2.1 Introduction

This chapter presents a survey of the investigations carried out on the analysis of cylindrical pressure vessel with torispherical end domes. This topic being a classical subject, quite a number of studies - analytical, experimental and numerical - have been reported in the literature. As it is difficult to cover the entire literature, a few notable ones on vessels with torispherical heads under internal pressure are discussed here. The earlier investigations are presented in chronological order in section 2.2, starting from the work of Galletly (1959). This section has three subdivisions: section 2.2.1 analytical studies, section 2.2.2 experimental studies and section 2.2.3 numerical studies. The above studies are summerised and the objectives and the scope of the present work are given in section 2.3.

2.2 Pressure vessel with torispherical head

The torispherical head geometry is composed of two circular arcs, crown of radius \( R_d \) at the top and knuckle of radius \( R_k \). This is fitted on the cylinder of radius \( R_c \). With reference to Fig. 2.1, \( O_d \) is the center of curvature of the crown and \( O_k \) is the center of curvature of the knuckle. The angle subtended by the knuckle at \( O_k \) is computed as:
\[ \alpha = \cos^{-1}\left( \frac{R_c - R_k}{R_d - R_k} \right) \] \hspace{1cm} \text{... (2.1)}

This ensures the continuity of slopes at ‘a’, the junction of crown and knuckle.

The rise of head H is computed by

\[ H = R_d - \sqrt{\left( (R_d - R_k)^2 - (R_c - R_k)^2 \right)} \] \hspace{1cm} \text{... (2.2)}

\( h_c, h_d \) and \( h_k \) are the thicknesses of cylinder, crown and knuckle respectively. For the case of uniform wall thickness, we have \( h_c = h_d = h_k \). In the description of the above geometry, the following two parameters are defined: (i) the ratio of crown-to-knuckle radii, \( \beta = \frac{R_d}{R_k} \) and (ii) the ratio of knuckle-to-cylinder thicknesses, \( \gamma = \frac{h_k}{h_c} \). For the case of pressure vessel with uniform wall thickness, \( \gamma \) is equal to 1. In the case of hemispherical dome, \( \beta \) is equal to 1.
In the view of the technological importance of pressure vessel with torispherical head, several studies have been carried out and reported in the literature. There are several codes of practice governing the design of pressure vessel, namely, IS 2825-1969, ASME Pressure vessel code 2004 Section VIII and BS 1515.

2.2.1 Analytical studies of torispherical vessels

It was Galletly (1959) who pointed out the deficiencies in the design as per the provisions of ASME code on unfired pressures vessels, when torispherical heads are used. According to him, the code predicted the stresses in the knuckle, which were less than one half of those actually occurring and gave no indication of possibility of failure by yielding and also by buckling due to circumferential compressive stress. Shield and Drucker (1961) carried out analytical studies of torispherical vessel under internal pressure and presented graphs and tables for the design and analysis of the knuckle region of a thin torispherical head of uniform thickness. Comparing their result and ASME codal provision, they have suggested approximate formula for the limit pressure,

Fig. 2.1 Geometry of a typical torispherical pressure vessel.
\[
\frac{nP_D}{\sigma_y} = \left(0.33 + 5.5 \frac{R_k}{D_c}\right) \frac{h}{R_d} + 28 \left(1 - 2.2 \frac{R_k}{D_c}\right) \left(\frac{h}{R_d}\right)^2 - 0.0006
\]  
\[ \text{... (2.3)} \]

where, \(\frac{R_k}{D_c} = 0.06, 0.08, 0.10, 0.12, 0.14, 0.16\) and \(\frac{R_d}{D_c} = 1.0, 0.9, 0.8, 0.7, 0.6\)

Ganapathy and Charles (1976) analyzed the stresses in torispherical pressure vessel by classical asymptotic expansion. They arrived at a simple design formula to find the maximum stress in the toroidal segment, instability pressure and optimum design of the toroidal knuckle.

