CHAPTER 1

Introduction

1.1 Introduction

“Each venture is a new beginning to explore something hidden.”

Statistical analysis aims to provide a satisfactory model for a given data set of observations, using valid assumptions about the underlying process. Usually a choice has to be made between special cases of distributions versus the more general versions. In this thesis, statistical properties of some newly introduced distributions are explored and their applications in various areas are considered. In particular, different autoregressive mini-

Extreme value distributions (EVD) are usually considered to comprise of three families namely Gumbel, Frechet and Weibull distributions, also known as type I, type II and type III extreme value distributions. The importance of EVD arises from the fact that they are ob-
tained as the limit distribution of the maxima of a sequence of independent and identically
distributed random variables. Similar results can be obtained for minima, and vice versa.
Of these families of distributions, type 1 is the most commonly referred to in discussions of
extreme values. It has been applied either as the parent distribution or as an asymptotic
approximation, to describe extreme wind speeds, sea wave heights, floods, rainfall, age at
death, minimum temperature, rainfall during droughts, electrical strength of materials, air
pollution problems, geological problems, naval engineering etc. The Extreme value distri-
butions were first derived by Fisher and Tippett (1928) in ‘Limiting forms of the frequency
distribution of the largest or smallest member of a sample’. Gumbel applied extreme value
theory on real world problems in engineering and in meteorological phenomena such as
annual flood flows. According to Gumbel (1958) “ it seems that the rivers know the theory.
It only remains to convince the engineers of the validity of this analysis.”

A time series is a sequence of observations taken sequentially in time. Examples of
data sets appear as time series of hourly observations made on the yield of a chemical
process, a weekly series of the number of road accidents, a monthly sequence of the
quantity of goods shipped from a factory, etc. Time series analysis comprises of methods
for analyzing time series data in order to extract meaningful statistics and other character-
istics of the data. Time series models are also useful in simulation studies. For example,
the performance of a reservoir depends heavily on the random daily inputs of water to the
system. If these are modeled as a time series, then we can use the fitted model to simulate
a large number of independent sequences of daily inputs. Knowing the size and mode of
operation of the reservoir, we can determine the fraction of the simulated input sequences
that cause the reservoir to run out of water in a given time period. This fraction can be
used as be an estimate of the probability of emptiness of the reservoir at some time in the
given period.

In autoregressive models, the current value of the process is expressed as a finite,
linear combination of previous values of the process. The standard form of an autoregres-
sive model of order p, denoted by AR(p) is given by
\[ X_t = a_1 X_{t-1} + a_2 X_{t-2} + \ldots + a_p X_{t-p} + \epsilon_t \]
where \( \{ \epsilon_t \} \) are independent and identically distributed random variables called innovations and \( a_1, a_2, ..., a_p \) are fixed parameters, with \( a_p \neq 0 \). Also \( \epsilon_t \) is independently distributed of \( X_{t-1}, X_{t-2}, ... \). A first order autoregressive time series model with exponential stationary marginal distribution was developed by Gaver and Lewis (1980). Sim (1986) conducted a simulation study of Weibull and gamma autoregressive stationary processes.

The theory of reliability deals with the failure law of systems. Whenever the system is put in operation, sometime or other it is bound to fail either due to external causes or due to internal structural defects. In reliability, the stress strength model describes the life of a component having a random strength (Y) subjected to a random stress (X). The component survives as long as Y is greater than X. The corresponding chance of occurrence, \( R = P(X < Y) \), is known as the reliability of the component. Reliability is the probability of a device performing its intended function satisfactorily for a specified period of time. Life-lengths of man-made devices or of biological organisms are the main focus of reliability and survival analysis. But waiting times for delays in traffic, intervals between earthquakes or floods, or time periods required for learning a task also arise in its applications. System elements are connected in series and/or in parallel. For example, in electric circuits, two resistors may be connected in series or in parallel. In electric power distribution, two generators may be used in parallel to supply energy.

