Chapter-V

Soft vortices in type-II superconductors: YNi$_2$B$_2$C and ErNi$_2$B$_2$C

We show that the apex angle of the rhombic cell of YNi$_2$B$_2$C decreases as a function of increasing magnetic field up to a certain constant field, $B_o \simeq 1130 \pm 70$ Oe. This dependence is consistent with the mean field theory of soft vortices. In ErNi$_2$B$_2$C, the magnetic penetration depth, also exhibits a soft vortex type behaviour below $B_o \simeq 250 \pm 10$ Gauss. The penetration depth is found to have a component which varies as $B^{1/2}[1 - (B/B_o)]^{-1/4}$ as predicted by the mean field theory and shows divergence at $B_o$. In both the samples, the theoretical predictions are in accord with the experimental measurements. This is the first time that London penetration depth associated with a soft vortex has been reported.
Introduction

Recently, it has been found that yttrium and rare-earth borocarbides upon cooling become conducting and upon further cooling become superconducting. There is a microscopic coexistence of antiferromagnetism and superconductivity in some of these compounds. In YNi$_2$B$_2$C, upon the application of a magnetic field, the hexagonal flux lattice changes into a square configuration above a certain field [1]. The apex angle, $\beta$, of the orthorhombic cell as a function of magnetic field decreases upon increasing field upto a certain field above which it becomes a constant. There is a vortex lattice reorientation transition from a state with the diagonal of the rhombic unit cell along [110] at low fields to [100] direction at high fields above the characteristic field. In ErNi$_2$B$_2$C, the high field square lattice transforms into a hexagonal lattice below a characteristic field [2]. The high field square lattice is aligned with [110] direction of the tetragonal crystal, while the low field hexagonal lattice is aligned with [100]. Within a certain range of fields both the square and hexagonal lattices are seen to coexist. In both the crystals YNi$_2$B$_2$C and ErNi$_2$B$_2$C the square configuration is stable at high field while hexagonal configuration is found at low fields. The area of the square lattice is equal to that of the hexagonal lattice in the coexistence region of the magnetic field for a given field in separate domains due to the flux quantization. These observations are analogous to that of a soft mode accompanied with a lattice distortion. The square lattice of higher symmetry is found at higher fields while lower symmetry occurs at lower fields.

Kogan et al. [3] have calculated the nonlocal corrections to the London model of a superconductor. In the case of LuNi$_2$B$_2$C they report that the apex angle $\beta$ of the rhombic unit cell shows two values for which the free-energy is a minimum. The apex angle as a function of magnetic field appears to increase with increasing magnetic field upto a certain field. Above this field the apex angle becomes independent of the field.
Kogan et al. [4] also report that theoretical models of nonlocal corrections to the London equations reproduce the symmetries of vortex lattices.

In this chapter we show that the apex angle of the orthorhombic cell exhibits a mean field exponent, $\beta = \beta_o[1 - (B/B_o)]^{1/2}$ with $\beta_o = 8^\circ$ and $B_o = 1130 \pm 70$ Oe as a function of field, in YNi$_2$B$_2$C. Since the magnetic penetration depth varies as the inverse square root of the frequency, we predicted that change in penetration depth varies as $\delta \lambda_L \propto B^{1/2}[1 - (B/B_o)]^{-1/4}$. For small fields, $B < 250$ Oe, we compare the experimental measurements of the change in London penetration depth in ErNi$_2$B$_2$C with the theory using mean field exponents. The measured values are in reasonable agreement with the theory. For large fields, $2250 > B > 250$ Oe, the penetration depth varies as the square root of the magnetic field in agreement with the theory which uses the viscous vortex oscillations.

**Soft vortices**

In the case of soft modes the higher temperature phase has higher symmetry and the lower temperature phase has lower symmetry. The soft lattice modes are often detected in the Raman spectra of solids. In the case of vortex lattices, we expect that the displacement of vortices should vary as

$$\delta x = (\delta x)_o[1 - (T/T_c)]^{1/2}$$

where mean field value for the critical exponent is chosen. In some of the studies, instead of the displacement an angle is measured as a function of temperature which also varies with temperature according to mean field exponent [5]. Similarly, in magnetic systems we expect soft modes so that the displacement of magnetic atoms or the angles vary with field,

$$\delta x = [\delta x]_o[1 - (B/B_o)]^{1/2}$$
where \( B_o \) is the critical induction. The frequency of the soft mode vanishes at the critical
temperature and at the critical field,

\[
\omega = \omega(0)[1 - (T/T_c)]^{1/2}
\]

and

\[
\omega = \omega_o[1 - (B/B_o)]^{1/2}
\]

The crystallographic angles are expected to vary as the distance between atoms as given
above so that in orthorhombic crystals,

\[
\beta = \beta_o[1 - (B/B_o)]^{1/2}
\]

Stephen and Bardeen [6,7] have shown that the vortices are subject to a viscous force
which contributes to the current. Coffey and Clem [8] have found the complex penetration
depth in terms of the field-dependent penetration depth, the normal-fluid skin depth and
the effective skin depth. We have solved [9] the equation of motion of a vortex properly
to define an effective penetration depth in a type-II superconductor. In addition to the
viscosity term in the force, we use the harmonic oscillations in the vortex and also include
the mass of the vortex to determine the current. The equation is then

\[
M \frac{dv}{dt} + \eta v + kx = \frac{1}{c} J \phi_o
\]

where \( M \) is the mass of the vortex, \( dv/dt \) is the acceleration, \( \eta \) is the coefficient of viscosity
of the vortex of velocity \( v \), force constant \( k \) and the critical current \( J \) [10]. It was found
that at large magnetic fields, the effective London penetration depth varies as,

\[
\lambda_{eff} \approx B^{1/2} \left[ \left\{ \left( \frac{k}{\omega} \right) - \omega M \right\}^2 + \eta^2 \right]^{-1/4} \left[ \phi_o/(4\pi\omega) \right]^{1/2}
\]

