Chapter 3: Design of Linear and Non Linear Controllers for Power Factor Correction using Boost Converter in Single Phase Systems

3.1 Introduction

This chapter discusses the analysis, modelling and design of Boost Converter for Power Factor Correction (PFC) in single-phase systems. Various Linear and Non Linear Controllers such as Linear Quadratic Controller (LQC), Hysteresis Controller (HC) and Non Linear Carrier (NLC) Controller have been employed to shape the input current wave to sinusoidal for power factor improvement. A Proportional-Integral (PI) based voltage controller is used for output voltage regulation. A series of simulation studies is carried out through MATLAB/SIMULINK in order to verify the effectiveness of all these controllers and the simulation results are presented.

3.2 The Power Factor

The input current drawn from the power line deviates from a sine wave if non linear loads are connected with the AC power line. However the non sinusoidal current can be decomposed into sine wave with fundamental frequency component and related higher order frequency components. The input power factor for such system can be expressed as

\[ \text{PF} = \text{Displacement Factor} \times \text{Distortion Factor} \]  \hspace{1cm} (3.1)

Displacement Factor = \( \cos \phi_i \) where \( \phi_i \) is the phase angle between supply voltage and fundamental frequency component of source current.

\[ \text{Distortion Factor} = \frac{I_{s1}}{I_s} \]  \hspace{1cm} (3.2)

\( I_{s1} \) is the rms value of fundamental component of the source current

\( I_s \) is the rms value of the source current
The amount of higher order harmonics relative to fundamental component known as Total Harmonic Distortion (THD) is defined as

\[
THD = \sqrt{\frac{I_{s2}^2 + I_{s3}^2 + \ldots + I_{sn}^2}{I_{s1}^2}} \tag{3.3}
\]

\(I_{sn}\) is the rms value of \(n^{th}\) harmonic component of source current

Based on the calculation of THD, the distortion factor which is a measure of deviation of source current waveform from a pure sinusoidal waveform is expressed as

\[
\text{Distortion Factor} = \frac{1}{\sqrt{1 + THD^2}} \tag{3.4}
\]

The overall power factor can be improved by increasing displacement factor and distortion factor. The distortion factor is improved by reducing THD i.e., lower input current distortion which means that the source current resembles a pure sine wave and the displacement factor is improved by reducing the phase angle \(\phi_1\) and thereby the source current is made to be in phase with the source voltage. As a result, in order to achieve effective source utilization and high power quality, it is desired to attain high power factor. The aim of PFC is to achieve unity power factor by making the source current nearly a sine wave and also in phase with source voltage with the help of boost converter.

### 3.3 Fundamentals of Boost Converter

A boost converter is a DC-DC power converter with an output voltage greater than its input voltage which makes it suitable for certain high voltage applications with less space and less number of components. The boost converter is provided with a filter inductor on the input side, which provides a smooth continuous input current waveform when operated in continuous mode. The continuous input current is much easier to
filter, which is a major advantage, since any additional filtering needed on the converter input will increase the cost and reduce the power factor due to capacitive loading of the line.

The circuit diagram of a Boost Converter is shown in Figure 3.1. Here $V_{in}$ is the input voltage, $L$ is the input inductor, $S$ is the active switch and $D_1$ is the diode. A load resistor $R_L$ is connected in parallel with a filter capacitor $C_0$. The output DC voltage is represented as $V_0$. The steady state analysis of the Boost Converter with Continuous Conduction Mode (CCM) is carried out by considering the two operating modes depending upon the switching positions. The switch $S$ operates with the duty cycle of $D$, switching frequency of $f_s$ and the time period of $T_s$ [64, 65].

![Figure 3.1: Boost Converter](image)

**Mode 1:** (During first sub interval $(0 < t < DT_s)$)

When the switch is in closed position as in Figure 3.2, the current starts flowing through the inductor $L$ and the switch. During this time interval $T_{ON}$, the current increases from $I_{L_{min}}$ to $I_{L_{max}}$ and the inductor stores the energy. At the same time the capacitor starts discharging through the load $R_L$. 
In mode 1, the inductor current rises from $I_{L,\text{min}}$ to $I_{L,\text{max}}$ during $T_{ON}$.

$$V_{in} = \frac{L(I_{L,\text{max}} - I_{L,\text{min}})}{T_{ON}}$$  \hspace{1cm} (3.5)$$

$$V_{in} = \frac{2\Delta I_{L}}{T_{ON}} L$$  \hspace{1cm} (3.6)$$

$$T_{ON} = \frac{2\Delta I_{L}}{V_{in}} L$$  \hspace{1cm} (3.7)$$

where $\Delta I_{L}$ is peak to average ripple of inductor current.

