Chapter 4

Numerical Study of Double-Diffusive Natural Convection from a Vertical Surface with Lateral Mass Flux

4.1 Introduction

The focus of the present study is on the fundamentals of the flow in a porous medium confined between two vertical walls maintained at different temperatures and different concentration of certain chemical species. The novel aspect of this research is the often antagonistic relationship between the two buoyancy effects that drive the flow, namely, the density difference caused by temperature variations and the density difference caused by concentration variations. The engineering applications of this phenomenon are important; for example, the migration of moisture through the air contained in fibrous insulations and grain storage installations, and the dispersion of chemical contaminants through water-saturated soil, thermal insulation engineering, solar power collectors, reactors cooling systems, under-ground disposal of wastes, spreading of pollutants, oceanography, geophysics, metallurgy and electro-chemistry etc.

Natural convection temperature and species transfer in a square porous cavity subjected to constant temperature and adiabatic wall conditions has been investigated by Trevisan and Bejan [1]. The numerical study has been carried out for a wide range of Darcy-
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Rayleigh number, Lewis number, and buoyancy ratio. In another paper the same authors [2] considered boundary conditions of constant heat flux on the vertical walls of the porous enclosure. In addition to the parameters considered in their previous article they also investigate the effect of aspect ratio. Mamou et al. [3] recently studied natural convective flow inside a rectangular enclosure with heat and mass flux boundary conditions on the vertical walls and adiabatic conditions on the horizontal walls. Alavyoon and co-workers [4,5] also reported similar studies with heat and mass flux boundary conditions. Angirasa et al. [6] presented natural convective flows adjacent to surfaces in confined buoyancies of heat and mass diffusion. Special attention is given to opposing buoyancy effects of the same order and unequal thermal and species diffusion coefficients. All the above studies are limited to Darcy's flow regime. A horde of literature is available on the prediction of natural convection flow and heat transfer in a porous medium, using specialized models corresponding to Darcy and non-Darcy regimes [7]. However, generalized models that are applicable for a wide range of Darcy and Rayleigh numbers are not available. In 1996 Nithiarasu et al. presented, such a generalized model, and numerical results predicted from the model have been validated with reference to a variety of available theoretical results for double diffusive natural convection.

It is remarkable that the effects of injection or withdrawal of fluid on heat transfer along vertical surfaces in porous media have been examined by several investigators. For example, Cheng [8] obtained similarity solutions for vertical plate in natural convection with varying surface injection/suction velocity and wall temperature. Merkin [9] used a series expansion method to solve the same problem. A similar type of problem with uniform surface injection/suction and uniform wall temperature was analysed by Minkowycz and Cheng [10] using the local non-similarity method. In a later paper, Minkowycz et al. [11] applied same solution method to study the problem of free convection thermal transport flow along a horizontal plate with effects of lateral mass flux. Lai and Kulacki [12] studied the effects of surface injection/suction on heat transfer in mixed convection over horizontal and inclined surfaces under similar boundary layer flow conditions. Hooper, Chen and Armaly [13] presented the effects of uniform surface blowing/suction on mixed
convection along an isothermal vertical plate embedded in a porous medium.

The main aim of the present analysis is to study the effect of injection/suction on combined heat and mass transfer numerically, along vertical surface embedded in a fluid saturated porous medium for aiding and opposing buoyancies. Both aiding and opposing flow are considered for wide range of parameters, the thermal Rayleigh number, the buoyancy ratio, Lewis number and mass flux parameter Varb. The problem has a number of geothermal and engineering applications. For example, the residual warm water discharged from a geothermal power plant is usually disposed of through subsurface reinjection wells which can be idealized as vertical plane sources in a porous medium. If the temperature of the injected fluid differs from that of the receiving groundwater in the rock formation, the injected fluid would experience a positive or negative buoyancy force (depending on the relative temperature and concentration differences) which results in a convective movement of ground water near the well. Similarly, convection of groundwater also occurs along the vertical fissures or cracks during the natural recharge of the aquifer, whenever the temperature and the concentration of the groundwater discharged from fissures and cracks differs from that of the receiving water in the aquifer. Recent literature displays much interest in the possibility of reducing heat transfer to high-speed aircraft and missiles with mass transfer or ablation cooling.