\[
(\sigma_\varphi)_{\text{max}} = \frac{pR_d}{2h} \left[1 + 0.354 \left(\frac{3}{1 - \nu^2}\right)^{1/2} \left(\frac{R_d}{R_k} - 1\right) \left(\exp\left(-\frac{\pi}{4}\right) g(\rho)\right)\right]
\]  
\[ \text{... (2.4)} \]

where,

\[
g(\rho) = \frac{\left\{\exp\left(\frac{\pi}{4} - \frac{\eta \cot \frac{\eta}{2}}{2}\right)\right\}}{\left\{(2)^{1/4} \cos(\eta/2)\right\}}, \quad \rho = \frac{Vr}{2Ec \sin^2 \Phi}, \quad \text{and} \quad \eta = \cos^{-1} \rho
\]

where \(V\) is the axial stress resultant, \(r\) is the radial distance of meridian from axis of revolution, \(c\) is the reduced thickness and \(\Phi\) is the angle between normal to the meridian and axis of revolution. They compared the numerical results with experimental results available in the literature and have reported a close agreement between the two.

Batchelor and Taylor (1979) carried out studies on lower bound to limit pressure of ellipsoidal and torispherical headed vessels using numerical optimization technique. They analysed (i) for the equivalence between torispherical and ellipsoidal heads, (ii) for torispherical heads of constant head height-to-cylinder radius ratio \(H/R_c\). (There was a peak limit pressure against knuckle radius, indicating optimum values of geometrical parameters for head heights in the range from 0.33 to 0.5) and (iii) the peak collapse pressures for torispherical heads were lower than that for ellipsoidal heads, even to the extent of 50 \%. 
2.2.2 Experimental studies of torispherical vessels

Fessler and Stanley (1965, 1966) carried out experimental studies on torispherical pressure vessel heads using photo elastic model. They pointed out the dependence of the elastic stress distribution on the shape and thickness parameters in a wide range of torispherical heads. The wall thickness of specimens were equal to that of cylinder ($h_d = h_c$). They fabricated 32 nos. of models having the wall thickness-to-cylinder radius ratio ($h/R_c$) in the range of 0.22 to 0.030 and the head height-to-cylinder radius ratio ($H/R_c$) were varied from 0.876 to 0.2. The peak principle stress indices were presented in the form of two contour plots and in terms of mean and bending stresses. They computed the maximum bending stresses in the knuckle region in all models and presented an empirical equation relating peak elastic stress, the head height and the wall thickness ratios in a range of torispherical heads. The empirical formula derived from their experimental investigations is:

$$I_k = 0.145 \left( \frac{H_i}{D_i} \right)^{-1.68} + 1.76 \frac{h}{D_i}$$

.. (2.5)

where $I_k$ is the peak stress index at knuckle and $D_i$ is the diameter of cylinder inner surface.

Fessler and Thorpe (1970) carried out experimental studies on torispherical headed vessel obtained by explosive forming. Explosive forming method which involves forcing flat plates repeatedly through ring dies produces approximately torispherical shapes with uniform wall thickness. Even though the models were not perfect in the geometrical shapes (there were deviations at the cylinder-knuckle junctions), the stresses obtained were reasonably in good agreement with that of earlier models (made by other methods).

Gwaltney (1972) carried out experimental and analytical studies on torispherical vessel and torispherical head with radial nozzle attached at its apex. He carried out stress and
buckling analyses. First model was subjected to internal pressure and second one to combined internal pressure, axial thrust and bending moment applied to the nozzle. He developed solutions for bending stresses, both under internal and external pressures. Membrane solutions were developed and the analytical results were compared with experimental results. Kitching and Lau (1973 and 1975) carried out experimental and numerical studies on model of two kinds: (i) without any openings on the head and (ii) with openings (3 inches diameter) on the head just above the knuckle region (with cover plate). The geometric parameters of the models were \( D_c = 18 \) inches, \( H = 3.025 \) inches \( L = 8.5 \) inches and \( R_k = 5.82 \% \) of the cylinder diameter. The stresses were found to be higher in the models with openings. Kirk and Gill (1975) carried out experimental studies on the failure of torispherical domes due to instability and plastic deformation. Twelve models were prepared in an aluminium alloy of three head heights \( (H = 24.5, 27.18 \text{ and } 32.08 \text{ mm}) \) and four internal diameter-thickness ratios. \( (D_i/h = 53, 106, 212 \text{ and } 530, \text{ where } D_i \text{-cylinder internal diameter } = 135 \text{ mm}) \). They pointed out that the spherical end is pushed outward and the knuckle is pulled inward with compressive hoop stresses. Peak bending moments in the meridional direction occur in the region of the sphere-knuckle and knuckle-cylinder junctions which are tensile on the outer surface and at about the mid-point of the knuckle with tension on the inner surface. The effect of change of geometry, increases in the knuckle radius as the deformation increases and this is of major importance.