Mode 2: (During second sub interval ($DT_{s} < t < T_{s}$))

When the switch is in open position as in Figure 3.3, the current is flowing through the inductor $L$, diode $D_1$, the load $R_L$ and the capacitor $C_0$. During this interval $T_{OFF}$, the current decreases from $I_{L,\text{max}}$ to $I_{L,\text{min}}$ and the energy that has been accumulated in the inductor gets transferred to the capacitor.
In mode 2, the inductor current decreases from $I_{L_{\text{max}}}$ to $I_{L_{\text{min}}}$ during $T_{\text{OFF}}$. The inductor current and capacitor voltage during both the intervals are shown in Figure 3.4

$$V_{in} - V_0 = \frac{-2\Delta I_L}{T_{\text{OFF}}}L$$  \hspace{1cm} (3.8)

$$T_{\text{OFF}} = \frac{2\Delta I_L}{V_0 - V_{in}}L$$  \hspace{1cm} (3.9)

From equations (3.7) to (3.9), we get:

$$\Delta I_L = \frac{V_{in}T_{\text{ON}}}{2L} = \frac{(V_0 - V_{in})T_{\text{OFF}}}{2L}$$  \hspace{1cm} (3.10)

The average output voltage is

$$V_0 = \frac{V_{in}T_S}{T_{\text{OFF}}} = \frac{V_{in}T_S}{T_S - T_{\text{ON}}} = \frac{V_{in}}{1 - \left(\frac{T_{\text{ON}}}{T_S}\right)} = \frac{V_{in}}{1 - D}$$  \hspace{1cm} (3.11)

Assuming a lossless circuit

$$V_{in}I_S = V_0I_0 = \frac{V_{in}}{1 - D}I_0$$  \hspace{1cm} (3.12)

The average input current is

$$I_S = \frac{I_0}{1 - D}$$  \hspace{1cm} (3.13)

The peak to average ripple current is

$$\Delta I_L = \frac{V_{in}D}{2f_cL} = \frac{V_{in}DT_S}{2L}$$  \hspace{1cm} (3.14)

The peak to average ripple voltage in the filter capacitor is

$$\Delta V_0 = \frac{I_0D}{2f_cC_0} = \frac{V_{in}DT_S}{2R_cC_0}$$  \hspace{1cm} (3.15)

The critical values of inductance and capacitance are
\[ L = \frac{V_{in}DT_S}{2\Delta I_L} \]  

(3.16)

\[ C_0 = \frac{V_0DT_S}{2R_L\Delta V_0} \]  

(3.17)

where \( \Delta V_0 \) is peak to average ripple of output voltage

\[ \text{Figure 3.4: The Inductor Current and Capacitor Voltage of Boost Converter} \]

### 3.4 Design Equations of Boost Converter

The boost converter is designed with the following specifications. DC input voltage \( (V_{in}) = 12 \text{ V} \), output voltage \( (V_0) = 24 \text{ V} \), Duty cycle \( (D) = 0.5 \), Load current \( (I_0) = (0 - 2.5) \text{ A} \), Switching frequency \((f_s) = 10 \text{ kHz} \). For \( \Delta I_L = 3\% \text{ of } I_L \). \( \Delta V_0 = 0.03\% \text{ of } V_0 \), values of reactive components are found to be

\[ L = \frac{V_{in}DT_S}{2\Delta I_L} = 2 \text{ mH} \]  

(3.18)

\[ C_0 = \frac{V_0}{2R_L\Delta V_0} DT_S = 9000 \text{ uF} \]  

(3.19)
3.5 Modelling of Boost Converter

When a DC-DC converter is operating under steady state, the input power and output power are equal in every switching period \( T_s \) of the converter. However, the same is not valid for single-phase resistor emulator rectifiers. The input voltage \( v_g \) (as shown in Figure 3.6) varies from 0 to \( V_m \) in a line cycle and the input current is proportional to the input voltage under steady state condition. Therefore the instantaneous input power \( \frac{v^2}{R} \) is not constant. So, unlike the DC-DC converter, the power balance condition between input and output cannot be satisfied at every switching period of the resistor emulator rectifiers. However, it may be noted that, the instantaneous input power is equal to the output power only at a period in which the input voltage is \( \frac{V_m}{\sqrt{2}} \). Assuming the input of the power stage is a DC voltage \( V_{in} \) equal to the rms value of the rectified input supply voltage \( V_g \) [20, 21].