1.2 Review of Literature

1.2.1 Marshall-Olkin Distributions

Adding parameters to a well established family of distributions is a time honored device for obtaining more flexible new families of distributions. For instance the family of Weibull distributions contains exponential distribution and is constructed by taking powers of exponentially distributed random variables. An ingenious general method of adding a parameter to a family of distributions is introduced by Marshall and Olkin (1997). For a random variable with a distribution function \( F(x) \), we can obtain a new family \( G(x) \) which contains one
more parameter is given by

\[ G(x) = \frac{F(x)}{\alpha + (1 - \alpha)F(x)}, \, x \in R \]

where \( F \) is a distribution function and \( \alpha > 0 \). If \( \alpha = 1 \) then we have \( G = F \).

The corresponding survival function is

\[ \overline{G}(x) = \frac{\alpha F(x)}{1 - (1 - \alpha)F(x)}; \, -\infty < x < \infty; \, 0 < \alpha < \infty. \]

In bivariate case if \((X, Y)\) be a random vector with joint survival function \( F(x, y) \), then

\[ G(x, y) = \frac{F(x, y)}{\alpha + (1 - \alpha)F(x, y)}; \, -\infty < x < \infty; \, -\infty < y < \infty; \, 0 < \alpha < \infty. \]

constitute Marshall-Olkin bivariate family of distributions.

Then the corresponding survival function is

\[ \overline{G}(x, y) = \frac{\alpha F(x, y)}{1 - (1 - \alpha)F(x, y)}; \, -\infty < x < \infty, -\infty < y < \infty; \, 0 < \alpha < \infty. \]

The new parameter \( \alpha \) results in added flexibility of distributions and influence the reliability properties.


1.2.2 Time Series

A time series is a set of observations $x_t$ each one being recorded at a specific time $t$. A discrete-time time series is one in which the set $T_0$ of times at which observations are made is a discrete set, as is the case, for example, when observations are made at fixed time intervals. Continuous time time series are obtained when observations are recorded continuously over some time interval, e.g., when $T_0 = [0, 1]$. An important part of the analysis of a time series is the selection of a suitable probability model (or class of models) for the data. To allow for the possibly unpredictable nature of future observations it is natural to suppose that each observation $x_t$ is a realized value of a certain random variable $X_t$. For the interpretation of economic statistics such as unemployment figures, it is important to recognize the presence of seasonal components and to remove them so as not to confuse them with long-term trends.

The time series $\{X_t\}$ is said to be stationary if, for any $t_1, t_2, \ldots, t_n \in Z$, and $n = 1, 2, \ldots$, and for all $k > 0$.

$$F_{x_{t_1}, x_{t_2}, \ldots, x_{t_n}}(x_1, x_2, \ldots, x_n) = F_{x_{t_1+k}, x_{t_2+k}, \ldots, x_{t_n+k}}(x_1, x_2, \ldots, x_n)$$

where $F$ denotes the distribution function of the set of random variables which appear as suffices. This is called stationarity in the strict sense. A process $\{X_n\}$ is weakly stationary if the mean and variance of $X_t$ remain constant over time and the covariance between any two values $X_t$ and $X_s$ depends only on the time difference and not on their individual time points.

To assess the degree of dependence in the data and to select a model for the data that reflects this, one of the important tools we use is the sample autocorrelation function (sample ACF) of the data. If we believe that the data are realized values of a stationary time series $X_t$, then the sample ACF will provide us with an estimate of the ACF of $X_t$. 
This estimate may suggest which of the many possible stationary time series models is a suitable candidate for representing the dependence in the data. Krishna and Jose (2011) introduced and studied the applications of Marshall- Olkin Laplace model in time series modeling.