Patel and Gill (1978) carried out experimental studies on the buckling behaviour of thin torispherical heads under internal pressure in the region of knuckle. Ten models were prepared in an aluminium alloy. All specimens were of constant internal cylinder diameter to thickness ratio equal to 531.5. The crown radius \( (R_d = 135 \text{ mm}) \) was equal to cylinder
diameter \( (D_c = 135 \text{ mm}) \) and only the knuckle radius \( (R_k) \) was varied from 7 to 30 mm. They concluded that for series of vessel of constant thickness, constant cylinder diameter (equal to crown radius) and various knuckle radii, the buckling pressure increases with knuckle radius and the two specimens with largest knuckle radii \( (27.94 \text{ and } 30.48 \text{ mm}) \) did not buckle.

Stanley and Campbell (1981a, 1981b and 1991) carried out experimental studies on torispherical head pressure vessels subjected to internal pressure. They prepared the models in an austenitic stainless steel material with nominal diameters equal to 54, 81 and 108 inches and radius-thickness ratios \( (R_c/h) \) equal to 21.1, 316 and 420. They predicted that the greatest stress index (stress index \( I \) is the ratio of the principal stress at a point to nominal hoop stress in the cylinder) in each case was that of the hoop compression in the knuckle. The inner surface meridional stress index reached the peak in the knuckle and was tensile in nature. The hoop stress index on the inner surface \( I_{\phi1} \) greater than that of the outer surface in the knuckle. They also concluded that no simple correlation was apparent between the peak stress indices and the thickness ratios for the different crown shapes. For an increase in knuckle-to-cylinder radius ratio, \( R_k/R_c \) (and \( h_d/R_c \) remaining constant), the peak stress indices were found to decrease. It was observed that as the ratio of knuckle-to-cylinder radius increases, the geometry of the crown tends to behave as that of hemispherical dome. The variations \( I_{\phi} \) and \( I_{c} \) with \( R_d/R_c \) \( (R_k/R_c \) and \( h_d/R_c \) remaining constant) followed no simple consistent trend.

Barton et al. (1981) prepared the torispherical head models in glass-reinforced plastic material for their experimental as well as analytical (using BOSORS program) studies. These models were of same nominal thickness \( (h = 0.010 \text{ m}) \) and crown radius, \( R_d \) equal to cylinder diameter, \( D_c \) of 1 m. Three different knuckle radii, \( R_k \) of 0.29, 0.19 and 0.1 m were used to
give head height, H of 0.32, 0.25, and 0.19 m, respectively and subjected to internal pressure of 1200 kPa. However, the actual thicknesses over the dome were measured (which were found to be varying) and these are the input in BOSORS program. Comparing the results of both their experimental and analytical studies, they arrived at the conclusions: (i) the stress index (I) increases with decrease of knuckle radius, (ii) while applying actual measured thickness from specimen in BOSOR program, there was a reduction in the magnitude of the peak stress compared to that corresponding to the nominal (uniform) thickness and (iii) the mode of failure was initiated in cylinder, when the knuckle radius was large and for smaller radius of knuckle, the failure was initiated in the knuckle portion.

Galletly (1981) and Bushnell (1982) investigated plastic buckling of pressure vessels torispherical and ellipsoidal heads subjected to internal pressure by experimental and numerical studies. They presented the design equation from the investigations as:

$$\ln\left( \frac{P_{cr}}{100} \right) = -0.6 + 0.89(A + B) - 1.49C + 0.42B^2 + 0.33A^2B + 0.305AC + 0.78ABC \quad \ldots (2.6)$$

where \( A = \ln\left( \frac{\sigma_{sp}}{60000} \right), B = \ln\left( \frac{R_k}{D_c} \right) \), and \( C = \ln\left( \frac{D_c}{h} \right) \).