The state equations are linearized to relate the AC components of the circuit variables about the operating point. The modelling of the system includes dynamic equations developed for each of the switching positions of the converter with inductor current \( i_L \) and capacitor voltage \( v_c \) as state variables and are given as follows [31, 66]

Mode 1

\[
\frac{di_L}{dt} = \frac{v_g}{L} \quad (3.20)
\]

\[
\frac{dv_c}{dt} = \frac{-v_c}{R_c C_0} \quad (3.21)
\]
\[
\frac{d}{dt} \begin{bmatrix} i_L \\ v_0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -1/R_L C_0 \end{bmatrix} \begin{bmatrix} i_L \\ v_0 \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} [v_g] 
\]  \hspace{1cm} (3.22)

\[A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -1/R_L C_0 \end{bmatrix}\]  \hspace{1cm} (3.23)

\[B_1 = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}\]

Mode 2:

\[
\frac{di_L}{dt} = \frac{v_g - v_0}{L}
\]  \hspace{1cm} (3.24)

\[
\frac{dv_0}{dt} = \frac{i_L - v_0}{C_0} - \frac{1}{R_L C_0}
\]  \hspace{1cm} (3.25)

\[
\frac{d}{dt} \begin{bmatrix} i_L \\ v_0 \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C_0 & -1/R_L C_0 \end{bmatrix} \begin{bmatrix} i_L \\ v_0 \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} [v_g] 
\]  \hspace{1cm} (3.26)

\[A_2 = \begin{bmatrix} 0 & -1/L \\ 1/C_0 & -1/R_L C_0 \end{bmatrix}\]  \hspace{1cm} (3.27)

\[B_2 = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}\]

\[A = DA_1 + (1-D)A_2 \]  \hspace{1cm} (3.28)

\[B = DB_1 + (1-D)B_2 \]  \hspace{1cm} (3.29)

The averaged system matrix is given as below

\[A = \begin{bmatrix} 0 & -(1-D) \\ (1-D) & -1 \end{bmatrix} \]  \hspace{1cm} (3.30)

\[B = \begin{bmatrix} 1/L \\ 0 \end{bmatrix} \]  \hspace{1cm} (3.31)
The dynamic behaviour of boost converter operating in continuous conduction mode can be represented in the small signal averaged state space model for every switching position. The expressions that model the equivalent converter controlled by the duty cycle in open loop are given as [31, 66]

\[
\frac{d\hat{x}}{dt} = A\hat{x} + B\hat{v}_g + [(A1 - A2)X + (B1 - B2)U]\hat{d}
\]

(3.32)

\[
\frac{d\hat{x}}{dt} = A\hat{x} + B\hat{v}_g + f\hat{d}
\]

(3.33)

where \( \hat{d} \) - small signal duty cycle

and \( f = [(A1 - A2)X + (B1 - B2)U] \)

(3.34)

Substituting corresponding state variables and relevant matrices \( A, B \) and \( f \) in (3.33), it becomes,

\[
\frac{d}{dt}\begin{bmatrix}
\hat{i}_L \\
\hat{v}_0
\end{bmatrix} = \begin{bmatrix}
0 & -(1-D) \\
\frac{(1-D)}{C_0} & -1
\end{bmatrix}\begin{bmatrix}
\hat{i}_L \\
\hat{v}_0
\end{bmatrix} + \begin{bmatrix}
\frac{1}{L} \\
-\frac{1}{R_L C_0}
\end{bmatrix}\hat{v}_g + \begin{bmatrix}
0 \\
\frac{1}{C_0}
\end{bmatrix}\begin{bmatrix}
I_L \\
V_o
\end{bmatrix}\hat{d}
\]

(3.35)

Expanding (3.35)

\[
\frac{di_L}{dt} = -\frac{(1-D)}{L}\hat{v}_0 + \frac{1}{L}\hat{v}_g + \frac{V_0}{L}\hat{d}
\]

(3.36)

\[
\frac{dv_0}{dt} = \frac{(1-D)}{L}\hat{i}_L - \frac{1}{R_L C_0}\hat{v}_0 + \frac{I}{C_0}\hat{d}
\]

(3.37)

In order to obtain the control transfer functions, [66, 67] \( \hat{v}_g \) is assumed to be zero, then the control - to - output transfer function \( G_{vd}(s) \) is derived and the equation (3.33) becomes

\[
\frac{d\hat{x}}{dt} = A\hat{x} + f\hat{d}
\]

(3.38)
\[ G_{vd}(s) = \frac{\hat{v}_0(s)}{\hat{d}(s)} = C(SI - A)^{-1} f \]  

(3.39)

where \( C = [0 \ 1] \)

Substituting the matrices \( A, C \) and \( f \), equation (3.39) becomes

\[ G_{vd}(s) = \frac{\hat{v}_0(s)}{\hat{d}(s)} = \frac{V_g}{(1-D)^2} \left( \begin{array}{c} \frac{-L}{R_l(1-D)^2} s + 1 \\ \frac{LC_0}{(1-D)^2} s^2 + \frac{L}{R_l(1-D)^2} s + 1 \end{array} \right) \]  

(3.40)

3.6 Open Loop and Closed Loop Control

In order to improve the supply side power factor and to regulate the low voltage DC in the load side, a PI based voltage controller is used in the outer voltage control loop and suitable current controllers are employed in the inner current control loop with necessary feedback signals.

