Chapter 7

CHANNELING RADIATION

7.1 INTRODUCTION

When charged particles propagating through a medium get accelerated or deccelerated through various physical processes, electromagnetic radiation is emitted. This is a well known fact in *electrodynamics theory*. As mentioned in Chapter 1, the longitudinal motion of a channeled particle [1,2] is assumed to nearly free particle type motion (neglecting energy losses for short longitudinal distance, compared to particle energy) and only the transverse motion of particles is governed by the continuum transverse potential. The channeled particles execute to and fro oscillatory motion in the transverse space and are accelerated and deccelerated in the process resulting in emission of electromagnetic radiation. Quantum mechanically, the bounded transverse motion in the field of continuum potential is quantized and the spontaneous transition among these levels result in the emission of relevant energy as electromagnetic radiation. The calculated transverse oscillation frequencies $\omega_o$ and the corresponding energies are generally very low and the observation of channeling radiation seemed to be impossible because the radiations due to other mechanisms (like *Bremsstrahlung*) are stronger and hence dominate in the same energy range. Later, towards the end of seventies, Kumakhov realized that when a relativistically fast charged particle is channeled in a crystal, the longitudinal relativistic motion results in Lorentz contraction along the axes or planes so that the strength of
continuum potential is enhanced by a factor $\gamma \left( \gamma = 1 / \sqrt{1 - (v^2/c^2)} \right)$. Since the emitted electromagnetic radiation is to be observed in the laboratory frame, the radiation frequency is Doppler shifted in the forward direction by a factor $2\gamma$, so overall enhancement in the frequency, of the order of $2\gamma^2$ results for forward emission [3,4]. Where $v$ is particle velocity and $c$ is velocity of light. The observed frequency is given by $\omega = 2\gamma^2\omega_0$, where $\omega_0$ is the original frequency due to spontaneous transition among the discrete eigenstates supported by the continuum planar or axial potential of the crystal. Experimental verifications of Kumakhov's prediction were followed promptly by the observation of channeling radiation from 56 and 28 MeV positrons [5] and electrons [6] channeled along the major crystallographic axis and planes of silicon single crystal respectively.

Electrons and positrons are of practical interest as projectiles in channeling radiation experiments because classically the instantaneous radiated power is proportional to acceleration which in turn is inversely proportional to mass of the projectile. Since the potential minima for electrons are at the centre of atomic strings or planes, channelled electrons have maximum probability for hard collision with atoms, contrary to positrons case. The planar potential for positrons can be approximated to harmonic type and frequency is almost independent of amplitude, so the energy levels are equidistant. Due to the negative charge of the electrons they cross the atomic planes during their motion, this results in anharmonic interaction between the electrons and atomic plane which gives nonequidistant energy levels and broad range of oscillation frequencies in the spectrum.

Eventhough channeling radiation occurs at higher energies, the transverse motion of particles can be still described by the usual non-relativistic Schrödinger equation. This is because the critical angle $\psi_c$ is inversely proportional to the square root of incident particle energy and consequently the transverse energy $E\psi_c^2$ remains non-relativistic even at high energies. Several theoretical methods using classical and quantum mechanical
treatments for channeling radiation were made and reported during last 15 years or so [8-14].

Most of the initial efforts on channeling radiation work have been towards its application as coherent source as well as to characterize the properties of crystal and channeled particle [15]. The photon energy can be tuned by changing the particle energy. The planar channeling radiation is linearly polarized. At high relativistic particle energies, the observed channeling radiation in the forward direction consists of hard x rays or \( \gamma \) rays concentrated inside a narrow cone of half angle \( 1/\gamma \). The radiation characteristics (frequency, line width etc) are also functions of crystal structure in addition to particle parameters. It is possible to investigate the imperfections in the crystals, to study various processes like defect formation which results in the dechanneling of channeled particles. So it is important to know the details about the radiation characteristics of channeling radiation emitted from a crystal and we have calculated the characteristics of the radiation emitted from relativistic positrons and electrons channeled along the (110) plane in Silicon single crystals using Shell planar potential and discussed in section 7.2 and 7.3 respectively. The effects of dislocations on channeling radiation is discussed in section 7.4 and the applications of channeling radiation are outlined in the last section.