Galletly (1982) carried out experimental and numerical studies on 30 nos. of thin stainless steel torispheres subjected to internal pressure (diameter of the vessels varied from 1.4 to 4.2 m). He proposed a design procedure for predicting plastic buckling pressure of torispherical shell. Comparison of experimental and analytical results indicated the safety margin greater than 1.5. Barton et al. (1982) prepared the torispherical head models in glass-reinforced plastic material for their experimental as well as analytical studies using BOSORS program. These models each having same head height (H = 0.25 m), cylinder inside diameter
(Dc = 1 m), nominal thickness (h = 0.01 m), and with three different knuckle radii (Rk = 0.1, 0.146 and 0.188 m) and crown radii (Rd = 0.71, 0.8 and 1.0 m) were subjected to internal pressure of 1200 kPa. They tested three vessels: (i) torispherical head (Barton et al. (1981)), (ii) ellipsoidal head and (iii) torispherical head, Rk = 10% of Dc and H = 0.19, Rd = Dc = 1 m. For all models, the material properties were: E = 7.0 GPa, ν = 0.3 and results were obtained for internal pressures 211, 411, 611, 811 and 1011 kPa. In all models, the peak stress index occurred at the knuckle portion and the stress and strain indices increased when knuckle radius was decreased. Barton et al. (1984) proposed the design procedure for hemispherical and torispherical headed vessels using the result of Barton et al. (1981 and 1982). It was based on the effect of change of shape of the head due to pressure and maximum stress and maximum strain. Head height ratios (H/Dc) of torispheres were equal to 0.20, 0.25, and 0.32. There were also cases of torispherical head of large knuckle radius and small crown radius which are more like hemispheres. They generated design data for torispherical head vessels from the above investigation.

Miller et al. (1986 and 2001) conducted experimental studies on two fabricated steel torispherical heads under internal pressure and investigated their buckling and rupture strengths. These two models were of same diameter = 192 inches, knuckle radius = 32.64 inches, crown radius = 172.8 inches and thickness was 0.196 inches for model 1 and 0.270 inches for model 2. They found that (i) the ratios of the rupture pressure to the first buckle pressure were 3.97 and 2.55 for models 1 and 2 respectively, (ii) the head rise of model 1 increased 17 % and that of model 2, increased 27 %. The corresponding H/Dc ratios were 0.29 and 0.31 compared with initial value of 0.25. These were all due to the elastic deformations. Galletly (1986a and 1986b) analysed the results of experimental and numerical
investigations on torispherical shells subjected to internal pressure and obtained design equations for preventing buckling as:

$$\frac{p_n}{\gamma F} = \frac{80\left(\frac{R_c}{D_c}\right)^{0.825}}{\left(\frac{D_c}{h}\right)^{1.5}\left(\frac{R_d}{D_c}\right)^{1.15}}$$  \quad \ldots (2.7)

where \( p_n \) = allowable internal design pressure (safety factor > 1.5),

\( \gamma \) - a constant, equal to 1.0 for crown and segment steel heads and 1.6 for cold spun steel heads

\( F = \sigma_y \text{ or } 0.2 \text{ percent proof stress of material} \)

Roche and Autrusson (1986) carried out experimental studies on buckling of uniform thickness torispheres by method of plastic bifurcation analysis. Brookfield et al. (1986) investigated experimental studies on Shakedown and cold creep of stainless steel type 316 of uniform thickness torispheres internally pressurized. Blachute (1995 and 1997) and Magnucki et al. (2000 and 2002) carried out experimental investigations on torispherical head models made of mild steel with five sets of geometry parameters, keeping cylinder diameter (= 200 mm) and thickness (= 6 mm) constant. The models were subjected to internal pressure, and deflection of the apex point was measured. From the plots of apex deflection vs. internal pressure, yield pressure was obtained and was compared with analytical values. They did not investigate the nature of stresses at the knuckle region where the yield was expected to occur. Theoretically calculated plastic collapse pressure differed marginally, not more than 12% from those obtained by experiments.

