Chapter-VII

Measurement of pinning frequency of a superconductor by a new method

The surface impedance of a system containing viscous damping and oscillating vortices is calculated, of which, in the limit of zero vortex mass, the real part is found to vary as 

$$\omega^2(\omega^2 + \omega_p^2)^{-1}$$

where $\omega_p$ is the pinning frequency. For finite mass of a vortex, a characteristic frequency $\omega_r$ is found. For $\omega_p < \omega_r$ the real part of the resistivity shows peaks and for $\omega_p > \omega_r$ it oscillates. For zero mass of the vortex, the experimental measurements of the surface resistance of 2H-NbSe$_2$ are found to be in reasonable agreement with the theory which is used to measure the pinning frequency, $\omega_p \simeq 145.7$ MHz.
Introduction

Some time ago, it was shown by Stephen and Bardeen [1,2] that the viscosity of the vortices can give rise to a normal state conductivity, $\rho_1$. The pinning frequency, $\omega_p$, is determined by the force constant of oscillations of a vortex and by the viscosity, which determines the force linear in the velocity,

$$\omega_p = \frac{k}{\eta} = \frac{k_0 \rho_n}{Hc \alpha} j_c$$

the force on the vortex is $kx$ with $x$ as the displacement of the vortex. The periodic potential in the 'wash board' model is given by $V(x) = V_0[1 - \cos(k_0 x)]$ with $k_0 = 2\pi/r_p$ where $r_p$ determines the characteristic length scale of the pinning interaction, usually taken as the lattice spacing [3]. For the flux line lattice (FLL) the wash board model gives the critical current as

$$j_c = \frac{k_0 V_0}{\phi_0}$$

where $\phi_0 = hc/2e$ is the unit flux. We have found [4] that large viscosity can cause a square root of magnetic field dependence in the penetration depth.

In this chapter we report that the resistivity depends on the frequency in such a way that the resistivity as a function of frequency can be used to measure the pinning frequency in the superconducting state of a type-II superconductor. We make use of the measured values [3] of resistivity as a function of frequency which are in reasonable agreement with the theory, leading to a measurement of $\omega_p \approx 145.7$ MHz in 2H-NbSe$_2$. Due to finite mass of the vortex, for $\omega_p < \omega_r$, we predict that there are several peaks in the resistivity and for $\omega_p > \omega_r$ there are oscillations in the resistivity.
Theory

Recently, Volovik [5] has found that the mass of the vortex is given by

\[ M \simeq m_e k_F^3 \xi^2 L \left( \frac{B_{c2}}{B} \right)^{\frac{1}{2}} \]  

(3)

where \( m_e \) is the mass of the electron, \( k_F \) is the Fermi wave vector, \( \xi \) the coherence length, \( L \) the length of the current loop, \( B_{c2} \) the upper critical field inside the superconductor and \( B \) the magnetic field. The mass is obtained from the volume \( \xi LR_v \) where \( R_v \) is the radius of the vortex in which the flux is quantized,

\[ \pi R_v^2 B = n \phi_0 \]  

(4)

so that \( R_v = (n \phi_0 / \pi B)^{1/2} \). The upper critical field is defined by \( \pi \xi^2 B_{c2} = n \phi_0 \) so that \( R_v = \xi (B_{c2}/B)^{1/2} \) and \( \xi R_v \simeq \xi^2 (B_{c2}/B)^{1/2} \). The mass of the vortex is thus several hundred times the mass \( m_e \) of the electron. The equation (3) due to Volovik applies for the superclean limit of d-wave superconductors. On the other hand NbSe\(_2\) is a conventional superconductor. Similar values were obtained by Suhl [6] who defined the inertial mass per unit length of a flux line of about 4000\( m_e \). If \( \xi = 30\text{Å} \) instead of 100\text{Å}, then the vortex mass is reduced to 444\( m_e \). Suhl mass is for a clean or dirty material and hence smaller than the superclean result by a factor \( (k_F \xi)^2 \sim (\Delta/\epsilon_F)^2 \sim 10^4 \). Thus while writing [7] the viscous and oscillatory forces, we include the Newtonian force on the vortex also so that

\[ M \frac{dv}{dt} + \eta v + kx = \frac{1}{c} J \phi_0 \]  

(5)

the intension being that there should be a mass term whether it is determined by Volovik or Suhl does not affect our equation. Therefore the vortex velocity is found to become

\[ v = \frac{J \phi_0}{c(\eta - i\omega M + (ik/\omega))} \]  

(6)
Chap. VII Measurement of...  