1.2.3 Autoregressive minification processes

One of the simplest and widely used time series models is the autoregressive models and it is well known that autoregressive processes of appropriate orders are extensively used for modeling time series data. Autoregressive models are developed with the idea that the present value of the series, $X_t$ can be explained as a function of past values namely, $X_{t-1}, X_{t-2}, ..., X_{t-p}$ where $p$ determines the number of steps in to the past, needed to forecast the current value. The standard form of an autoregressive model of order $p$, denoted by AR(p) is given by $X_t = a_1 X_{t-1} + a_2 X_{t-2} + \ldots + a_p X_{t-p} + \epsilon_t$ where $\epsilon_t$ are independent and identically distributed random variables called innovations and $a_1, a_2, ..., a_p$ are fixed parameters, with $a_p \neq 0$. Also $\epsilon_t$ is independently distributed of $X_{t-1}, X_{t-2}, ...$. A first order autoregressive time series model with exponential stationary marginal distribution was developed by Gaver and Lewis (1980). Sim (1986) conducted a simulation study of Weibull and gamma autoregressive stationary processes. Tavares (1980) introduced an autoregressive process of the form

$$X_n = k \min(X_{n-1}, \epsilon_n); n \geq 1 \quad (1.2.1)$$

where $k > 1$ is a constant and $\{\epsilon_n\}$ is a sequence of i.i.d random variables such that $\{X_n\}$ is a stationary Markov process with a specified marginal distribution function $F(x)$. The process $\{X_n\}$ is called an AR (1) minification process.

The work of Tavares was motivated by hydrological considerations, for example, modeling of run-off data. These data tend to have long tails and thus cannot be modeled by the linear exponential autoregressive EAR(1) processes of Gaver and Lewis (1980). Weibull or extreme value random variables are commonly used for modeling the marginal distribu-
tion functions of run-off series, but processes with these marginal distributions cannot be generated with linear autoregressive models. Thus autoregressive minification processes are another class of time series models useful for modeling many real contexts.

Lewis and McKenzie (1991) gave a detailed account on the theory and applications of minification processes. If the survival function of \( X_n \) is \( \bar{F}_x(x) \) for \( n > 0 \), then the survival function of \( \epsilon_n \) is obtained such that

\[
\bar{F}_{\epsilon_n}(x) = \frac{\bar{F}_{X(kx)}(x)}{\bar{F}_X(x)}; x \geq 0, k > 1.
\]

is a proper survival function. But if \( \bar{F}_{\epsilon_n}(x) \) is not strictly a proper survivor function having an atom of probability \( p \) located at infinity, then we can write (1) as

\[
X_n = \begin{cases} 
kX_{n-1} & \text{with probability } p \\
\min(X_{n-1}, \epsilon_n^*) & \text{with probability } 1 - p; 0 < p < 1 \end{cases}
\]  

(1.2.2)

In this case we have,

\[
\bar{F}_{\epsilon_n^*}(x) = \frac{\bar{F}_{\epsilon_n}(x) - p}{1 - p}
\]

All the properties of the process (1.2.1) can be derived similarly, provided

\[
\bar{F}_{\epsilon_n}(x) = p + (1 - p)\bar{F}_{\epsilon_n^*}(x)
\]

The Pareto distribution provides an example in which the form (1.2.2) is required rather than (1.2.1). Little John (1992) considers discrete minification processes in the case of integer valued time series having discrete stationary marginal distributions and discusses on time reversibility.

Arnold and Robertson (1989) constructed a minification process having logistic marginal
distribution. Such minification processes are having the general structure given by

\[
X_n = \begin{cases} 
\epsilon_n & \text{with probability } p \\
\min(X_{n-1}, \epsilon_n) & \text{with probability } 1 - p; 0 < p < 1 
\end{cases}
\]

where \(\{\epsilon_n\}\) is a sequence of i.i.d random variables such that \(\{X_n\}\) is a stationary Markov process with a given marginal distribution. Jayakumar and Thomas (2004) developed autoregressive semi-logistic processes. Balakrishna (1998) proposed different estimates for the parameters of semi-Pareto and Pareto autoregressive minification processes. Minification processes with discrete marginals is discussed by Kalamkar (1995).

Kuttykrishnan and Jayakumar (2006) and Ristic (2006) independently generalized the two parameter semi-Pareto minification process to a semi-Pareto minification process with three parameters. This model is defined by

\[
X_n = \begin{cases} 
\epsilon_n & \text{w.p. } q \\
p^{-1/\alpha_1}X_{n-1} & \text{w.p. } p(1-q) \quad , \quad n \geq 1. \\
\min(p^{-1/\alpha_1}X_{n-1}, \epsilon_n) & \text{w.p. } (1-p)(1-q)
\end{cases}
\]

where w.p. means “with probability”.

