functions of infrastructures, the density of which is directly proportional to human population density.

Therefore in this chapter, we propose and analyze the spread of carrier dependent infectious diseases by considering the effect of infrastructure. The growth rate and carrying capacity of the carrier population density are assumed as functions of cumulative density of infrastructures, which is wholly dependent on human population density.

4.2 AN SIS MODEL

Let $X(t)$ and $Y(t)$ denote densities of susceptible and infective classes respectively of total human population density $N(t)$, in a region under consideration. Let $C(t)$ be the
carrier population density which affects all susceptibles and $I(t)$ be the cumulative density of infrastructures. In view of the above consideration and assuming simple mass action interaction, an SIS model proposed as follows:

\[
\begin{align*}
\frac{dX}{dt} &= A - \beta XY - \lambda XC + \nu Y - eX \\
\frac{dY}{dt} &= \beta XY + \lambda XC - (\nu + \alpha + e)Y \\
\frac{dC}{dt} &= sC - \frac{s_0 C^2}{L} + s_1 IC + s_2 C^2 I \\
\frac{dI}{dt} &= \theta N - \theta_0 I
\end{align*}
\]

(4.2.1)

where $X + Y = N$ with initial conditions $X(0) > 0, Y(0) \geq 0, C(0) \geq 0, I(0) \geq 0$.

**List of Coefficients in the model (4.2.1):**

- $A$: Rate of immigration of susceptibles in the host population from outside the region
- $e$: Rate of emigration from the host population
- $\beta$: Transmission coefficient due to infective human population
- $\lambda$: Transmission coefficient due to carrier population
- $\alpha$: Disease related death rate
- $\nu$: Recovery rate of infectives
- $s$: Intrinsic growth rate of carrier population
- $L$: Carrying capacity of carrier population in the absence of human activities
- $s_1$: Growth rate coefficient of carrier population
- $s_2$: Growth coefficient of the carrying capacity caused by the growth of cumulative density of infrastructures.
- $\theta$: Growth rate coefficient of infrastructural development due to human population density related factors
$\theta_0$: Depletion rate coefficient of infrastructural development.

All the coefficients in the model (4.2.1) are assumed to be positive and constant. In the modelling process, the following assumptions are made:

(i) The birth and natural death rates of host population are equal.
(ii) The disease spread due to direct contact between infectives and susceptibles.
(iii) There exists a carrier population in the environment which provides an additional indirect mode of transport for the spread of an infectious disease.
(iv) The growth rate of carrier population density follows logistic model, the intrinsic growth rate and carrying capacity of which increase due to human activities, assumed to be proportional to its population density.

In the following we analyze the model (4.2.1) by the stability theory of differential equations:

### 4.3 EQUILIBRIUM ANALYSIS

Since $X + Y = N$, the model (4.2.1) can be written as follows:

\[
\frac{dY}{dt} = \beta(N-Y)Y + \lambda(N-Y)C - (\nu + \alpha + e)Y
\]
\[
\frac{dN}{dt} = A - eN - \alpha Y
\]
\[
\frac{dC}{dt} = sC - \frac{s_0 C^2}{L} + s_1 IC + s_2 C^2 I
\]
\[
\frac{dI}{dt} = \theta N - \theta_0 I
\]

The following lemma establishes region of attraction for the system (4.3.1).

**Lemma 4.3.1:** The set

$$\Omega = \left\{ (Y, N, C, I) : 0 < Y \leq N \leq \frac{A}{e}, \frac{A}{e + \alpha} \leq N \leq \frac{A}{e}, 0 < C \leq C_m, 0 < I \leq I_m \right\},$$
attracts all the solutions initiating in the positive orthant, where

\[ C_m = \frac{s_0 (s_1 \frac{A \theta}{e \theta_0})}{(s_0 / L) - s_2 (A \theta / e \theta_0)} \quad I_m = (A \theta / e \theta_0) \quad (4.3.2) \]

provided,

\[ \frac{s_0}{L} > s_2 I_m \quad (4.3.3) \]