4.2 Analysis

The physical configuration and coordinate system under consideration are illustrated in Fig. 1. A permeable vertical surface of height L is embedded in fluid saturated porous medium, with surface injection/suction at uniform velocity varb. The problem of natural convection from a permeable vertical plate embedded in a fluid saturated porous medium, with surface injection/suction at uniform velocity \( v \) is considered here. The surface is maintained at uniform temperature \( T_w \) and at uniform concentration \( C_w \). The free stream temperature and concentration are \( T_\infty \) and \( C_\infty \) respectively, where \( T_w > T_\infty \) and \( C_w > C_\infty \). Assuming low velocity and porosity, the Darcy model is employed in an
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An isotropic porous medium. We assume that (1) the properties of the fluid and porous medium are homogeneous and isotropic, (2) the convective fluid and porous medium are everywhere in thermodynamic equilibrium, (3) the Boussinesq approximation is invoked (4) the temperature is everywhere below boiling point and (5) properties of the fluid and the porous medium such as viscosity, thermal conductivity, thermal and concentration expansion coefficients, specific heats and permeability are constant. The buoyancy driven Darcy flow and transport, adjacent to a vertical surface due to the combined effects of the thermal and species diffusion can be described by the following two-dimensional conservation equations in nondimensional form

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{4.1}
\]

\[
\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} = \frac{\partial T}{\partial Y} + N \frac{\partial C}{\partial Y} \tag{4.2}
\]

\[
\sigma \frac{\partial T}{\partial \tau*} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Ra} \left( \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) \tag{4.3}
\]

\[
\phi' \frac{\partial C}{\partial \tau*} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{RaLe} \left( \frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right) \tag{4.4}
\]

where \(\phi'\) is the porosity of the medium and \(\sigma\) is the ratio of heat capacities of the stagnant medium of the fluid. The buoyancy ratio, \(N\), is defined as

\[
N = \frac{\beta_c \Delta C}{\beta_T \Delta T} \tag{4.5}
\]

where \(\beta_T\) is the thermal volumetric expansion coefficient and \(\beta_c\) is the volumetric coefficient due to concentration. \(\beta_T = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p, \beta_c = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial c} \right)_p\). The magnitude of the buoyancy ratio, \(N\), indicates the relative strengths of the two buoyant forces and the algebraic sign provides information on the relative direction of the two forces. While the thermal buoyancy always acts vertically upward, the species buoyancy may act in either direction depending on the relative molecular weights, with a heavier species contributing a negative buoyant force that acts vertically downward, resulting in a negative sign on the buoyancy ratio. For the case where the buoyancy ratio is equal to zero, i.e. \(N = 0\), the flow is driven by thermal buoyancy alone.
Two other parameters which appear in the non-dimensional conservation equations are the Rayleigh number, $Ra = \frac{Kg\beta\gamma\Delta t}{\nu \alpha}$ and the Lewis number, $Le = \frac{\alpha}{D}$. Here, $K$ is the permeability, $g$ is the gravitational acceleration, $\alpha = \frac{\kappa}{\rho C_f}$ is the thermal diffusivity and $D$ is the species diffusion coefficient. The following non-dimensional variables were defined in the development of equations (4.1) - (4.4).

\[ X = \frac{x}{L}, Y = \frac{y}{L}, \tau^* = \frac{\tau V_c}{L}, U = \frac{u}{V_c}, V = \frac{v}{V_c}, T = \frac{t - t_\infty}{t_w - t_\infty}, C = \frac{C - C_\infty}{C_w - C_\infty} \]  