Several bivariate minification processes are also defined by many authors in recent years. Bivariate minification processes with Marshall-Olkin bivariate semi-Pareto and Pareto distributions are introduced and their properties are studied by Alice and Jose (2004). Balakrishna and Jayakumar (1997) introduced a bivariate minification process of first order \(\{(X_n, Y_n), n \geq 0\}\) given by

\[
X_n = \min(p^{-1/\alpha_1}X_{n-1}, \epsilon_n)
\]

\[
Y_n = \min(p^{-1/\alpha_2}Y_{n-1}, \eta_n)
\]

where \(\{(\epsilon_n, \eta_n)\}\) is a sequence of i.i.d. nonnegative random vectors, \((X_0, Y_0)\) and \(\{(\epsilon_i, \eta_i),\) \(i \geq 1\}\) are independent random vectors, \(0 < p < 1, \alpha_1 > 0, \alpha_2 > 0\).
Ristic (2006) considered a class of stationary bivariate minification processes and the process \((X_n, Y_n)\) defined by
\[
X_n = k_1 \min(X_{n-1}, Y_{n-1}, \epsilon_n) \\
Y_n = k_2 \min(X_{n-1}, Y_{n-1}, \eta_n)
\]
where \((\epsilon_n, \eta_n)\) is a sequence of i.i.d nonnegative non-degenerate random vectors with common survival function \(G(x, y)\), random vectors \((X_0, Y_0)\) and \((\epsilon_1, \eta_1)\) are independent and \(k_1 > 1\), \(k_2 > 1\). Jose et al. (2011) consider a bivariate Marshall Olkin Weibull processes. Krishana et al. (2011) discuss various applications of Marshall Olkin Frechet distribution and process.

1.2.4 Acceptance Sampling Plan

Acceptance sampling is the process of evaluating a portion of the product/material in a lot for the purpose of accepting or rejecting the lot as either conforming or not conforming to a quality specifications. Inspection for acceptance purpose is carried out at many stages in manufacturing. There are generally two ways in which inspection is carried out: (i) 100% inspection. (ii) Sampling inspection. Sampling inspection can be defined as a technique to determine the acceptance or rejection of a lot or population on the basis of number of defective parts found in a random sample drawn from the lot. If the number of defective items does not exceed a predefined level, the lot is accepted, otherwise it is rejected. In acceptance sampling inspection a defective article is defined as one that fails to conform to specifications in one or more quality characteristics. A common procedure in acceptance sampling is to consider each submitted lot of product separately and to base the decision on acceptance or rejection of the lot on the evidence of one or more samples chosen at random from the lot. If the quality of the product inspected is the lifetime of the product that is put for testing, after the completion of sampling inspection what we have is a sample of life times of the sampled products. If the genuine products are rejected on the basis of sample information, this error is called type-1 error. On the other hand, if the genuine
products are not accepted by the consumer, this error is type-2 error. If a decision to accept or reject the lot are subjected to the risks associated with the two types of errors, this procedure is termed as ‘acceptance sampling based on life tests’ or ‘reliability test plans’. In judging various acceptance sampling plans it is desirable to compare their performance over a range of possible quality level of submitted product.


1.2.5 Reliability and Stress-strength analysis

Let $T$ be a continuous non negative random variable that represents lifetime of individual items in a specified population. The probability that an individual item will survive until at least time $t$ is given by the reliability function

$$R(t) = P(T \geq t) = \int_t^{\infty} F(x)dx$$

The instantaneous failure rate often referred to as the hazard function

$$h(t) = \frac{f(t)}{R(t)}$$
reverse hazard rate is defined as

\[ r(t) = \frac{f(t)}{F(t)} \]