**Proof:** Here we give only a brief outline of the proof; the detail proof can be seen in Freedman and So (1985).

From the second equation of model (4.3.1), we have

\[ \frac{dN}{dt} = A - eN - \alpha Y \leq A - eN \quad \text{and} \]

\[ \frac{dN}{dt} = A - eN - \alpha Y \geq A - (e + \alpha)N \]

Using comparison theorem, we get

\[ 0 < Y \leq N \leq \frac{A}{e}, \quad \frac{A}{e + \alpha} \leq N \leq \frac{A}{e}. \]

From the last equation of model (4.3.1), we have

\[ \frac{dI}{dt} \leq \theta \frac{A}{e} - \theta_0 I \]

which gives \( 0 < I \leq \frac{A \theta}{e \theta_0} \)

From the equation for carrier population density of model (4.3.1), we have

\[ \frac{dC}{dt} \leq sC - \frac{s_0}{L} C^2 + s_1 \frac{A \theta}{e \theta_0} C + s_2 \frac{A \theta}{e \theta_0} C^2 = \left( s + s_1 \frac{A \theta}{e \theta_0} \right) C - \left( \frac{s_0}{L} - s_2 \frac{A \theta}{e \theta_0} \right) C^2 \]

65
which gives, \(0 < C \leq C_m = \frac{s + s_1 A\theta}{s_0 - s_2 \frac{A\theta}{L}}\), which is positive provided \(s_0 > s_2 \frac{A\theta}{L}\).

In the following we analyze the model (4.3.1), under the conditions (4.3.3):

**Theorem 4.3.1:** The system (4.3.1) has following equilibria:

(i) \(E_0\left(0, \frac{A}{e}, 0, I_m\right)\): the disease free and carrier free equilibrium

(ii) \(E_1(\bar{V}, \bar{N}, 0, \bar{I})\): the carrier free equilibrium

where \(\bar{N} = \frac{\beta A + a(v + \alpha + e)}{\beta(\alpha + e)}\), \(\bar{V} = \frac{\beta A - e(v + \alpha + e)}{\beta(\alpha + e)}\), \(\bar{I} = \frac{\theta N}{\theta_0}\).

Here \(\bar{V}\) exists if \(R_0 = \frac{\beta A}{e(v + \alpha + e)} > 1\). Here \(R_0\) is the basic reproduction number.

(iii) \(E^*(Y^*, N^*, C^*, I^*)\): The equilibrium point \(E^*\).

**Proof:** The existence of \(E_0\) or \(E_1\) is obvious. We prove the existence of \(E^*\). The equilibrium point \(E^*\) is given as the solutions of system of following equations, which are obtained after some simplification from (4.3.1) by putting left hand sides to zero:

\[
\beta Y^2 + Y\left[(v + \alpha + e) - \beta N + \lambda C\right] - \lambda NC = 0 \quad (4.3.4)
\]

\[
Y = \frac{A - eN}{\alpha} \quad (4.3.5)
\]

\[
s - \frac{s_0 C}{L} + s_1 I + s_2 CI = 0 \quad (4.3.6)
\]

\[
\theta N - \theta_0 I = 0 \quad (4.3.7)
\]

Now eliminating \(Y\) between equations (4.3.4) and (4.3.5) we get
\[ F(N) = \left( \frac{\beta}{\alpha^2} \right) (A-eN)^2 + \left( \frac{A-eN}{\alpha} \right) \left[ (v+\alpha+e) - \beta N + \lambda C \right] - \lambda NC = 0 \] (4.3.8)

where \( C \) is given in terms of \( N \) by (4.3.6) and (4.3.7). From equation (4.3.8) we note the following

\[ F\left( \frac{A}{\alpha + e} \right) = \frac{A}{\alpha + e} (v + \alpha + e) > 0 \] (4.3.9)

\[ F\left( \frac{A}{e} \right) = -\frac{\lambda A}{e} C_m < 0 \] (4.3.10)

Thus, it is clear that there exists a root \( N^* \) of \( F(N) = 0 \) in the interval \( \frac{A}{e+\alpha} \leq N \leq \frac{A}{e} \).