7.2 CHANNELING RADIATION FROM RELATIVISTIC POSITRONS

The positrons move between the crystallographic planes with small amplitude of oscillation and the potential due to the two planes is given by [16,17]

\[
Y_2(x) = Y(l - x) + Y(l + x)
\]

(7.1)

where \( l \) is given by \( l = d_p/2 \) and \( x \) is measured from the halfway point between two planes. We have numerically evaluated equation (7.1) using shell planar potential and curve fitted it to a fourth order polynomial of the form, \( Y_{2}^{\text{poly}}(x) = Y_2(0) + (1/2) k_1 x^2 \)
\[ + \frac{1}{4} k_2 x^4. \]
The coefficients \( k_1 \) and \( k_2 \) (both are in atomic units) are given by \( k_1 = 0.354918 \), \( k_2 = 0.026305 \) respectively. Using harmonic approximation for potential and keeping only up to quadratic term the channeling radiation frequency is given by

\[ \omega = 2 \gamma^2 \sqrt{\frac{k_1}{m_0 \gamma}} \]  \hspace{1cm} (7.2)

where \( m_0 \) is the rest mass of positron. The finite line width due to slightly different energies of the radiation because of the anharmonic effect is calculated using perturbation theory [15] and is determined in terms of a parameter \( \epsilon \) and is proportional to \( n_{\text{max}} \), where \( n_{\text{max}} \) is the maximum number of bound states supported by transverse continuum potential. The transitions are calculated using first order perturbation theory, (This number corresponds to \( n_{\text{max}} = 11 \), \( \epsilon = 0.00885612 \) for \( \gamma = 111 \) and \( n_{\text{max}} = 10 \), \( \epsilon = 0.00937775 \) for \( \gamma = 99 \)).

\[ \hbar \omega_{n,n-1} = \frac{\hbar \omega_0}{1 - \beta \cos \theta} (1 + n \epsilon) \]  \hspace{1cm} (7.3)

For forward emission \( \theta = 0 \) and \( \epsilon \) is given by

\[ \epsilon = \frac{3 \hbar k_2}{4 k_1^3 \sqrt{m_0 \gamma}} \]  \hspace{1cm} (7.4)

The spectral peaks were calculated for 56 MeV and 50 MeV positrons channeled along (110) plane of Silicon single crystal and is compared with experimental results. This is shown in Table 7.1.

7.3 CHANNELING RADIATION FROM RELATIVISTIC ELECTRONS

The planar potential experienced by an electron is attractive and can be approximated to a form [17-19]

\[ Y(y) = Y_0 \exp(-k|y|) + A \]
where $y$ is the distance from the plane, $k$ is the effective screening length, and $A$ is a constant added, so that energy can be fixed to zero at the midway point between two planes. Solving the \textit{Schrodinger equation} using the above mentioned potential, we get the wave function.

$$
\psi_p(y) = J_{\nu_p} \left( Q_o \exp \left( \frac{-k |y|}{2} \right) \right)
$$

(7.6)

where $J_{\nu_p}$ is the \textit{Bessel function} of the first kind and of order $\nu_p$, $Q_o$ and $\nu_p$ are respectively given by

$$
Q_o = \sqrt{\frac{8 m_o \gamma |Y_o|}{k^2 h^2}}
$$

$$
\nu_p = Q_o \sqrt{\frac{E_p - A}{Y_o}}
$$

(7.7)

where $m_o$ is the rest mass of electron, $h$ is the planck constant, $E$ is the energy eigenvalue associated with $\psi_p$. The continuity condition for both odd and even parity states are imposed as

$$
J_{\nu_p}(Q_o) = 0 \quad \text{odd state}
$$

(7.8)

$$
\frac{d}{dy} J_{\nu_p}(Q_o) = 0 \quad \text{even state}
$$

(7.9)

The odd and even parity states are derived from (7.8) and (7.9) respectively and the photon energy can be calculated using the relation

$$
\hbar \omega_{obs} = 2 \gamma^2 \Delta E_p
$$

(7.10)

where $\Delta E_p$ is the difference in eigenvalue for the transition of interest. Equation (7.5) can not provide a good approximation to the potential function when thermal vibration of atoms are considered. The eigen values are calculated using equation (7.5) and the first order perturbation theory has been used to correct these eigen values to account for thermal vibrations of atoms which is dominant near the region $y = 0$ (i.e close to planes).
The shell planar potential is therefore curve fitted to a function of the form [18]

\[ Y(y) = Y_o \exp(-ky) + A; \quad \text{for} \quad u_1 \leq |y| \leq \frac{d_p}{2} \]  
\[ (7.11) \]

\[ Y(y) = Y_1 y^2 - B; \quad \text{for} \quad |y| \leq u_1 \]  
\[ (7.12) \]