Newby (1986) gave some simple explanations and examples of analysis in terms of mean residual life, hazard and hazard rate average. The theory of reliability deals with the failure law of systems. Whenever the system is put in operation, sometime or other it is bound to fail either due to external causes or due to an internal structural defect. In reliability, the stress strength model describes the life of a component having a random strength (Y) and is subjected to a random stress (X). The component survives as long as Y is greater than X. The corresponding chance of occurrence, \( R = P(X < Y) \), is known as the reliability of the component. The germ of this idea was introduced by Birnbaum (1956) and developed by Birnbaum and Mc Carty (1958). The formal term “stress-strength” appears in the title of Church and Harris (1970). The situations where (X,Y) follows bivariate exponential (BVE) distribution has been discussed by Jana (1994) and Hanagal (1995). Hanagal (1996) estimated the reliability of a two component parallel system, subjected to a common stress. Inference on reliability in two parameter exponential stress-strength model is discussed by Krishnamoorthy et al. (2006). Stress-strength reliability under two component stress system is discussed by Mukherjee and Maiti (2005). Estimation of reliability in multi-component stress-strength model following Log logistic distribution is considered by Rao and Kantam (2010). Reliability of stress-strength models with a bivariate exponential distribution is discussed by Mokhlis (2006). Kundu and Gupta (2006) studied estimation of \( P(Y < X) \) for the generalized exponential distribution. Bindu (2011) studied reliability of double Lomax distribution. Jose and Rani (2010) introduced Gumbel models for stress-strength analysis.
1.3 Bivariate failure rate

The bivariate failure rate at \((x_1, x_2)\) defined by Basu (1971) is

\[
a(x_1, x_2) = \frac{f(x_1, x_2)}{R(x_1, x_2)}; \text{ for } x_1 > 0, x_2 > 0
\]

Another approach to defining bivariate failure rate is by Johnson and Kotz (1975), where it is taken as the vector valued function,

\[
h(x_1, x_2) = \left( \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2} \right) H(x_1, x_2)
\]

\[
= \left( \frac{\partial H}{\partial x_1}, \frac{\partial H}{\partial x_2} \right)
\]

\[
= \left( h_1(x_1, x_2), h_2(x_1, x_2) \right)
\]

where \( H = -\log R(x_1, x_2) \).

According to Galambos and Kotz (1978), \( a(x_1, x_2) \) defined above can be expressed as

\[
a(x_1, x_2) = \frac{\partial H}{\partial x_1} + \frac{\partial H}{\partial x_2} + \frac{\partial^2 H}{\partial x_1 \partial x_2}
\]

where

\[
H = H(x_1, x_2) = -\log R(x_1, x_2)
\]

1.3.1 Morgenstern family of distributions

The Morgenstern family of distributions or Farlie- Gumbel- Morgenstern system of distributions discussed by Kotz et al. (2000) provides a flexible family that can be used in construction of bivariate distributions with given marginals and is specified by the distribution function

\[
F(x, y) = G(x)H(y) \left[ 1 + \alpha(1 - G(x))(1 - H(y)) \right]
\] (1.3.1)
having $G(x)$ and $H(y)$ as marginal distribution functions. The corresponding probability density function is

$$f(x, y) = g(x)h(y) \left[ 1 + \alpha (2G(x) - 1)(2H(y) - 1) \right]$$

where $g(x)$ and $h(y)$ are the densities of marginal distributions. This system of distributions has a considerable appeal in model building since it provides a convenient way to construct a joint distribution with specified marginals. Johnson and Kotz (1975) introduced a multivariate extension of bivariate system of distributions. The distribution of concomitants of record values arising from Morgenstern type bivariate logistic distribution is studied by Chacko and Thomas (2006).

### 1.3.2 $q$-type Distributions

In order to enable a transition from one functional form to another, a pathway parameter $q$ is introduced and a pathway model is created in Mathai (2005). For the real scalar case the pathway density can be written in the form

$$f_1(x) = c_1 |x|^{\gamma \left[ 1 - a(1 - q)|x|^\delta \right]} \frac{\eta^\frac{\gamma}{\gamma-q}}{\Gamma (\frac{\gamma}{\gamma-q})},$$