3.6.1 Open Loop Transfer Function

The open loop control to output transfer function equation (3.40) can be written in the normalized form as [64]

\[ G_{vd}(s) = G_{d0} \left( \begin{array}{c} 1 - \frac{s}{\omega_c} \\ \frac{s}{Q_0 \omega_0} + \frac{s^2}{\omega_0^2} \end{array} \right) \]  

(3.41)

where the DC gain \( G_{d0} = \frac{V_0}{(1-D)^2} \)  

(3.42)

Corner frequency \( f_0 = \frac{\omega_0}{2\pi} = \frac{1-D}{2\pi \sqrt{LC_0}} \)  

(3.43)

\( Q \) factor \( Q_0 = R_l \frac{C_0}{L} (1-D) \)  

(3.44)
\[ \omega_z = \frac{R_L}{L}(1 - D)^2 \]  

(3.45)

To obtain a output voltage of 24 V from 12 V input, the required duty ratio is 0.5. A Pulse train of fixed duty cycle is applied to the converter and the corresponding waveforms are shown in Figures 3.5 (a) to (c). From the open loop control, it is clear that the output voltage is not maintained constant for load variations and the wave shape of source current is not exactly sinusoidal because of high THD. In order to obtain the desired output voltage, regardless of variations in load and component values, the converter has to be operated in closed loop with negative feedback. This feedback circuit makes use of a compensator that adjusts the duty cycle \( \hat{d} \) automatically for any disturbances and keeps the output voltage constant under all conditions.

**Figure 3.5**: (a) Output Voltage and Load Current for Step Load Change (b) Source Current (c) THD (Total Harmonic Distortion)
3.6.2 Closed Loop with Voltage Mode and Current Mode Control

The PFC in single phase is considered with the design and implementation of DC to DC boost converter using linear and non linear controllers [1, 28, 68]. In most of the single phase systems, for achieving PFC, it requires sensing of rectified input voltage, inductor current and output load voltage. In the outer voltage control loop, the output voltage is compared with the reference voltage. This error is processed in the PI controller. The output of this controller is multiplied by the rectified input voltage which acts as the reference current template. In inner current loop, the inductor current is compared with this reference current. The resulting error acts as a modulating signal in the PWM circuit and changes the duty ratio which enables the converter to attain the desired control objectives. The closed loop control with voltage mode and current mode control is shown in Figure 3.6.

![Closed Loop Control of Boost Converter](image)

**Figure 3.6**: Closed Loop Control of Boost Converter
3.7 Design of Voltage Controller

In order to obtain the desired output voltage regardless of variations in load, the voltage controller is designed where the actual output voltage is sensed and compared with the reference value which is then processed to the PI controller. The voltage controller is designed by considering the numerical control to output transfer function of the boost converter which is obtained based on the specifications, the input voltage, duty cycle and the component values. The numerical control to output transfer function obtained from (3.40) is as follows

\[ G_{vd}(s) = \frac{V_0(s)}{d(s)} = 48 \left( \frac{-8e^{-4}s + 1}{7.2e^{-5}s^2 + 8e^{-4}s + 1} \right) \]  

(3.46)

The outer voltage control loop includes control to output transfer function \( G_{vd}(s) \) and voltage controller transfer function \( G_c(s) \) as in Figure 3.7(a). The voltage controller has two parts: PI controller and low pass filter. The PI controller is included to obtain satisfactory dynamic response and to reduce the steady state error. Since the outer voltage loop requires low bandwidth, it is reasonable to assume the current loop to be ideal at low frequencies around 15Hz. The voltage controller transfer function \( G_c(s) \) has a PI controller \( G_{c1}(s) \) and a low pass filter \( G_{c2}(s) \) connected in cascade. The purpose of the low pass filter is to eliminate high frequency components, noise and allows only DC component. The controller transfer function \( G_c(s) \) [69] follows the general form as shown below:

\[ G_c(s) = G_{c1}(s)G_{c2}(s) \]  

(3.47)

\[ G_{c1}(s) = k_p + \frac{k_i}{s} \]  

(for PI controller)  

(3.48)
\[ G_{c2}(s) = \frac{w_p}{s + w_p} \quad \text{(for low pass filter)} \quad (3.49) \]

Substituting \( G_{c1}(s) \) and \( G_{c2}(s) \), the controller transfer function \( G_c(s) \) follow the general form as shown below:

\[
G_c(s) = \frac{k_i}{s} \left( \frac{1 + \frac{s}{w_z}}{1 + \frac{s}{w_p}} \right) \quad (3.50)
\]

where \( w_z = \frac{k_i}{k_p} \) \quad (3.51)