The vortex moving with velocity $v$ in a magnetic field $B$ produces the electric field,

$$E_\varphi = -\frac{1}{c}vB$$

(7)

The London penetration depth is defined by the relation

$$\frac{dJ}{dt} = \frac{c^2}{4\pi\lambda_L^2}(E + E_\varphi)$$

(8)

where $E + \nabla \varphi = -\partial A/\partial t$. Substituting (6) in (7) and the resulting relation into (8) and the time dependence of the current as $J = J_0 e^{-i\omega t}$ we find

$$J = E \left[ \frac{\phi_0 B}{c^2 \left\{ \eta - i\omega M + (ik/\omega) \right\}} - \frac{4\pi i\omega \lambda_L^2}{c^2} \right]^{-1}$$

(9)

which gives the resistivity as,

$$\rho = \frac{\phi_0 B \eta}{c^2 (\eta^2 + m^2)} - i \left[ \frac{m(\phi_0 B/c^2)}{\eta^2 + m^2} + \frac{4\pi \omega \lambda_L^2}{c^2} \right]$$

(10)

with $m = (k/\omega) - \omega M$. It may be noted that $M$ is the vortex mass, $m_e$ the electron mass and $m$ is the quantity which has the dimensions of force constant divided by frequency which is the same as frequency multiplied by mass. The real part of resistivity is

$$Re\rho = \frac{\phi_0 B \eta \omega^2}{c^2 [\omega^2 \eta^2 + k^2 + \omega^4 M^2 - 2k \omega^2 M]}$$

(11)

For zero vortex mass, $M \approx 0$, and the pinning frequency $\omega_p = k/\eta$, the above becomes

$$Re\rho = \frac{\phi_0 B \omega^2}{c^2 \eta (\omega^2 + \omega_p^2)}$$

(12)

In dimensionless units, the resistivity is thus predicted to vary as

$$\frac{\rho(\omega)}{\rho(\omega = \text{const})} = \frac{\omega^2}{\omega^2 + \omega_p^2}$$

(13)

Therefore a comparison of measured values of the resistivity as a function of frequency leads to a determination of $\omega_p$. 
The denominator of (11) is zero at the roots of the equation,
\[
\frac{M^2}{\eta^2} \omega^4 + \omega^2 (1 - 2 \omega_p \frac{M}{\eta}) + \omega_p^2 = 0
\]  
which are given by
\[
\omega_{1,2}^2 = \frac{2 \omega_p (M/\eta) - 1 \pm [(1 - 2 \omega_p \frac{M}{\eta})^2 - 4 \frac{M^2}{\eta^2} \omega_p^2]^{1/2}}{2 \frac{M^2}{\eta^2}}
\]
so that four peaks are predicted in the real part of the resistivity as a function of frequency. The roots are real only as long as,
\[
\omega_p < \omega_r
\]
where \( \omega_r = \eta/(4M) \). When pinning frequency \( \omega_p \) is less than \( \omega_r \), there are four real roots of (14) so that four peaks are predicted in the resistivity as a function of frequency. When \( \omega_p > \omega_r \) the solutions become complex which means that the resistivity oscillates. The time of oscillations can be expressed as
\[
\tau = \left[ \frac{\tau_p^2 - (Im \tilde{\rho}) \tau_p^2}{Im \tilde{\rho}} \right]^{1/2}
\]
where \( \omega_p = \tau_p^{-1} \) and \( Im \tilde{\rho} = (Im \rho) (c^2 \eta / \phi_0 B) \) which is described in dimensionless units. Thus at large frequencies, the resistivity shows new type of oscillations caused by the pinning and the vortex oscillations.

**Comparison with experiments**

The frequency dependence of the real part of the resistivity of 2H-NbSe\(_2\) at a field of 0.5T and a temperature of 3K has been measured by Henderson et al. [3]. They have plotted the resistivity relative to the value at 2.8 GHz in dimensionless units as a function of logarithm of frequency of measurement from 10 MHz to 1000 MHz. According to our calculation the dimensionless resistivity varies as
\[
\frac{r(\omega)}{r(2.8 \text{GHz})} = \frac{\omega^2}{(\omega^2 + \omega_p^2)}
\]
without the logarithmic divergence at $\omega = 0$. In Fig.1 we have plotted the dimensionless resistivity as a function of frequency from equation (18) which does not have any logarithmic term. The calculated values for $\omega_p = 145.7$ MHz, 212.7 MHz, and 327 MHz are shown as there is no other parameter. At low frequencies, the calculated curve with $\omega_p = 145.7$ MHz fits the data very well. At higher frequencies there are deviations from the predicted value. Thus we have obtained a new method of measuring the pinning frequency. The expression (18) is independent of the vortex mass.

Fig.1 The relative surface resistivity of 2H-NbSe$_2$ as a function of frequency. The dots are taken from the experimental measurements performed by Henderson et al. [3]. The calculated curves are shown for three different values of the pinning frequency, $\omega_p$. At low frequencies the curve calculated for $\omega_p \approx 145.7$ MHz agrees with the experimental measurements.
New effects are predicted at very small magnetic fields. In particular it is found that the resistivity oscillates if the field is less than $16(m^*k_F^2L^2\xi^2)^2B_{c2}/\eta^4$. It may be noted that only the frequency is being varied to measure the pinning frequency but the temperature is kept constant. The symmetry of the system may change from $s$ wave to $d$ wave in varying the temperature and the flux lattice may melt [8,9].

Conclusions

The vortex oscillations, viscosity and their kinetic energy all contribute to the current. When the contribution of the kinetic energy is small, the measurement of resistivity leads to the evaluation of the pinning frequency. Our theoretical expression for the resistivity is compared with the experimental measurements of the resistivity of $2H$-$\text{NbSe}_2$ as a function of frequency which gives for the pinning frequency, $\omega_p \simeq 145.7\text{MHz}$. For finite mass of a vortex interesting effects are predicted at small fields.