$$a > 0, 1 - a(1 - q)|x|^\delta \geq 0, \eta > 0, q < 1$$

and

$$f_2(x) = c_2 |x|^{\gamma \left[ 1 + a(q - 1)|x|^\delta \right]} \frac{\eta^\frac{\gamma}{\gamma+1}}{\Gamma (\frac{\gamma}{\gamma+1})},$$

$$a > 0, -\infty < x < \infty, \eta > 0, q > 1$$

where $c_1$ and $c_2$ are normalizing constants. This distribution includes type-1 beta, type-2 beta, gamma, Weibull, Gaussian, Cauchy, exponential, Rayleigh, Student-$t$, logistic, etc. As $q \rightarrow 1$, $f_1(x)$ and $f_2(x)$ tend to $f_3(x)$, which is the generalized gamma distribution where $f_3(x)$ is given by

$$f_3(x) = \frac{\delta (a\beta)^\frac{\gamma}{\delta}}{2\Gamma (\frac{\gamma}{\delta})} |x|^{\alpha-1} \exp(-a\beta |x|^\delta); -\infty < x < \infty; a, \alpha, \beta, \delta > 0.$$  (1.3.2)
For different values of the parameters in pathway model, we get different distributions like Weibull, gamma, beta type-1, beta type-2, etc. By taking $\delta = \alpha, \beta = 1, a = \lambda \alpha$ in $f(x)\delta$ the pathway model reduces to the $q$-Weibull distribution. The $q$ - exponential distribution tends to the exponential distribution as $q$ tends to 1. Recently various authors have introduced several $q$-type distributions namely $q$-exponential, $q$-Weibull, $q$-logistic etc. Mathai and Provost (2006) developed models that generalize the type-1 and type-2 beta distributions along with their logistic counterparts. Jose et al. (2010) introduced Marshall Olkin $q$-Weibull distribution and Max-Min processes. Semi $q$- Weibull distribution and autoregressive processes are discussed by Naik et al. (2010). Jose et al. (2009) introduced and studied the properties of $q$- Weibull distribution.

1.3.3 Gumbel distribution

Extreme value distributions are usually considered to comprise the following three families Gumbel, Frchet and Weibull also known as type I, type II and type III extreme value distributions. The extreme value distributions were first derived by Fisher and Tippett (1928) in ‘Limiting forms of the frequency distribution of the largest or smallest member of a sample’. Gumbel (1958) made several significant contributions to the extreme value analysis, most of them are detailed in his book ‘Statistics of Extremes’. Of these families of distributions, Type 1 is the most commonly referred to in discussions of extreme values. Indeed some authors call type 1, the extreme value distribution. In probability and statistics, the Gumbel distribution is used to model the distribution of the maximum (or the minimum) of a number of samples of various distributions. It is useful in predicting the chance that an extreme earthquake, flood or other natural disaster will occur. Its importance arises from the fact that it is the limit distribution of the maxima of a sequence of independent and identically distributed random variables. The importance of the Gumbel distribution in practice is due to its extreme value behavior. It has been applied either as the parent distribution or as an asymptotic approximation, to describe extreme wind speeds, sea wave heights, floods, rainfall, age at death, minimum temperature, rainfall during droughts, electrical strength of materials, air pollution problems, geological problems, naval engineering etc. A gener-
alization of the Gumbel distribution is discussed by Adeyemi and Ojo (2003). Inference for \( P(Y < X) \) in exponentiated Gumbel distribution is studied by Kakade et al. (2008) and exponentiated Gumbel distribution for estimation of return levels of significant wave height is considered by Persson and Ryden (2010). Estimation of the extreme value type 1 distribution by the method of L-Q moments is discussed by Shabri and Jemain (2009).

### 1.4 Summary of the Present Work

The present study has been undertaken with the following specific objectives:

1. To introduce and study various generalizations of probability distributions using Marshall-Olkin technique.

2. To study various properties of these new distributions and apply them to simulated as well as real data sets.

3. To develop autoregressive minification processes using these distributions and explore their properties.

4. To estimate reliability under stress-strength model and examine the validity of the estimate by extensive simulation studies.

5. To apply these distributions in various areas such as time series modeling, stress-strength analysis, reliability analysis, acceptance sampling plans etc.