In order to have high loop DC gain and zero steady state error, the controller transfer function must have a pole at the origin. The pole zero pair provides the necessary phase boost and hence the specified phase margin (approximately 60°). For designing the outer voltage loop, the crossover frequency is selected as 15 Hz and the design equations are given as

\[
\phi_{boost} = -90^\circ + \phi_{PM} - \angle G_{vd}(s) \bigg|_{f = f_c} = -90^\circ + 60^\circ - (-15.45^\circ) = -14.55^\circ \quad (3.52)
\]

Substituting the value obtained in (3.52) in the following equation (3.53)

\[
K_{boost} = \tan(45^\circ + \frac{\phi_{boost}}{2}) = \tan(45^\circ - \frac{14.55^\circ}{2}) = 0.7735 \quad (3.53)
\]

The two parameters viz., the phase boost \( K_{boost} \) and \( \phi_{boost} \), determine the pole zero locations of the Transfer Function \( G_c(s) \)

\[
f_z = \frac{f_c}{K_{boost}} = 15/0.7735 = 19.39 \text{ Hz} \quad (3.54)
\]

\[
f_p = K_{boost} f_c = 11.6 \text{ Hz} \quad (3.55)
\]
From equations (3.54) and (3.55), the value of \( w_z \) and \( w_p \) are calculated as 121 rad/s and 72 rad/s. Then, the integral gain is selected as 0.5 and the proportional gain is calculated as 0.004 (\( w_z = \frac{k_i}{k_p} \)) and \( w_p \) is found to be approximately 70 rad/s (with a phase margin of 83.9°). Since the required phase margin is 60°, the \( w_p \) can be further adjusted to 30 rad/s. Hence the selected values of \( k_p \), \( k_i \) and \( w_p \) are 0.004, 0.5 and 30 rad/s respectively (with a phase margin of 61.8°). The bode plot of uncompensated and compensated system are shown in Figures 3.7 (b) and (c).

**Figure 3.7:** Stability Analysis (a) Block Diagram of Voltage Controller (b) Bode Plot of Uncompensated System (c) Bode Plot of Compensated System
3.8 Design of Hysteresis Controller for PFC Boost Converter

To achieve input current wave shaping which makes the power factor nearer to unity and to obtain a regulated DC output voltage, the Boost Converter employing a simple Hysteresis Controller is proposed here. The major advantages of this control technique are there is no need of compensation ramp as in peak current mode control and the input current is less distorted [3]. In spite of its simplicity, variable switching frequency operation of this controller leads to more chattering noises, EMI and complicated filter design.

3.8.1 Principle of Hysteresis Controller

In Hysteresis Control scheme as in Figures 3.8 (a) and (b), the output of the PI controller decides the amplitude of the input reference current which is then multiplied with the sine template extracted from the input sinusoidal voltage. Here, two sinusoidal current references are generated corresponding to maximum and minimum boundary limits and the switch is turned on when the actual inductor current goes below the lower reference $I_{L(ref)}$ and is turned off when the inductor current goes above the upper reference $I_{U(ref)}$ giving rise to a variable frequency control [23, 24]. To achieve smaller ripple in the input current, a narrow hysteresis band of ±0.0005 is needed. The potency of the Hysteresis Controller (HC) is proved by simulation results.
3.8.2 Simulation Results

To study the system performance, simulations have been performed using MATLAB/SIMULINK tool and the MATLAB schematic is shown in Appendix C. The following parameters are considered for simulation:

AC input voltage $= 12$ V (RMS), output voltage $V_o = 24$ V, Inductor $L = 2$ mH, Capacitor $C_o = 9000 \mu$F, Load resistance $R_L = 10$ $\Omega$, Switching frequency $f_s = 10$ kHz.

Figure 3. 8: (a) Closed Loop Control using Hysteresis Controller (HC)
(b) Pulse Generation using HC
Figure 3.9 (a) shows the actual DC output voltage \( (V_o) \) for rated load current of 2.4 A. It is seen from Figure 3.9 (b) that the source current wave shape is sinusoidal and also in phase with the source voltage. From Figure 3.9 (c), it is evident that constant DC output voltage is maintained for step change in load from 1.6 A to 2.0 A at \( t = 0.3 \) s. The output voltage settles to 24 V after a transient interval of 100 ms. The output voltage is well regulated for abrupt increase (1.6 A to 2 A) and decrease in load (2 A to 1.6 A) which are shown in Figure 3.9 (e). It is seen from Figures 3.9 (d) and (f) that the source current is sinusoidal and the same wave shape is continued with different amplitudes for load variation. It is concluded that the recommended controller makes the input current wave shape to be sinusoidal and also in phase with the source voltage.