(4.6)

where $V_c$ is a convective velocity defined as $V_c = \frac{Kg\beta\gamma\Delta t}{\nu}$. It may be noted that the present approach of formulating the problem and introducing the dimensionless parameters reduces the present problem to an extension of the rectangular cavity case. The appropriate boundary conditions in non-dimensional form are expressed as

\[
\begin{align*}
Y &= 0, & T &= 1, C &= 1, V &= varb \\
Y &= \infty, & T &= 0, C &= 0, U &= 0
\end{align*}
\]

(4.7)

A non-dimensional vorticity, defined as

\[ \omega = \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \]

(4.8)

allows equation (4.2) to be written as

\[ \omega = \frac{\partial T}{\partial Y} + N \frac{\partial C}{\partial Y} \]

(4.9)

and a non-dimensional stream function $\psi$ to be defined such that

\[ U = \frac{\partial \psi}{\partial Y} \quad \text{and} \quad V = \frac{\partial \psi}{\partial X} \]

(4.10)

Combining equations (4.8) and (4.10), the stream function equation can be written as

\[ \frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = \omega \]

(4.11)

The average Nusselt number is obtained as

\[ Nu = \frac{hL}{k} = \int \left(-\frac{\partial T}{\partial Y}\right) dX \]

(4.12)

where $h$ is the average heat transfer coefficient. Similarly, the average Sherwood number can be expressed as

\[ Sh = \frac{mL}{D} = \int \left(-\frac{\partial C}{\partial Y}\right) dX \]

(4.13)

where $m$ is the average mass transfer coefficient.
4.3 Numerical Methods

The heat and mass conservation equations, equations (4.3) and (4.4) are solved first using the alternating direction implicit scheme (ADI) of Peaceman and Rachford. The convective terms are discretized with first-order upwind differencing and diffusion terms with central differencing. The vorticity is then evaluated using equation (4.9) and central difference formulation. The stream function equation, equation (4.11), is then solved iteratively with in each time-step using a successive order of relaxation (SOR) method. It should be noted that the wall vorticities are not required to obtain the vorticity field. The vorticity is merely evaluated with the field solutions T and C by means of equation (4.9). The average Nusselt and Sherwood numbers are calculated through numerical integration of equations (4.12) and (4.13) using an open-ended formulation of Simpson’s rule. The initial conditions used to march the discretized equations, equations (4.3) and (4.4), are

\[ \tau^* = 0, T = 0, C = 0, U = 0, V = 0 \]  

(4.14)

for all X and Y

The boundary conditions for equations (4.3), (4.4) and (4.11) are shown in Fig. 1. Both the absolute and relative error criteria with a value less than $10^{-5}$ were used to check for steady state solution. Constant grid spacing were employed in each direction. The number of grid points was varied to make the grid-dependent error less than one percent in field variables. This required 101 grid points in each direction. The time-step was chosen to be 0.001 after checking that the value had no impact on the final steady-state solution. The detailed discussion on the effects of upwind differencing, mesh sizes, relaxation parameters, convergence criterion etc. on the accuracy and the computational time are presented in reference [6].
4.4 Results and Discussions

Extensive calculations have been performed to obtain steady-state flow, Nusselt and Sherwood numbers for wide range of parameters, buoyancy ratio, $N$, Lewis number, $Le$, Rayleigh number $Ra$ and mass flux parameter $vrb$. The literature survey discussed above revealed that similarity solutions could not obtained, when $-1 < N < 0$, for low values of Rayleigh number and Lewis number less than unity when $N < 0$. These ranges of parameters are given special consideration here because the limitations of boundary layer analysis have been removed.

Since the energy and species conservation, equations (4.3) and (4.4), respectively, are marched in time to the asymptotic steady-state solutions, influence of $\sigma$ and $\phi'$ on the steady state solutions was first ascertained. Very little difference is observed between the two sets of Nu and Sh. Although the transient results are influenced by $\sigma$ and $\phi'$, steady state solution are not. Hence, the values of $\sigma$ and $\phi'$ each was set to unity without introducing any error in the results to follow.