The relationship between the effective magnetic penetration depth and the London pen-
etration depth is also given in [9]. It may be noted that the original London penetration
depth is independent of field. However, we have found that the penetration depth changes when magnetic field is applied to a vortex. The relation (7) gives the field dependent penetration depth. For large viscosity, \( \eta >> (k/\omega) - \omega M \), we retain only one term in the large bracket and substitute (4) in (7) and leave out the remaining factors in a constant consistent with Stephen-Bardeen theory [6,7] so that the effective London penetration depth becomes.

\[
\lambda_{\text{eff}} = k_1 \frac{B^{1/2}}{\omega_0^{1/2} [1 - (B/B_o)]^{1/4}}
\]  

(8)

where the constant \( k_1 \) can be determined by comparing (8) with (7). If peaks are absent in the resistivity the mass term is small. However, the largest contribution of soft vortices to the London penetration depth varies with field as given by (8) and the structural parameters vary as given by (2) and (5). The result (8) ignores the mass of the vortex as small so that the first term of (6) which gives the force depending on the acceleration is neglected.

**Comparison with experimental measurements**

We report two experimental measurements which agree with the ideas of soft vortices which are linked with two minima in the free energy when one parameter is varied. In the present case the parameter is the magnetic field. This means that when field is varied, the system changes from one minimum value to another. The higher field minimum has higher symmetry and the lower field minimum has a lower symmetry. Therefore, the two states have a critical field, \( B_o \). Since both phases are superconducting, \( B_o < B_{c2} \).

The experimental values of the apex angle, \( \beta \), of the rhombic unit cell of \( \text{YNi}_2\text{B}_2\text{C} \) are taken from [1] and compared with expression (5). It is found that the data fit very well with

\[
\beta = 8[1 - (B/B_o)]^{1/2} + 52 \quad \text{(deg)}
\]  

(9)
with $B_0 = 1130 \pm 70$ Oe as shown in Fig. 1.

![Diagram](image)

**Fig. 1** The apex angle $\beta$ of the rhombic unit cell as a function of $[1 - (B/B_0)]^{1/2}$ showing linear dependence. The experimental points are taken from the small angle neutron scattering measurements performed on YNi$_2$B$_2$C given in [1]. The theoretical interpretation in terms of a critical field and soft vortex is as given in the text in the present work.

We therefore conclude that in YNi$_2$B$_2$C there is a soft vortex with critical field of $B_0$. The angle $\beta$ has a large value at $B = 0$. There is a component in this angle which reduces with increasing field and becomes zero at $B = B_0$.

In our earlier work [9] the penetration depth as a function of magnetic field in ErNi$_2$B$_2$C was explained only for magnetic fields larger than $\sim 250$ Gauss. For small fields, the penetration depth increases slowly with increasing magnetic field but the measured values did not agree with the predicted $B^{1/2}$ or $B^2$ dependence. We are now able to explain the
change in the penetration depth as a function of magnetic field for small fields, $B < 250$ G. These changes are now assigned to soft vortex modes. According to the expression (8) for magnetic fields smaller than $\sim 250$ G, we calculate the change in London penetration depth as a function of magnetic field from

$$\delta \lambda_L = C_1 B^{1/2}[1 - (B/B_o)]^{-1/4}$$

(10)

for $C_1 = 4.73$ and $B_o = 250 \pm 10$ G as shown in Fig. 2.

Fig. 2 The change in London penetration depth as a function of magnetic field calculated for a soft vortex with critical field $B_o = 250$ G. There is a divergence in the calculated value at 250 G. The experimental point deduced from the measurements on ErNi$_2$B$_2$C given in [1] are also shown.

The experimental values deduced from the work of Eskildsen et al. [2] for ErNi$_2$B$_2$C are also shown. It is seen that the soft vortex interpretation works well and the calculated
values are in agreement with the experimental data.

Conclusions
The free energy as a function of field has two minima separated by a barrier. When the applied magnetic field energy, $B^2/8\pi$, is equal to that of the barrier, there is a transition from one minimum value to another accompanied by a soft vortex. In the apex angle in YNi$_2$B$_2$C, such a soft vortex gives rise to a critical behaviour which we have described by the mean field theory. It is found that in YNi$_2$B$_2$C the apex angle varies as $[1 - (B/B_0)]^{1/2}$ with $B_0 \simeq 1130 \pm 70$ Oe which is in agreement with the value, $B \sim 112$ mT found in [1]. The theoretical predictions are in accord with the experimental measurements. The London penetration depth is found to vary with the magnetic field. A critical field exists such that the London penetration depth diverges at this field, $\lambda_L \propto B^{1/2}[1 - (B/B_0)]^{-1/4}$. In ErNi$_2$B$_2$C the measurements of penetration depth are in good agreement with the theory of soft vortices. For $B > B_0$, the vortex lattice has the symmetry of a square lattice while for $B < B_0$ the structure is hexagonal.
REFERENCES

References