Further, this root will be unique if \( F'(N) < 0 \) for \( \frac{A}{e+\alpha} \leq N \leq \frac{A}{e} \).

To show this, we differentiate (4.3.8) to get

\[ F'(N) = -\frac{2\beta e}{\alpha^2} (A-eN) - \frac{e}{\alpha} \left[ (v+\alpha+e) - \beta N + \lambda C \right] - \frac{\beta}{\alpha} (A-eN) - \lambda \frac{C}{\alpha} [N(\alpha+e) - A] \frac{dC}{dN} \] (4.3.11)

Using (4.3.8), in (4.3.11), we get on simplification,

\[ F'(N) = -\frac{\beta e}{\alpha^2} (A-eN) - \frac{e}{(A-eN)} \lambda NC - \frac{\beta}{\alpha} (A-eN) - \lambda \frac{C}{\alpha} [N(\alpha+e) - A] \frac{dC}{dN} \] (4.3.12)

Which is negative in \( \frac{A}{e+\alpha} \leq N \leq \frac{A}{e} \) as \( \frac{dC}{dN} = \left( \frac{s_1 + s_2 C}{s_0} \right) \frac{dl}{dN} \) and \( \frac{dl}{dN} = \frac{\theta_1}{\theta_0} \) are positive.

Now, knowing the value of \( N^* \), the value of \( Y^* \), \( C^* \) and \( l^* \) can be uniquely determined from (4.3.5), (4.3.6), (4.3.7).
Remark: We can check analytically that \( \frac{dY}{d \theta} > 0 \) and \( \frac{dY}{d \theta} < 0 \). These conditions imply that as the cumulative density of infrastructures increases (decreases), the density of infectives increases (decreases).

First we show that at \( E' \), \( \frac{dY}{d \theta} < 0 \).

From (4.3.5) we have

\[
\frac{dN}{d \theta} = \frac{\alpha}{e} \frac{dY}{d \theta}
\]

(4.3.13)

Now on differentiating (4.3.4) w. r. t. \( \theta \) and using (4.3.4) and (4.3.5) we get

\[
\frac{dY}{d \theta} \left[ \beta Y + \lambda \frac{NC}{Y} + \frac{\alpha \beta}{e} Y + \frac{\lambda \alpha}{e} C \right] = \lambda(N - Y) \frac{dC}{d \theta}
\]

(4.3.14)

From (4.3.7)

\[
\frac{dl}{d \theta} = \frac{\theta \alpha}{e} \frac{dY}{d \theta} + I
\]

(4.3.15)
From (4.3.6)
\[
\frac{dC}{d\theta_0} = \frac{(s_1 + s_2 C)}{\left(\frac{s_0}{L} - s_2 I\right)^2} \frac{dI}{d\theta_0}
\]

(4.3.16)

Using (4.3.15) and (4.3.16) in (4.3.14), we get at \( E^* \), \( \frac{dY}{d\theta_0} < 0 \).

Thus it is seen here that as the depletion rate coefficient of cumulative density of infrastructures \( \theta_0 \) increases, the infective human population density decrease at \( E^* \).

In the similar manner as above, we can show that at \( E^* \), \( \frac{dY}{d\theta} > 0 \). Thus it is seen here that as the growth rate coefficient of infrastructural development due to human population density related factors \( \theta \) increases, the infective human population density increases at the equilibrium point \( E^* \).

From the above discussion it may be concluded that the spread of the carrier dependent infectious disease increases as infrastructures increase in a habitat.