Likhman et al. (1996) investigated the strength of torispherical headed vessels and tanks for cryogenic rocket engines by experimental and numerical procedures. They prepared twenty two models in different combinations of geometric parameters with different
materials and applied monotonic loading and repetitive (cyclic) loading of internal pressure. The actual safety factor \( n = 5.2 \) was considerably greater than the values specified in the standards. Updike and Kalnin (1998) and Kalnin and Updike (1998) carried out experimental and numerical studies on pressure at a tensile plastic instability and burst pressure of torispherical head vessel subjected internal pressure. Tensile plastic instability is defined as when applied pressure is plotted versus internal volume change, it is expected that the plot will tend to a condition at which an increase in volume requires no increase in pressure. If this condition is reached, it is called a tensile plastic instability of the vessel. They prepared 39 models and tested. They prescribed the mathematical model for the calculation of the loads at tensile plastic instabilities of axisymmetric thin walled geometries. The burst pressure obtained from the empirical equation was within 4 % of the test pressure. Blachute and Vu (2007) carried out experimental and numerical investigations on burst pressures of torispheres and shallow spherical caps. The models were made of aluminum and steel. The burst pressure was obtained from numerical method (finite element) smaller than experimental values with errors varying from -0.4 % to -14 % for aluminum vessel and higher than experimental values, with errors ranging from 15.2 % to 37.9 %.

2.2.3 Numerical studies of torispherical vessels

Gotsulyak (1972) carried numerical studies on torispherical heads. He considered shell parameters \( D_c/h = 1000, R_d/D_c = 0.7414, R_k/D_c = 0.173 \) and length 2.0 m. He found that an increase in thickness of the knuckle by a factor of 1.2 compared to the thickness of the cylindrical and spherical parts reduces the compressive circumferential stress in the knuckle part by 34 % and increases the critical load by 70 to 80 %. With an increase in \( h_k \) by a factor
of 1.3, (stepped variation) a cylinder withstood 2.5 times higher internal pressure than that of uniform knuckle thickness. Ramesh et al. (1974) carried out stress analysis of cylindrical vessels with hemispherical, torispherical and ellipsoidal heads, using numerical integration (Runge-Kutta-Gill algorithm) procedure on the classical of thin shell theory. Comparison of results of the three heads, for \( \frac{D_c}{h} \) = 100, showed that the hoop (outer) compressive stresses in the ellipsoidal head were two and half times greater than that in the torispherical head. High compressive stresses were induced in the hoop direction in the region of 45° to 50° from the apex of the crown, whereas in ellipsoidal heads it was in the region of 60° to 65°.

Aggarwal et al. (1978 and 1982) carried out the parametric studies on cylindrical pressure vessel with hemispherical, torispherical, semi-ellipsoidal and toriconical heads, internally pressurised by finite element method. They used axisymmetric solid finite element. The numerical data used were: \( D_c = 0.25 \) m, \( p = 1 \) kgf/cm² \((\approx 0.1\text{MPa})\), cylinder diameter - wall thickness ratios, \( \frac{D_c}{h} = 25, 50, 100, 200, 300 \) and \( 400 \), \( \alpha = 60° \) (for all value of \( \frac{D_c}{h} \)), \( \nu = 0.3 \) and \( \frac{\sigma_y}{E} = 0.001607 \). They have plotted the stress pattern in axial and hoop directions. They also investigated the dilation or radial growth of different heads by using membrane solution, and obtained an expression for the same as:

\[
\delta_r = \frac{p \ D_c \ (2-\nu)}{8hE} \quad \cdots (2.8)
\]

