Chapter 2

Review of Literature

The oscillation theory for the solution of differential equations is one of the conventional trends in the qualitative theory of differential equations.

2.1 Oscillatory Behavior of Ordinary and Delay Differential Equations

For the oscillatory behavior of ordinary differential equations, one can refer the monographs [20, 30, 64]. The oscillation theory of delay differential equations was initiated in a pioneering paper [25] of Fite. Subsequently, there has been much research work regarding the oscillatory behavior of different kinds of delay differential equations. In [29], Graef et al. studied the existence of oscillatory and nonoscillatory solutions of neutral differential equations with positive and negative coefficients, in which the authors obtained some necessary and sufficient conditions for the existence of bounded and unbounded positive solutions. In [77], Zhou et al. established some existence results for neutral type differential equations with positive and negative coefficients. In [72], Zhang et al. studied the existence of nonoscillatory solutions of first-order neutral delay differential equations. In [73], Zheng et al. discussed the new oscillation criteria for second-order quasi-linear differential equations with an oscillatory forcing term by using the Riccati
technique and the variational principle. In [63], Sun et al. established an interval oscillation results for the second-order forcing type ordinary differential equations with mixed nonlinearities. In [74], Zheng et al. developed the oscillation criteria for forced second order differential equations with mixed nonlinearities by using Riccati transformation technique. In [12], Candan presented some new sufficient conditions for the existence of nonoscillatory solutions for the system of higher order neutral differential equations by using the Banach contraction principle. Recently, many authors have been considered the oscillatory behavior of integer order differential equations, see [4, 13, 26, 49].

For more details, one can refer the monographs [2, 3, 7, 23, 31, 45] and the references cited therein.

2.2 Oscillatory Behavior of Fractional Differential Equations

In recent years, many researchers found that the fractional differential equations are more accurate in describing some practical models. Today it has been used widely in physics, electrochemistry, control theory and electromagnetic fields [19, 21, 43, 48, 54]. The study of the oscillatory problem with a view on the fractional differential equation is just being initiated. As a new cross-cutting area, recently some attention has been offered to oscillations of fractional differential and difference equations [16, 17, 28, 34, 46, 55, 75].

In 2012, Grace et al. [28] initiated the study of oscillation theory for fractional differential equations. The authors obtained the oscillation criteria for a class of nonlinear fractional differential equations of the form

$$D^q_a x + f_1(t, x) = v(t) + f_2(t, x), \quad \lim_{t \to a^+} J_a^1 x(t) = b_1,$$
where $D^q_a$ is the Riemann-Liouville fractional differential operator of order $q$, $0 < q \leq 1$.

After that, in 2012, Chen [16] studied the oscillatory behavior of the following fractional differential equation

$$[r(t)(D^\alpha_a y(t))]' - q(t)f\left(\int_t^\infty (v-t)^{-\alpha}y(v)dv\right) = 0, \quad t > 0,$$

where $D^\alpha_a$ denotes the Liouville right-sided fractional derivative of order $\alpha \in (0, 1)$.

The authors obtained some oscillation criteria by using a generalized Riccati transformation technique.

Using the same technique, in 2013, Chen [17] discussed the oscillatory behavior of the fractional differential equation of the form

$$(D^{1+\alpha}_{-\infty} y)(t) - p(t)\left(D^\alpha_a y\right)(t) + q(t)f\left(\int_t^\infty (v-t)^{-\alpha}y(v)dv\right) = 0, \quad t > 0,$$

where $D^\alpha_a y$ denotes the Liouville right-sided derivative of order $\alpha \in (0, 1)$.

Using the integral transformation and inequalities technique, in 2013, Shao et al. [61] established some new oscillation criteria for the following fractional differential equation with mixed nonlinearities

$$D^t_a x - p(t)x(t) + \sum_{i=1}^m q_i(t) |x(t)|^{\lambda_i-1}x(t) = v(t), \quad \lim_{t \to a^+} J_a^{1-q}x(t) = a_1,$$

where $p(t)$, $v(t)$ and $q_i(t)$ ($1 \leq i \leq m$) are continuous functions on $[a, +\infty)$, and $\lambda_i$ ($1 \leq i \leq m$) are ratios of odd positive integers with $\lambda_1 > \ldots > \lambda_l > 1 > \lambda_{l+1} > \ldots > \lambda_m$.

In 2014, Wang et al. [66] obtained some oscillation results for the following fractional differential equation by using the Riccati technique:

$$D^a_a x(t) + q(t)f(x(t)) = 0, \quad t \in [a, \infty), \quad a > 0,$$

where $D^a_a$ is the standard Riemann-Liouville differential operator of order $\alpha$ with $0 < \alpha \leq 1$. 
Recently, many authors have been studied the oscillatory behavior of fractional differential equations, see [10, 18, 53, 65, 68, 76, 79, 80].

The above fact motivated us to do our research in the area of oscillatory behavior of solutions of various kind of fractional differential equations using Riccati transformation, integral averaging technique and some inequalities.