where \( u_1 \) is the root mean square displacement due to the thermal vibrations. In actual practice the shell planar potential was curve fitted to the function of the form \( Y_o \exp(-k|y|) \) (this is shown in Fig 7.1) and the constant \( A \) added afterwards so that energy can be normalized to zero at the midway point between two planes. In order to find the value of the parameter \( B \) we convoluted the fitted potential to write [20]

\[ Y_c(y) = \frac{Y_o \exp(k^2 u_1^2/2)}{2} \left( \exp(-k|y|) \ \text{erfc} \left[ \frac{1}{\sqrt{2}} \left( ku_1 - \frac{y}{u_1} \right) \right] 
+ \exp(k|y|) \ \text{erfc} \left[ \frac{1}{\sqrt{2}} \left( ku_1 + \frac{y}{u_1} \right) \right] \right) \]  
\[ (7.13) \]

where \( Y_c(y) \) is the convoluted potential and \( \text{erfc}(y) \) is the complementary error function given by

\[ \text{erfc}(y) = \frac{2}{\sqrt{\pi}} \int_y^\infty \exp(-t^2) \ dt \]  
\[ (7.14) \]

The value of convoluted potential for \( y = 0 \) is taken to be the value of the parameter \( B \), and since the values of parameters \( Y_o, A, B \) are known, the value of the parameter \( Y \) can be found out by equating the potential given in (7.11) and (7.12) at \( y = u_1 \).

The spectral peaks were calculated (Table 7.2) using equations (7.8) (7.9) and (7.10) respectively for shell planar potential for both 56 MeV and 28 MeV electrons channeled along (110) direction of silicon single crystal using IMSL MATH/LIBRARY™, IMSL MATH/LIBRARY Special functions™ and cross checked with Mathematica™ software package, and is compared with experimental results. Corresponding eigenvalues were calculated and are shown in Fig.7.2. Here we have used the following values for the parameters \( Y_o, k, A, Y_1 \) and \( B \). \( Y_o = -27.539568 \ eV, \ k = 25.24414 \ \text{nm}^{-1}, A = 2.440461 \ eV, Y_1 = 184.55627 \times 10^2 \ eV/\text{nm}^2 \) and \( B = 21.387006 \ eV \).
7.4 EFFECTS OF DISLOCATIONS

As mentioned in the section 7.1, the channeling radiation can be used to study imperfections in the crystal. The presence of defects in crystals affects channeling phenomena in general and channeling radiation in particular. The effects are of two types; The obstruction effects like those created by point defects, stacking faults etc and distortion effects like those resulting from dislocations. Here we present the study of effects of dislocations on channeling radiation from 56 \(MeV\) positrons channelled along \(<110>\) axial channel of Silicon single crystals using Shell axial potential taking into account the geometry of the \(<110>\).

Around a dislocation the atomic rows and planes exhibit curvature which will alter the trajectory of the channeled particle and can dechannel the particle altogether if the curvature is large enough to modify the trajectory. The distortion produced due to the presence of dislocation axis decreases as one moves away from the dislocation axis. So the curvature in channel decreases as its distance \(d\) from the dislocation axis increases. All the particles passing through regions where the centrifugal force \(2E\pm /R\) (due to distortion) is greater than the restoring force due to the continuum potential will be dechanneled. The axial potential due to all six strings surrounding an \(<110>\) axial channel of Silicon is given by

\[
U(r') = U(R_o - r') + U(R_o + r') + 2U(\sqrt{R_o^2 + r'^2} + R_o r') + 2U(\sqrt{R_o^2 + r'^2} - R_o r') \tag{7.15}
\]

where \(R_o\) is the radius of the channel, \(r'\) is measured from the channel axis and the cross section of \(<110>\) axial channel has been approximated by a regular hexagon. Thus \(R_o\) is simply taken as distance between channel axis and one of the strings of regular hexagon. This is curve fitted to a polynomial of the form

\[
U_S^{poly}(r) = U_o(p_0 + p_2 r^2 + p_4 r^4) \tag{7.16}
\]
where $r'$ has been replaced by $r$. The Schrodinger equation for transverse motion of a positron channeled along $<110>$ axial channel is a two-dimensional harmonic oscillator equation with potential given by equation (7.15). The observed channeling radiation frequency for perfect crystal case is given by [5]

$$\omega_{obs} = 2 \gamma^{3/2} \sqrt{\frac{2U_p p_2}{m_0}}$$  \hspace{1cm} (7.17)

where $m_0$ is the rest mass of positron. Our calculations are based on Constant curvature model (shown in Fig. 7.3.) for dislocation affected channel proposed by Pathak [23,24]. The dislocations are assumed to introduce a continuous and constant curvature with radius of curvature $R$ in the channel which is given by

$$R = \frac{2\pi^2 d_0^2}{b \cos^3 \phi}$$  \hspace{1cm} (7.18)

where $\phi$ is the angle between channel axis and plane perpendicular to the dislocation axis. The half length $z_c$ of curved part of dislocation affected channels situated at a distance $d_0$ from the dislocation of Burgers vector $b$ are given by [24],

$$z_c = \frac{\pi d_0}{\cos \phi}$$  \hspace{1cm} (7.19)