where \( \delta_r \) = dilation of cylindrical vessel (increase in radius). They concluded that: (i) the effect of increased thickness was reflected by an increase in bending action, (ii) the peak stress occurred near the junction, (iii) the cylindrical wall always experienced hoop stresses causing longitudinal cracking and (iv) the head portion experienced peak meridional stresses near the junction which were likely to cause circumferential cracking.
Aylward and Galletly (1979) carried out numerical studies on buckling of very thin torispheres using finite difference method. They assumed that the shells were geometrically perfect and without any residual stresses and used the numerical data: $R_{k}/D_{c} = 0.06, 0.08, 0.1, \text{ and } 0.15$, $R_{d}/D_{c} = 0.75, 1.0, 1.25 \text{ and } 1.5$, $D_{c}/h = 500, 1000, \text{ and } 2000$, $\nu = 0.3$ and $E = 207 \text{ GPa (Steel)}$. They have plotted maximum equivalent stress curve and initial yield curves and concluded that elastic buckling occurred for shells with large $D/h$ ratios of the order of 2000.

They have proposed an empirical relation for the elastic buckling pressure as:

$$\frac{P_{cr}}{E} = K \alpha_1 \alpha_2 \alpha_3 \left( \frac{h}{D_{c}} \right)^{\beta} \quad \ldots \text{(2.9)}$$

where $K = 0.167 \times 10^{3}$,

$$\alpha_1 = 1.1 \left( R_{d}/D \right)^{2} - 1.5 \left( R_{d}/D \right) + 1.0,$$

$$\alpha_2 = 48.0 \left( R_{k}/D_{c} \right)^{2} - 6.0 \left( R_{k}/D_{c} \right) + 1.0,$$

$$\alpha_3 = \{-3.5 \left( R_{k}/D_{c} \right)^{2} + 8.0 \left( R_{k}/D_{c} \right) - 0.32 \right\} \left( R_{d}/D_{c} \right)^{2} + 1.0 \text{ and } \beta = 2.058 + 0.4 \left( R_{d}/D_{c} \right).$$

Sori‘c (1990, 1995a and 1995b) investigated stability of the torispherical shells subjected to internal pressure (buckling in the knuckle region) by finite element method treating it as a geometrically nonlinear problem. He used doubly curved finite element with 48 dof per element using the software, FEMAS. The imperfection shape was assumed to be that of first buckling mode. The magnitude of imperfection was taken to be $1/10^{th}$ of wall thickness. The internal pressure which defines the minimum post buckling value was 27% lower than the critical pressure in the primary bifurcation point. Torispherical shells failed to demonstrate any significant imperfection sensitivity for the imperfection pattern analysed.
The load carrying capacity of imperfect shell was found to be about 8% less than that of the perfect geometry. For the imperfection amplitude equal to one half of the wall thickness, the reduction in the load was found to be about 15%. Hamada et al. (1989) carried out investigation on buckling of torispherical vessels by finite element method. They used axisymmetric shell element. They use Aylward and Galletly (1979) numerical data in non-linear prebuckling analysis. The finite element results were in good agreement with results of Aylward and Galletly (1979) obtained using finite difference method.

Yeom and Robinson (1996) carried out numerical studies on elastic-plastic behaviour of pressure vessels with torispherical heads. They assumed uniform thickness. The vessels were subjected to internal pressure. The numerical parameters of torispherical heads were $R_c/H = 1.0, 1.5625, 2.0, 2.5, 3.33333$, $R_k/R_c = 0.12, 0.2, 0.3$ and $R_c/h = 10, 14.2857, 25, 60$. They used eight node axisymmetric finite solid element (ABAQUS code) and carried out non-linear analysis. The material was assumed elastic-perfectly plastic, $E = 70$ GPa, $\nu = 0.3$ and uniaxial yield strength of 20MPa. The results indicated that the error in first yield pressure was 10% in comparison with earlier studies, when $R_c/h$ is larger than 25 and limit pressure was within 5% when $R_c/h = 50$

Tafreshi and Thorpe (1996) and Tafreshi (1997) carried out investigations on the design sensitivity of geometric imperfection of thin torispherical headed vessel, subjected to an internal pressure by finite element method. They used the data available in literature [Stanley and Campbell (1981a, 1981b)] as input. Three node axisymmetric shell element, designated as SAX2 in the ABAQUS was used for modeling. The deformed profiles showed
inward displacement in the knuckle part. They analysed the stresses in perfect and imperfect models and concluded that there was a distinction between the real and nominal dimensions (imperfect and perfect geometries). Finite element models of end domes considering thickness imperfection was analysed and stresses were compared with the results of perfect domes and with the corresponding experimental results. With imperfection, the stress indices were shown to differ by significant amounts (up to 60 %) from those in perfectly formed vessel. The effects of thickness variation had significant influence.