4.4 STABILITY ANALYSIS

Now we shall study the stability behavior of equilibrium points \( E_0 \), \( E_1 \) and \( E^* \) in the following theorems:

Theorem 4.4.1: The equilibrium point \( E_0 \) and \( E_1 \) are unstable and \( E^* \) is locally asymptotically stable provided the following conditions are satisfied,
\[
\alpha \lambda^2 C^{*2} < e\beta^2 Y^{*2}
\]
(4.4.1)
\[
2\alpha \theta^2 \lambda^2 (N^{*} - Y^{*})^2 \left(s_1 + s_2 C^{*}\right)^2 < e\beta^2 Y^{*2} \theta_0^2 \left(\frac{s_0}{L} - s_2 I^{*}\right)^2
\]
(4.4.2)
Proof: The local stability behavior of each of the two equilibria $E_0$ or $E_1$ is studied by computing corresponding variational matrices for system (4.3.1) and for the nontrivial equilibrium point $E^*$ it is studied by using Lyapunov's theory.

The variational matrix $M_i$ corresponding to equilibrium points is given by:

$$
M_i = \begin{bmatrix}
\beta N - 2\beta Y - \lambda C - (v + \alpha + e) & \beta Y + \lambda C & \lambda(N - Y) & 0 \\
-\alpha & -e & 0 & 0 \\
0 & 0 & s - \frac{2s_0}{L} C + s_1 I + 2s_2 CI & s_1 C + s_2 C^2 \\
0 & \theta & 0 & -\theta_0
\end{bmatrix}
$$

**Local Stability Behaviour of $E_0 \left(0, \frac{A}{e}, 0, I_m\right)$:**

The variational matrix corresponding to equilibrium point $E_0$ is given by:

$$
M_0 = \begin{bmatrix}
\beta \frac{A}{e} - (v + \alpha + e) & 0 & \lambda \frac{A}{e} & 0 \\
-\alpha & -e & 0 & 0 \\
0 & 0 & s + s_1 I_m & 0 \\
0 & \theta & 0 & -\theta_0
\end{bmatrix}
$$

Here one of the eigen value $s + s_1 I_m$ is positive and hence $E_0$, if exists, is unstable.

**Local Stability Behaviour of $E_1 \left(\bar{Y}, \bar{N}, 0, \bar{I}\right)$:**

In this case the variational matrix will be

$$
M_1 = \begin{bmatrix}
\beta \bar{N} - 2\beta \bar{Y} - (v + \alpha + e) & \beta \bar{Y} & \lambda(\bar{N} - \bar{Y}) & 0 \\
-\alpha & -e & 0 & 0 \\
0 & 0 & s + s_1 \bar{I} & 0 \\
0 & \theta & 0 & -\theta_0
\end{bmatrix}
$$

This variational matrix has a positive eigen value $s + s_1 \bar{I}$ and hence $E_1$, if exists, is unstable.
Local Stability Behaviour of $E'(y', N', C', I')$:

We study the stability behavior of $E'$ by Lyapunov’s method. For this we linearise the system (4.3.1) by using following transformations

\[ y = Y - Y', \quad n = N - N', \quad c = C - C', \quad i = I - I' \]

and use following positive definite function to find the sufficient conditions for local stability

\[ V = \frac{1}{2} y'^2 + \frac{k_1}{2} n'^2 + \frac{k_2}{2} c'^2 + \frac{k_3}{2} i^2 \] (4.4.3)

(where $k_1$, $k_2$, and $k_2$ are positive constants to be chosen appropriately).

Differentiating (4.4.3) w.r.t. $'t'$ and using the linearized version of (4.3.1) \( \frac{dV}{dt} \) can be written as:

\[ \frac{dV}{dt} = -\left( \beta Y' + \gamma \frac{N' C'}{Y'} \right) y'^2 - (k_1 e)n'^2 - k_2 \left( \frac{s_0}{L} C' - s_1 i' C' \right) c'^2 - k_3 \theta_0 i^2 \\
+ (\beta Y' + \lambda C' - k_1 \alpha) y n + \lambda \left( (N' - Y') \right) y c + k_3 \theta \ n + k_2 C' \left( (s_1 + s_2 C') \right) c i \]

\[ = (\beta Y' - k_1 \alpha) y n - \lambda \left( \frac{N' C'}{Y'} \right) y' \left[ \left( \frac{\beta}{2} Y' \right) y^2 - \lambda C' y n + \frac{k_1 e}{2} n^2 \right] \]