6. To develop algorithms and R programs for generating the random variables as well as processes.

7. To develop inference procedures for estimation of parameters and validate these with respect to real data sets.

8. To extend the models to bivariate and multivariate contexts.
The research concentrates on developing various extensions of probability distributions and applying these distributions in the area of stochastic modeling especially with respect to autoregressive minification processes. The thesis is organized into 8 chapters.

Chapter 1 deals with introduction and a brief summary of the thesis. In chapter 2, as a generalization of Gumbel distribution, the Marshall-Olkin Gumbel distribution is introduced and its properties are studied. Minification processes with Marshall-Olkin Gumbel marginal distribution are also developed and studied. We apply the models to a real data set and compare Marshall Olkin Gumbel maximum distribution with Gumbel maximum distribution and establish that the Marshall Olkin Gumbel maximum distribution is a better fit. The new distributions and processes are used for modeling extreme value data on climate changes and environmental statistics.

In chapter 3, as generalizations of Gumbel distribution, the Marshall-Olkin Gumbel maximum and minimum distributions as well as the Marshall-Olkin q-Gumbel distribution are introduced and their properties are studied. Minification processes with Marshall - Olkin q-Gumbel marginal distribution is also developed and studied. In chapter 4, we develop reliability test plans for acceptance/rejection of a lot of products submitted for inspection with lifetimes governed by the Marshall-Olkin Gumbel maximum distribution. The results are illustrated by a numerical example on ordered failure times associated with the release of a software. In chapter 5, some generalizations of Gompertz distribution is considered. Minification processes with Marshall-Olkin Gompertz distribution is also developed. Various results are obtained.

In chapter 6, Extended Marshall-Olkin bivariate exponential distribution is introduced and its properties are studied. Expressions for stress- strength reliability are obtained. Estimation of parameters is done. Multivariate extensions as well as computation of R for various parameter combinations are also done. We introduce three different forms of minimification processes and necessary and sufficient conditions for stationarity are established.

In chapter 7, Marshall-Olkin Gumbel bivariate exponential distribution is introduced and its properties are studied. Expressions for stress- strength reliability $R = P(X < Y)$
of a two component system is derived, where stress \((X)\) and strength \((Y)\) are independently distributed and stress \((X)\) consists of two components namely \(X_1\) and \(X_2\). Components of stress are assumed to be in series or in parallel. The reliability of the system when the components \(X_1\) and \(X_2\) have Morgenstern Gumbel bivariate and Marshall-Olkin Gumbel bivariate exponential distributions are derived and evaluated for various parameter values. Two different minification processes with Marshall-Olkin Gumbel bivariate exponential marginal distribution are also developed and studied. The reliability values are also computed for different parameter values. Expression for the bivariate failure rate is also derived. Using these, bivariate system reliability can be estimated and also multi-component systems having optimum reliability properties can be designed.

In chapter 8 a new bivariate distribution called Marshall-Olkin Morgenstern bivariate Weibull distribution is introduced and studied. Two different models of minification processes with the above bivariate distribution as stationary marginal distribution are developed. It is shown that the process is strictly stationary. The properties of the process are derived. The expressions for reliability under stress-strength analysis when the components are in series and parallel are obtained. The process is extended to \(p^{th}\) order as well as \(k\)-variate cases.

Thus the thesis deals with a number of univariate and bivariate distributions and their applications in autoregressive time series modeling through minification processes, reliability theory, stress-strength analysis, acceptance sampling etc.

The following research papers have been published/presented in conferences/communicated for publication.

Presentations


2. Presented a paper titled “Marshall-Olkin Morgenstern Weibull distribution: Generalizations and Applications” in the International Conference on Actuarial Statistics,
Bio Statistics and Stochastic Modeling at Kannur University, during January 10-14, 2011.


5. Accepted for presentation the paper titled “Generalizations of Gumbel distributions and their Applications” in the 31st annual Conference of Indian Society for Probability and Statistics (ISPS) and International Conference on Statistics, Probability and Related Areas, at Cochin University of Science and Technology, during December 19-22, 2011.

Publications


References