Here we assume that particles are still outside the dechanneling cylinder [25,26] so that the ions do not get dechanneled but their state of motion is appreciably modified due to the curvature of the channels [24]. The assumption of constancy of curvature is valid only for those channels which are not too close to dislocation core. Thus for an initially (i.e., before encountering the dislocation affected channel) well channeled particle the equation of motion becomes [21]

$$m \ddot{r} + \frac{dU_S^{poly}}{dr} - \frac{2E}{R} = 0$$  \hspace{1cm} (7.20)

second order equation which after integration gives

$$\dot{r} = \sqrt{\left(\frac{2}{m}\right)\left[U_S^{poly}(0) - U_S^{poly}(r) + \left(\frac{2E}{R}\right)r\right]}$$  \hspace{1cm} (7.21)
Because of the additional centrifugal force in the transverse direction, the equilibrium axis shifts from $r = 0$ to $r = r_o$ given by

$$\frac{2E}{R} - \frac{dU_S^{\text{poly}}}{dr} \bigg|_{r=r_o} = 0$$  \hspace{1cm} (7.22)$$

The maximum oscillation amplitude $r_m$ gained by a particle is given by

$$U_S^{\text{poly}}(r_m) - \frac{2E}{R} r_m = U_S^{\text{poly}}(0)$$  \hspace{1cm} (7.23)$$

Substituting the potential (equation 7.16) in equation (7.22) and (7.23), we get cubic equations for $r_o$ and $r_m$ as

$$r_o^3 + \frac{p_2}{2p_4} r_o - \frac{E}{2p_4 RU_o} = 0$$

$$r_m^3 + \frac{p_2}{p_4} r_m - \frac{2E}{p_4 RU_o} = 0$$  \hspace{1cm} (7.24)$$

These equations can be solved analytically [27] and positive roots are given by

$$r_o = 2 \sqrt{\frac{p_2}{6p_4} \sinh \left[ \frac{1}{3} \ln \left( \frac{3E}{2RU_o p_2 \sqrt{p_2}} \right) \right]}$$

$$r_m = 2 \sqrt{\frac{p_2}{3p_4} \sinh \left[ \frac{1}{3} \ln \left( \frac{3E}{RU_o p_2 \sqrt{p_2}} \right) \right]}$$  \hspace{1cm} (7.25)$$

For initially well channeled particles, the oscillation amplitude $r_{\text{amp}}$ gained in harmonic approximation, after crossing the dislocation affected channel is given by [24]

$$r_{\text{amp}} = \frac{4 R_o^2 E b}{3 \pi^3 d_o^2 U_o}$$  \hspace{1cm} (7.26)$$

The corresponding change in the period is obtained from the equation of motion in the transverse space

$$\frac{1}{2} mr^2 + U_S^{\text{poly}}(r) = U_S^{\text{poly}}(r_{\text{amp}})$$  \hspace{1cm} (7.27)$$
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The period of oscillation for transverse motion is then

$$T = 4 \int_0^{r_{\text{amp}}} \frac{dr}{r}$$

(7.28)

The observed channeling radiation frequency is given by

$$\omega_{\text{obs}}^{d} = \frac{4\pi \gamma^2}{T}$$

$$= \omega_{\text{obs}} \sqrt{\left[ \frac{p_2 + 2r_{\text{amp}}^2p_4}{p_2} \right]} \frac{\pi}{2F\left( r_{\text{amp}}/\sqrt{2r_{\text{amp}}^2 + (p_2/p_4)} \right)}$$

(7.29)

where $F$ is the complete elliptic integral of the first kind. Fractional increase in channeling radiation frequency is given by

$$f = \frac{\omega_{\text{obs}}^{d} - \omega_{\text{obs}}}{\omega_{\text{obs}}}$$

(7.30)

we have calculated the fractional increase in channeling frequency using shell model axial potential for $(r_{\text{amp}} = R_o/2)$ and $(r_{\text{amp}} = R_o/4)$ and obtained the values 0.286 and 0.042 respectively. This means that frequencies increase due to concentration of dislocation.