\[ - \left[ \left( \frac{\beta}{2} Y' \right) y^2 - \lambda \left( (N' - Y') \right) y c + \frac{k_2}{2} C' \left( \frac{s_0}{L} - s_1 i' \right) c^2 \right] \]

\[ - \left[ \frac{k_2}{2} C' \left( \frac{s_0}{L} - s_1 i' \right) c^2 - k_2 C' \left( (s_1 + s_2 C') \right) c i + \frac{k_3}{2} \theta_0 i^2 \right] - \left[ \frac{k_3}{2} \theta_0 i^2 - k_3 \theta n + \frac{k_1 e}{2} n^2 \right] \]

Choosing $k_1 = \frac{\beta Y'}{\alpha}$, the conditions for $\frac{dV}{dt}$ to be negative definite can be written as follows:

71
Now if we choose \( k_3 = \frac{1}{2} \frac{\beta Y^* \theta_0 e}{\alpha \theta^2} \), then inequality (4.4.8) will satisfy automatically. Now we can choose \( k_2 \) satisfying inequalities (4.4.6) and (4.4.7) provided

\[
\frac{\lambda^2 (N^* - Y^*)^2}{\beta Y^* C^* \left( \frac{s_0}{L} - s_2 I^* \right)} \leq \frac{1}{2} \frac{\theta_0 \left( \frac{s_0}{L} - s_2 I^* \right)}{C^* \left( s_1 + s_2 C^* \right)^2} \frac{\beta Y^* \theta_0 e}{\alpha \theta^2}
\]

or

\[
2 \alpha \theta^2 \lambda^2 (N^* - Y^*)^2 \left( s_1 + s_2 C^* \right)^2 < \epsilon \beta^2 Y^* \theta_0^2 \left( \frac{s_0}{L} - s_2 I^* \right)^2
\]

Hence \( \frac{dV}{dt} \) is negative definite if (4.4.5) and (4.4.9) are satisfied. Thus, \( E^* \) is locally stable if (4.4.1) and (4.4.2) are satisfied.

**Theorem 4.4.2:** The equilibrium point \( E^* \) is nonlinearly asymptotically stable in \( \Omega \) provided the following inequalities are satisfied:

\[
\alpha \lambda^2 C^* \leq \epsilon \beta^2 Y^* \quad (4.4.10)
\]
We prove the theorem by using the following positive definite function:

\[ V = \left(Y - Y' - Y' \ln \frac{Y}{Y'}\right) + \frac{m_1}{2} (N - N')^2 + m_2 \left(C - C' - C' \ln \frac{C}{C'}\right) + \frac{m_3}{2} (I - I')^2 \]  

(4.4.12)

(Where \(m_1, m_2\) and \(m_3\) are positive constants to be chosen appropriately).

Differentiating (4.4.12) w.r.t. \(t\) and using (4.3.1), we get

\[
\frac{dV}{dt} = \left(\frac{Y - Y'}{Y}\right) \frac{dY}{dt} + m_1 (N - N') \frac{dN}{dt} + m_2 \left(\frac{C - C'}{C}\right) \frac{dC}{dt} + m_3 (I - I') \frac{dI}{dt} 
\]

\[
= -\left(\beta + \frac{NC}{YY'}\right)(Y - Y')^2 - m_1 e (N - N')^2 - m_2 \left(\frac{s_0}{L} - s_2 I\right) (C - C')^2 
\]

\[
- m_3 \theta (I - I')^2 + \left(\beta - m_1 \alpha + \frac{C}{Y'}\right)(Y - Y')(N - N') + \left(\frac{N'}{Y' - 1}\right)(Y - Y')(C - C') 
\]

\[
+ m_2 \left[s_1 + s_2 C\right] (C - C')(I - I') + m_3 \theta (I - I')(N - N') 
\]