To observe the effect of various parameters on rate of heat and mass transfer coefficients the results are presented through graphs. To examine the structure of thermal and concentration fields, Nu and Sh are plotted against buoyancy ratio in Figure 2 and Figure 3. Trends show how the thermal and solutal boundary layers react to change in the buoyancy ratio $N$ for various values of mass flux parameter $vrb$, $Le = 1$ and $Ra = 200$. The magnitude of buoyancy ratio $N$, indicates the relative strengths of the two buoyant forces and the algebraic sign provides information of relative direction of the two forces. While the thermal buoyancy always acts vertically upward, the solutal buoyancy may act in either direction depending on the relative molecular weights, with a heavier species contributing a negative buoyant force that acts vertically downward, resulting in negative sign on the buoyancy ratio. For $N < 0$, the buoyancy due to species diffusion is negative and acts vertically downward thereby opposing the vertically upward thermal buoyancy. Figures predict that Nusselt and Sherwood numbers decrease as mass
flux parameter goes from suction to injection domain and $N$ goes from $N = 5$ to $N = -1$ but after $N = -1$ it increases from injection to suction domain. For $N = -1$, there is no flow condition, and the transport takes place only by diffusion. $N = 0$ indicates that convection is due to thermal buoyancy alone.

The rate of diffusions ($Le = \frac{D_t}{D_c}$) determine the relative extent of the thermal and concentration fields from vertical surface through fluid saturated porous media. A value of $Le > 1$ implies that the extent of the two thermal diffusion from the vertical surface is larger than that of solutal diffusion. $Le < 1$ indicates that these thicknesses are reversed. Figure 4 and 5 indicate that for positive buoyancy ratio the rate of heat transfer $Nu$ decreases with $Le$ while $Sh$ increases, however for negative buoyancy ratio the results are same (see figures 6,7).

Figures 8, 9 are plotted against mass flux parameter. They predict that $Nu$ and $Sh$ increase as mass flux parameter move from injection to suction domain. As in the case of thermally driven flow, suction of the fluid at the wall leads to thinner boundary layer such that rate of heat transfer increased while injection of the fluid at the wall leads to thicker boundary layer such that it is decreased than no suction or injection limit.
Bibliography


Fig. 1 The Regime in which the Phenomenon of Free Convection Heat and Mass Transfer exist.
- Fig. 2 Heat Transfer Coefficient as a Function of Buoyancy Ratio.
Fig. 3 Mass Transfer Coefficient as a Function of Buoyancy Ratio.
Fig. 4 Heat Transfer Coefficient as a Function of Lewis Number for $N = 0.5$. 

$N = 0.5, \; Ra = 200$

- $\varphi = -1$
- $\varphi = 0$
- $\varphi = +1$
Fig. 5 Mass Transfer Coefficient as a Function of Lewis Number for $N = 0.5$. 

$N = 0.5$, $Ra = 200$

- $\varphi = -1$
- $\varphi = 0$
- $\varphi = +1$
Fig. 6 Heat Transfer Coefficient as a Function of Lewis Number for $N = -1.5$. 

$N = -1.5$, $Ra = 200$

- $\text{varb} = -1$
- $\text{varb} = 0$
- $\text{varb} = +1$
Fig. 7 Mass Transfer Coefficient as a Function of Lewis Number for $N = -1.5$. 

N = -1.5, Ra = 200

- $\text{varb} = -1$
- $\text{varb} = 0$
- $\text{varb} = +1$

- $Sh$
- $Le$
Fig. 8 Heat Transfer Coefficient as a Function of Mass Flux Parameter Varb.
Fig. 9 Mass Transfer Coefficient as a Function of Mass Flux Parameter Varb.