It is evident from waveforms shown in Figure 3.9 (g) that the voltage controller acts effectively and brings the output voltage to the reference exactly. The servo response of the closed loop system is tested by changing the reference voltage and it is observed that output voltage is able to follow the change in the reference voltage from 20 V to 24 V and then to 28 V. Figure 3.9 (h) shows the Harmonic spectrum of source current and the Total Harmonic Distortion (THD) is found to be 8.4%. 


Figure 3.9: Performance of PFC Boost Converter with Hysteresis Controller (HC)
3.9 Design of Linear Quadratic Controller for PFC Boost Converter

In closed loop control, the performance of the system can be improved by the selection of state feedback gain matrix \( (K) \). Linear Quadratic Regulator (LQR) defines the optimal pole location based on the cost function; it has a set of differential equations which illustrates the paths of the control variables that minimize the cost function [25-27]. The optimal solution is to find the control input \( u_c \) to minimize the quadratic cost function

\[
J = \int_0^\infty (x_e^T Q x_e + u_c^T R u_c) dt
\]

(3.56)

The matrices \( Q \) and \( R \) are selected such that closed loop poles are at the desired location. If the weightage assigned to the elements of the \( Q \) matrix are higher, then the poles of the closed loop system matrix \( A_c = A - BK \) will move towards the origin in the \( s \)-plane. Hence, the error in the state variables will decay faster. For well defined performance index, the selection of \( Q \) has to be positive semi definite and \( R \) to be positive definite. The feedback gain matrix \( K \) is calculated based on the appropriate selection of matrices \( Q \) and \( R \) [28-30]. To achieve perfect wave shaping of input current, more weightage is given for inductor current in the selection of state weighting matrix \( Q \) which is as follows:

\[
Q = \begin{bmatrix}
615 & 0 \\
0 & 345
\end{bmatrix} \quad R = [1] \quad (3.57)
\]

The control law chosen to minimize the performance index \( J \) is

\[
u_c = -Kx_e
\]

(3.58)

where \( x_e = x_{ref} - x_{actual} \)
The LQR control strategy using the state feedback has been applied to allocate the poles of the closed loop system in any position in order to meet the design specifications. Assuming the small variations in the input voltage \( \dot{v}_{g} = 0 \), the equation (3.33) becomes

\[
\frac{d\hat{x}}{dt} = A\hat{x} + f\hat{d}
\]

(3.60)

Let us consider a system defined by the following state space equation

\[
\dot{x} = Ax + Bu_c
\]

(3.61)

The quadratic optimal regulator aims to find matrix \( K \) for the optimal control vector \( u_c = -Kx = -[K_1 \ K_2]x \) so that the cost function is minimized. By comparing (3.60) with (3.61), \( u_c \) is equivalent to the small signal duty cycle \( \hat{d} \) and \( x \) is the state vector. Matrix \( A \) is already known [31]. Then, matrix \( B = f \) is obtained as follows:

\[
B = [(A_1 - A_2)X + (B_1 - B_2)U] = \begin{bmatrix} V_0 \\ L \\ \frac{-I}{C_0} \end{bmatrix} = \begin{bmatrix} 12 \times 10^3 \\ -555.5 \end{bmatrix}
\]

(3.62)

In the design of optimal control system, reduced matrix Riccati equation is written as

\[
A^T P + PA - PBR^{-1}B^T P + Q = 0
\]

(3.63)

\[
\begin{bmatrix} 0 & 55.55 & \begin{bmatrix} P_{11} & P_{12} \end{bmatrix} \\ -250 & -11.11 & \begin{bmatrix} P_{21} & P_{22} \end{bmatrix} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & -250 \\ -55.55 & -11.11 \end{bmatrix} - \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 12 \times 10^3 \\ -555.55 \end{bmatrix} [1 \begin{bmatrix} 12 \times 10^3 & -555.55 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} 615 & 0 \\ 0 & 345 \end{bmatrix} = 0
\]

(3.64)

Solving the above equation yields the positive definite matrix \( P \), which is as follows:

\[
\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} 0.0129 & 0.2336 \\ 0.2336 & 5.0253 \end{bmatrix}
\]

(3.65)

Then, the optimal gain feedback matrix \( K \) is obtained as
\[ K = R^{-1}B^TP \]  
(3.66)

\[
K = \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 12 \times 10^3 & -555.55 \\ 0.2336 & 5.0253 \end{bmatrix}
\begin{bmatrix} 0.0129 \\ 0.2336 \end{bmatrix}
\]  
(3.67)

\[
K = \begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 25.3 & 10.7 \end{bmatrix}
\]  
(3.68)

