Abstract of the Proposed Ph.D. Thesis

Title: Some Topological Properties with Selection Principles and Star Covering

1 Origin of the Problem

For any two collections $\mathcal{A}$ and $\mathcal{B}$ of subsets or family of subsets of a set $X$, a selection principle is a procedure applied to $\mathcal{A}$ to arrive at $\mathcal{B}$. Selection principle theory is a field of mathematics having a rich history going back to the papers by Borel, Menger, Hurewicz, Rothberger (1920 to 1930’s). A systematic investigation of Scheepers attracted a large number of Mathematicians in this area and the investigations increased very rapidly.

The method of star operator has been used to study the problem of metrization of topological spaces and is used in the definitions of several important classical topological notions. Kočinac is the first person who used the star operator for defining the selection principles. Classical selection principles, selective versions of classical selection principles and star-selection principles has been studied extensively by many mathematicians in recent days. Some important selection principles in the field of topology are $S_1(\mathcal{A}, \mathcal{B}), S_{fn}(\mathcal{A}, \mathcal{B}), S_f^1(\mathcal{A}, \mathcal{B}), S_{fn}^1(\mathcal{A}, \mathcal{B})$ etc. Nowadays, this theory has deep connections with many branches of Mathematics such as set theory and general topology, set theoretic topology, game theory, ramsey theory, function spaces and hyperspaces, cardinal invariants, dimension theory, uniform structures, topological groups and relatives etc.

In the definition of star operator, it is observed that starting from a collection $\mathcal{A}$ to arrive to a collection $\mathcal{B}$ another collection is required with respect to which star operator will operate. So it is natural to study star-selection principles which deal with three collections, which is almost ignored. $SS_{K}^1(\mathcal{A}, \mathcal{B})$ is the only such selection principle introduced by Kočinac. We are interested to investigate some unrevealed theories and corresponding topological properties in this direction.
2 Aims and Objectives of the Proposed Research Work:

The main objects of the proposed research work are:

- To introduce some new types of selection principle.
- To introduce topological properties with the introduced selection principle with star covering.
- To relate the new theories with some of the existing theories.
- To introduce new types of topological games and to study them.

3 Abstract:

For a set $X$, if $A \subseteq X$ and $U \subseteq \mathcal{P}(X)$ we denote and define the star of $A$ with respect to $U$ as $St(A, U) = \cup\{U \in U : A \cap U \neq \emptyset\}$.

In the proposed thesis two new selection principles $\mathcal{U}_1(A, B)$ and $\mathcal{U}_{fin}(A, B)$ with star operator as defined above were introduced with two collections $A$ and $B$ which are collection of families of subsets of a set $X$.

Another two new selection principles $SS_{C,1}(A, B)$ and $SS_{C,fin}(A, B)$ with three collections $A$, $B$ and $C$ and star operator were introduced, where $A$ is family of subsets and $B, C$ are family of family of subsets of a set $X$.

Some set properties of the above selection principles are studied along with properties when $X$ is a topological space and $A, B, C$ topologically significant collections like collection of all open covers ($\mathcal{O}$), collection of all $\gamma$ covers ($\Gamma$), maximal cellular family ($\mathcal{MC}$), collection of dense sets ($\mathcal{D}$) etc.

The topological spaces satisfying the selection principles $SS_{C,fin}(\mathcal{D}, \mathcal{O})$ is termed as selectively star-Lindelöf or $M$-star Lindelöf spaces, $SS_{C,1}(A, B)$ as $SS_{O,fin}(\mathcal{M}_C, \mathcal{O})$ $R$-star Lindelöf a selectively star-ccc spaces. We have studied some topological properties as well some heredity and preservation were studied for these spaces.
Next, we have generalized the notion of selectively star-Lindelöfness, $R$ star-Lindelöfness and selectively star-ccc-ness to $k$-star-Lindelöfness, $R$ $k$-star-Lindelöfness and selectively $k$-star-ccc-ness where $k \in \omega \setminus \{0\} = \mathbb{N}$ a positive integer using iterated star operator $St^{k+1}(A \mathcal{U}) = St(St^k(A \mathcal{U}))$ where $St^1(A \mathcal{U}) = St(A \mathcal{U})$. Also we have studied some topological properties of such spaces.

Lastly we have defined the topological games or simply games directly associate to the selection principle defined in this proposed thesis. The games are $^*G_1(\mathcal{A}, \mathcal{B})$ and $^*G_{fin}(\mathcal{A}, \mathcal{B})$, $SG_{\mathcal{C},1}(\mathcal{A}, \mathcal{B})$ and $SG_{\mathcal{C},fin}(\mathcal{A}, \mathcal{B})$. We have studied some properties of these games as well as some properties as topological games.

At the end of the proposed thesis a concluding chapter is made with the outcome of the research work along with a list of unsolved problems for further research.

Signature of the candidate.  
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