Yamamoto et al. (1997) performed elastic, plastic and limit analyses by finite element analysis in pressure vessel when internally pressurized (p = 8.6MPa). The vessel head was composed of a knuckle, a conical shell and shallow spherical crown with uniform thickness. The inner dimensions of the model was $D_c = 3 \text{ m}$, $R_d = 4.5 \text{ m}$, $R_k = 0.36 \text{ m}$, $h = 0.225 \text{ m}$, length of shell = 3 m, length of conical shell = 0.6582 m, $h = 0.225 \text{ m}$. The material properties of the model were $E = 1.75 \times 10^5 \text{ MPa}$ and $\nu = 0.3$. In the limit and plastic analysis, the calculation was performed by double elastic slope method (DESM). From elastic analysis, they arrived at the following: (i) maximum value and its von Mises equivalent stress along the inner surface and (ii) maximum value of averaged and linearised stress across thickness. The calculated collapse loads are within 5 % difference compared to the other in the small deformation theory. Reicheng Tong and Xucheng Wang (1997) investigated the structural limit analysis on crown with nozzle (subjected internal pressure and radial load) and torispheres (subjected internal pressure) by finite element procedure. They used quadrilateral element and in the thickness direction 1, 2, 4, and 5 elements were used and 3, 18, 20, 25 and 50 elements were used along the meridional direction. The results showed that a quite coarse mesh density is sufficient to obtain very satisfactory results.
Mahendra and Arturs Kalnins (2000) analysed the code cases on design of ellipsoidal and torispherical heads as given in ASME section VIII vessels. They suggested that (i) ellipsoidal heads are specified to be designed as equivalent torispherical heads. The design formulae for ellipsoidal and torispherical heads are the same, (ii) the knuckle radius must be greater than or equal to 0.06 \( D_c \) or 3h. The crown radius \( R_d \) must be less than or equal to \( D_c \), (iii) the Code Cases have been developed to prevent the failure mode of burst and low cycle fatigue. The specified lower limit on \( h/R_d = 0.002 \) preclude knuckle buckling. José et al. (2006) carried out finite element analysis (using two node axisymmetric shell element) for limit analysis of torispherical pressure vessels using object oriented programming (OOP). They applied this technique in (Shield and Drucker (1961)) ASME torispherical problem and got good agreement with their result.

2.3 Objectives and scope of this work

From the above literature review, it may be observed that the pressure vessel with torispherical head has remained as a topic of active research interest during the last five decades, starting from Galletly (1959) till date, Blachute et al. (2007). It is due to its technological importance in several areas, analytical (based on shell theory solutions), numerical (finite element and finite difference methods) and experimental investigations have been carried out. The focal point in most of these investigations has been the severe stress field in the knuckle region. For certain values of knuckle-to-crown radius, the hoop stress turns out to be compressive, giving rise to the possibility of buckling in the knuckle. Also, it may be observed that all earlier investigators have assumed uniform wall thickness in the three sections: cylinder, knuckle and the crown.
The objective of the present study is to analyse the pressure vessel with torispherical heads with variable thickness in the knuckle region. The increased thickness profiles are obtained in two ways: (i) tangent method and (ii) spline method. It is of interest to investigate the reduction in the stress levels due to the increased (variable) thickness in the knuckle region. In this respect, the present work differs from all other earlier investigations.

The scope of this work is restricted to the study of stress fields in the knuckle region by numerical method and does not include the analysis for buckling. The possibility of occurrence of buckling failure is remote, in view of the increased wall thickness. The analyses are carried out using (i) axisymmetric shell element (two node and three node elements) and (ii) axisymmetric solid element (four node quadrilateral, four node incompatible mode option and eight node quadratic elements). The von Mises stress indices are taken for comparison of performance and numerical accuracy of various elements used.