### 3.9.1 LQR Control Scheme

In LQR, the output of the PI controller decides the amplitude of the input reference current which is then multiplied with the sine template extracted from the input sinusoidal voltage. The voltage error \( e_2 \) is multiplied by the gain \( K_2 \). The actual inductor current is sensed and compared with the reference current template and this error is \( e_1 \) multiplied with \( K_1 \). The control input \( u_c \) is calculated by using the equation

\[
u_c = K_1(e_1) + K_2(e_2) = K_1(i_{ref} - i_{actual}) + K_2(V_{ref} - V_0)
\]
where \( K_1 \) and \( K_2 \) are the feedback gains. The control input modifies the duty ratio of the boost converter. The LQR for current wave shaping is shown in Figure 3.10.
3.9.2 Simulation Results

To observe the system performance, simulations have been performed using MATLAB/SIMULINK tool and the MATLAB schematic is shown in Appendix C. The following parameters are considered for simulation:

AC input voltage = 12 V (RMS), output voltage $V_0 = 24$ V, Inductor $L = 2$ mH, Capacitor $C_0 = 9000$ µF, Load resistance $R_L = 10$ Ω, Switching frequency $f_s = 10$ kHz.

Figure 3.11 (a) shows the actual DC output voltage ($V_0$) for rated load current of 2.4 A. It is seen from Figure 3.11 (b), that the source current wave shape is sinusoidal and also in phase with the source voltage. From Figure 3.11 (c), it is evident that a constant DC output voltage is maintained for step change in load from 1.6 A to 2.0 A at $t = 0.3$ s. The output voltage settles to 24 V after a transient interval of 100 ms. The output voltage is well regulated for an abrupt increase (1.6 A to 2 A) and decrease in load.
(2 A to 1.6A) which are shown in Figure 3.11 (e). It is seen from Figures 3.11 (d) and (f) that the source current is sinusoidal and the same wave shape is continued with different amplitudes for load variation. It is inferred that the LQR improves the power factor by wave shaping.

It is evident from waveforms shown in Figure 3.11 (g) that the voltage controller acts efficiently and brings the output voltage to the reference value precisely. The servo tracking is verified by changing the reference voltage and it is seen that output voltage follows the change in the reference voltage from 20 V to 24 V and then to 28 V. Figure 3.11 (h) shows the Harmonic Spectrum of source current and the Total Harmonic Distortion (THD) is found to be 4.14%.
Figure 3.11: Performance of PFC Boost Converter with Linear Quadratic Controller
3.10 Design of Non Linear Carrier (NLC) Controller for PFC Boost Converter

In most of the PFC circuits, sensing of input voltage, input current and output voltage is essential for reference current generation and the number of sensors are increased in three phase circuits which are responsible for additional losses. The proposed method with Non linear carrier controller eliminates tedious calculations and desired performance can be achieved without input voltage and current sensors.

It is desired to operate the PFC boost converter (Figure 3.12) as an ideal rectifier which presents only a resistive load to the AC power line which in turn makes both supply voltage and current to be in phase with each other and identical in shape. As the input current should be proportional to the input voltage, they are correlated by the following equation [32, 64, 67]

\[
i_{ac}(t) = \frac{v_{ac}(t)}{R_e} = \frac{V_m \sin \omega t}{R_e}
\]

(3.69)

The key feature of UPF rectifier is to make the low frequency region of \(i_g(t)\) to follow \(v_g(t)\) and to maintain the output voltage \(v_0\) equal to the reference voltage \(v_{ref}\)

\[
i_g(t) = \frac{v_g(t)}{R_e} = \frac{V_m |\sin \omega t|}{R_e}
\]

(3.70)

Since the emulated resistance \(R_e\) mainly depends on the control signal \(v_{control}(t)\) which results from the PI controller, it can be termed as \(R_e(v_{control})\). The system is described with boost converter in continuous conduction mode with NLC. The switch current is sensed and the averaged switch current for one switching period \((T_s)\) is calculated as follows:
The switch current is integrated with capacitor and the voltage is given as

\[ v_i(t) = \frac{1}{C_i} \int_0^{r+T} i_s(\tau) d\tau \]  

(3.72)

In order to apply input resistor emulation, the average value of \( i_g(t) \) is found as

\[ \langle i_s(t) \rangle_{T_s} = \frac{\langle v_s(t) \rangle_{T_s}}{R_e(v_{\text{control}})} \]  

(3.73)

In NLC, the switch current is sensed instead of input current and hence the average value of switch current for one switching cycle is derived in terms of input current. The DC output voltage is sensed for designing a low bandwidth feedback loop in order to balance the average input and output powers and expressing the average value of input voltage in terms of output voltage

\[ \langle v_s(t) \rangle_{T_s} = (1-d(t)) \langle v_0(t) \rangle_{T_s} \]  