Taking \(m_1 = \frac{\beta}{\alpha}\),

\[
\frac{dV}{dt} = -\lambda \frac{NC}{YY'}(Y - Y')^2 - \left[\frac{\beta}{2}(Y - Y')^2 - \lambda \frac{C}{Y'}(Y - Y')(N - N') + \frac{me}{2} (N - N')^2\right] 
\]

\[
- \left[\frac{\beta}{2}(Y - Y')^2 - \lambda \left(\frac{N'}{Y' - 1}\right)(Y - Y')(C - C') + \frac{m_2}{2} \left(\frac{s_0}{L} - s_2 I\right) (C - C')^2\right] 
\]

\[
- \left[m_3 \left(s_0 - s_2 I\right) (C - C')^2 - m_2 \left(s_1 + s_2 C\right) (C - C')(I - I') + \frac{m_3 \theta_0}{2} (I - I')^2\right] 
\]

\[
- \left[m_3 \theta (I - I')(N - N') + \frac{m_3 \theta_0}{2} (I - I')^2\right] 
\]

Now \(\frac{dV}{dt}\) will be negative definite if following conditions holds:
\[ \alpha \lambda^2 C^2 < e \beta^2 Y^* \quad (4.4.13) \]

\[
\frac{\lambda^2 \left( \frac{N^*}{Y^*} - 1 \right)^2}{\beta \left( \frac{s_0}{L} - s_z I^* \right)}
\]

\[
m_2 > \frac{\beta}{2} \left( \frac{s_0}{L} - s_z I^* \right) \theta_0
\]

\[
m_2 < \frac{\left( \frac{s_0}{L} - s_z I^* \right) \theta_0}{(s_1 + s_2 C)^2} - m_3
\]

\[
m_3 < \frac{\beta e \theta_0}{\alpha \theta^2}
\]

If we choose \( m_3 = \frac{1}{2} \frac{\beta e \theta_0}{\alpha \theta^2} \), the inequality (4.4.16) will satisfy automatically and we can choose \( m_2 \) satisfying inequalities (4.4.14) and (4.4.15) provided

\[
\frac{\lambda^2 \left( \frac{N^*}{Y^*} - 1 \right)^2}{\beta \left( \frac{s_0}{L} - s_z I^* \right)} < \frac{\left( \frac{s_0}{L} - s_z I^* \right) \theta_0}{2} \frac{\beta e \theta_0}{(s_1 + s_2 C)^2 \alpha \theta^2}
\]

or

\[ 2\alpha \theta^2 \lambda^2 \left( \frac{N^*}{Y^*} \right)^2 (s_1 + s_2 C)^2 < e \beta^2 Y^* \theta_0^2 \left( \frac{s_0}{L} - s_z I^* \right)^2 \quad (4.4.17) \]

Now on maximizing the left hand sides and minimizing right hand side of (4.4.13) and (4.4.17), we get

\[ \alpha \lambda^2 C_m^2 < e \beta^2 Y^* \quad (4.4.18) \]

\[
2\alpha \theta^2 \lambda^2 \left( \frac{N^*}{Y^*} \right)^2 (s_1 + s_2 C_m)^2 < e \beta^2 Y^* \theta_0^2 \left( \frac{s_0}{L} - s_z I^* \right)^2 \quad (4.4.19) \]
Hence $\frac{dV}{dt}$ is negative definite if above inequalities are satisfied. Thus $E^*$ is nonlinearly asymptotically stable if (4.4.9) and (4.4.10) are satisfied, as stated in theorem 4.4.2.

**Remark:** It is noted here that if $\lambda = 0$ or $\alpha = 0$, the above inequalities are satisfied automatically, which shows that $\lambda$ and $\alpha$ have destabilizing effects on the system.