To the best of our knowledge, the only experiment of this kind, ie, study of crystal defects using channeling radiation was reported by Park et.al [28]. The experiment was done in Diamond crystals and the effects of dislocationlike structures (along (100) planar direction) have indicated an increase in channeling radiation peak frequencies from positrons. The concentration of dislocations was too high in type I diamond crystals. Most of the positrons are dechanneled at high dislocation concentration, moreover at these high dislocation concentrations they interact with each other and one sees formation of dislocation loops. The model presented above is for single noninteracting dislocation, ofcourse the results are quantitatively in right direction namely increase in channeling radiation frequency as observed experimentally. But quantitative comparison
is not possible at this stage. We suggest an experiment of channeling radiation in single crystals with moderate dislocation density of the order of $10^9 / \text{cm}^2$.

The other area in which channeling radiation can be used as a tool to study the crystal defects and strains is Semiconductor Superlattices in general and Strained Layer Superlattices (SLS) in particular. As discussed in the last chapter Catastrophic Dechanneling Resonance is one of the methods in ion channeling experiments used extensively for this purpose. It has been already theoretically predicted that [29,30] the channeling radiation peak frequency decreases by a factor of $\cos \Delta \psi$ and line width decreases by a larger factor $\cos^2 \Delta \psi$ in SLS (where $\Delta \psi$ is the tilt at the interface and is a measure of strain). The theory is based on the experimental fact [31] that due to the tilt at successive interfaces, the continuum planar potential is weakened as evidenced by the decrease in the width of channeling dips. Hence if the strains in SLSs become too large (due to increase in layer thickness), they relax to create dislocations resulting in increase of channeling radiation frequencies and line widths. Therefore, one can immediately conclude from channeling radiation experiments, whether or not the misfit defects have been created at the interfaces.

7.5 CONCLUDING REMARKS

The application of channeling phenomena to get monochromatic (and possibly coherent) radiation has essentially revived the interest in coherent radiation sources and their application in Solid State, Nuclear and Atomic Physics [12]. Presently the use of channeling radiation is only of speculative nature [32] and the most exciting and challenging task is the realization of the possibility to some how induce a population inversion mechanism to cause the channeled electrons or positrons to get pumped to higher transverse energy levels. If this can be achieved, either by using a part of the emitted radiation to
re-excite the transverse energy levels or any other way, one can produce a coherent \( x \) or \( 7 \) ray laser source, tunable by varying the channeled particle energy [30]. As mentioned earlier the radiation characteristics are dependent on the crystal structure, one has to characterize the properties of crystals and study the parameter like *channeling length* [33] etc inside a crystal. Systematic accumulation of channeling radiation data on variety of crystal species has already been reported [33-35]. One can expect that very soon *Channeling Radiation techniques* will emerge as potential alternative to *EPR* and electron microscopy techniques for the determination of structure and properties of both imperfect and perfect crystals.
References


References


### Spectral feature of planar channeled positrons

**Table - 7.1**

<table>
<thead>
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<th>Crystal plane</th>
<th>Beam energy (MeV)</th>
<th>γ</th>
<th>Energy of emitted channeling radiation (keV)</th>
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<td>110</td>
<td>50</td>
<td>99</td>
<td>36.5 ± 1</td>
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<tr>
<td>110</td>
<td>56</td>
<td>111</td>
<td>42.5 ± 0.05</td>
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### Spectral peaks for planar channeled electrons

**Table - 7.2**

<table>
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<th>Energy of emitted channeling radiation (keV)</th>
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<td></td>
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<td>E (exp) [6]</td>
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Fig 7.1: The dashed curve represents the shell planar potential and the solid curve represents the shell planar potential fitted to the form $Y_0 \exp(-ky)$ in the region $|y| \leq d_y/2$
Fig 7.2: The curve represents the shell planar potential curve fitted to a function of the form $Y = Y_0 \exp(-ky) + A$, the eigenvalues of this function are shown by horizontal lines.
Fig 7.3: (a) A typical channel at some finite distance $d_0$ from a dislocation. (b) The model channel replacing the actual channel of the part (a), and showing the co-ordinates used in the text. Here $R_0$ is the distance between the axis of the channel and one of the strings, $r_0$ is the equilibrium position about which the particle will oscillate and $r_1$ and $r_2$ are the positions at which the particle arrives after traversing the first and second parts of the channel, respectively (i.e. left and right to AB, respectively). (Ref [24])