(3.74)

The average switch current is given as

\[ \langle i_s(t) \rangle_{T_s} = d(t) \langle i_s(t) \rangle_{T_s} \]  

(3.75)

Rearranging equations (3.73) and (3.74) and substituting the relevant terms in equation (3.75) results

\[ \langle i_s(t) \rangle_{T_s} = d(t) \frac{\langle v_s(t) \rangle_{T_s}}{R_e(v_{\text{control}})} = d(t)(1-d(t)) \frac{\langle v_0(t) \rangle_{T_s}}{R_e(v_{\text{control}})} \]  

(3.76)

Equation (3.76) implements the average value of the switch current over one switching period and equation (3.77) exactly resembles the parabolic waveform produced
from Non Linear Carrier generator. Both the equations are identical when $d(t)$ is replaced by $\frac{I}{T_s}$

$$v_c(t) = v_{control}\left(\frac{I}{T_s}\right)\left(1 - \frac{I}{T_s}\right) \text{ for } 0 \leq t \leq T_s$$

(3.77)

$$v_c(t + T_s) = v_c(t)$$

(3.78)

When $t = dT_s$ the derived signal of switch current $v_i(t)$ equals $v_c(t)$ and the comparator output becomes high which turns OFF the active switch.

$$v_i(dT_s) = v_c(dT_s) = v_{control}d(t)(1 - d(t))$$

(3.79)

The emulated resistance calculated from equation (3.73) is as follows:

$$R_e(v_{control}) = d(t)(1 - d(t))\frac{\langle v_0(t) \rangle_{T_s}}{\langle i_s(t) \rangle_{T_s}}$$

(3.80)

### 3.10.1 NLC Control Scheme

Figure 3.12 (a) shows the block diagram of NLC Controller for single module Boost Converter. Figure 3.12 (b) illustrates the waveforms relevant to various functional blocks behind the NLC Controller. It is desired to regulate the output voltage and to improve the input power factor [33, 34]. The clock pulse of short duration sets the flip flop at the beginning of each switching period which turns ON the switch. The DC output voltage is compared with the reference voltage and the resulting error signal is processed by the PI Controller which produces the actuating signal $v_{control}(t)$ for the carrier generator from which periodic non linear parabolic carrier is generated. The switch current is sensed and the integral of the switch current is compared with the non linear carrier waveform. The signal proportional to the switch current can also be used
for comparison instead of the integral of switch current. At \( t = T_s \), when \( v_i(t) = v_e(t) \), the comparator output goes high which resets the flip flop and turns OFF the switch. This process is repeated for all switching periods.

**Figure 3.12:** (a) Closed Loop Control using Non Linear Carrier (NLC) Controller (b) Pulse Generation

### 3.10.2 Simulation Results

Figure 3.13 (a) depicts the actual DC output voltage \((V_o)\) for rated load current of 2.4 A. It is clear from Figure 3.13 (b) that the source current wave shape is sinusoidal and also in phase with the source voltage. From Figure 3.13 (c), it is seen that a constant DC output voltage is maintained for step change in load from 1.6 A to 2.0 A at \( t = 0.3 \) s. The output voltage settles to 24 V after a transient interval of 120 ms. The output voltage is well regulated for an abrupt increase (1.2 A to 1.6 A) and decrease in load (1.6
A to 1.2 A) which are shown in Figure 3.13 (e). It is observed from Figures 3.13 (d) and (f) that the source current is sinusoidal and the same wave shape is retained with different amplitudes for load variation. This proves that the suggested controller makes the input current wave shape to be sinusoidal and remain in phase with the source voltage.

It is understood from the waveforms shown in Figure 3.13 (g) that the servo response of the closed loop system is verified by changing the reference voltage and it is observed that the change in reference voltage from 20 V to 24 V and then to 28 V is accurately tracked by the output voltage. Figure 3.13 (h) shows the Harmonic spectrum of source current and the Total Harmonic Distortion (THD) is found to be 9.61%.
Figure 3.13: Performance of PFC Boost Converter with Non Linear Carrier (NLC) Controller
3.11 Conclusion

In this chapter, the design, analysis and control algorithms for fixed frequency Linear Quadratic Controller and variable frequency controllers namely, HC and NLC Controllers are presented. The effectiveness of these controllers is verified by the simulation results. Hysteresis Controller is suitable for low power applications because of its simplicity. But the variable frequency operations invite problems like more EMI and complicated filter design. The NLC Controller reduces the number of sensors in spite of problems associated with variable frequency operation. In commercial applications, linear or fixed frequency LQC is preferred due to low power loss. Hence, it is concluded that linear controllers are the best choice for output voltage regulation and input current wave shaping.