### 4.5 NUMERICAL SIMULATION

Here we discuss the existence and stability of the nontrivial equilibrium point $E^*$ by taking the following set of parameter values and using MAPLE.

$$
A = 200, \quad e = 0.02, \quad \alpha = 0.03, \quad \beta = 0.000005, \quad \lambda = 0.000001, \quad \nu = 0.05, \quad s = 0.2, \quad s_0 = 0.9,
$$

$$
L = 50000, \quad s_1 = 0.002, \quad s_2 = 0.00000001, \quad \theta = 0.01, \theta_0 = 0.2.
$$

For these values of parameters, the nontrivial equilibrium point $E^*$ corresponding to (4.3.1) is obtained as follows

$$
Y^* = 2516, \quad N^* = 6225, \quad C^* = 55253, \quad I^* = 311
$$

The variational matrix for the above values of parameters at $E^*$ is

$$
M^* = \begin{bmatrix}
-0.1492 & 0.0678 & 0.0037 & 0 \\
-0.03 & -0.02 & 0 & 0 \\
0 & 0 & -0.8225 & 141.0353 \\
0 & 0.01 & 0 & -0.2
\end{bmatrix}
$$

The eigenvalues of the variational matrix corresponding to the equilibrium point $E^*$ are:

-0.0582, \quad -0.0945, \quad -0.2168, \quad -0.8220

All of which are negative. Hence $E^*(Y^*, N^*, C^*, I^*)$ is locally stable.

Now numerical simulation is performed for $N$ vs. $Y$ for the different initial starts and displayed in the figure 4.2, which indicates nonlinear stability of the point $E^*$. 

75
The model (4.3.1) has also been solved by using MAPLE and the graphs of the variable $Y$ with respect to $t$ for various values of different parameters have been plotted in figure 4.3 - 4.7.

(i) From figure 4.3, it is noted that $Y(t)$ increases as intrinsic growth rate of carrier population in absence of human activities $s$ increases.

(ii) From figure 4.4, it is seen that $Y(t)$ increases as growth rate coefficient of carrier population $s_1$ increases.

(iii) From figure 4.5, we note that $Y(t)$ increases as growth rate coefficient for carrying capacity of carriers $s_2$ increases.

The above results are expected, as the carrier population increases with the parameters $s$, $s_1$, $s_2$. Further, we also note that
From figure 4.6, it is seen that $Y(t)$ increases as growth rate coefficient of infrastructural development due to human population density related factors $\theta$ increases.

From figure 4.7, we note that $Y(t)$ increases as depletion rate coefficient of infrastructural development, $\theta_0$ decreases. These results are again expected as increase (decrease) in the cumulative density of infrastructure causes increase (decrease) in the density of carrier population, resulting increase (decrease) of the density of infectives.

![Figure 4.3](image_url)

**Figure 4.3**
Plots between $Y$ and $t$ for different values of intrinsic growth rate of carrier population in absence of human activities $s$
Figure 4.4
Plots between $Y$ and $t$ for different values of growth rate coefficient of carrier population $s_1$

Figure 4.5
Plots between $Y$ and $t$ for different values of growth rate coefficient for carrying capacity of carriers $s_2$
Figure 4.6
Plots between $Y$ and $t$ for different values of growth rate coefficient of infrastructural development due to human population density related factors $\theta$

Figure 4.7
Plots between $Y$ and $t$ for different values of depletion rate coefficient of infrastructural development $\theta_0$
4.6 CONCLUSIONS

In this chapter, a four dimensional SIS non-linear model with immigration of human population has been proposed and analyzed to study effect of infrastructure which is wholly dependent on human population density on the spread of infectious diseases. The density of carriers has been assumed to be governed by a logistic model, the intrinsic growth rate and carrying capacity of which are increasing functions of cumulative density of infrastructures. In the modelling process, the cumulative density of infrastructure has been assumed to be directly proportional to the human population density. The model has been analyzed by using stability theory of differential equation as well as by computer simulation. The following main conclusions have been drawn from the analysis:

(i) The density of infectives increases, as the parameters related to increase in infrastructural development due to human population density related factors, increase.

(ii) The density of infectives increases with the increase in the cumulative density of infrastructures.

It may then be concluded that the spread of carrier dependent infectious diseases increases due to increase in cumulative density of infrastructures in the habitat changed by human actions.