Vessels are in use in many of the technological applications: process equipments in chemical industries, boilers in mechanical Engineering and rockets in aerospace, containment vessels in nuclear engineering and also in hydrospace applications (submersibles and seabed-based off-shore oil extraction facilities). The major load in these applications is either internal pressure (former three applications) or external pressure (the latter applications). Also vessels are used to store liquids (storage tanks), the load being only the hydrostatic pressure and there is no additional pressure load. In all cases, vessels (thin shells) of spherical geometry are considered appropriate from structural efficiency point of view, as the deformations and stresses are lower due to doubly curved geometry. However, from the applications point of view, vessel of circular geometry is in use in many cases and hence assumes technological importance. Cylindrical shell is closed at both ends in order to convert it as ‘pressure vessel’. The end-closures are of different forms: (i) circular diaphragms (circular plate welded), (ii) hemispherical (iii) torispherical (iv) hemiellipsoidal and (v) toriconical. The end closures are referred to as either ‘domes’ or ‘heads’ in the literature.
In view of the high stresses developed at the junction between flat plate and cylinder, the circular diaphragm type of end-closure is not found suitable for high pressure applications. However, this is used for storage tanks. Pressure vessels with hemispherical heads are extensively used in ground-based applications (chemical and mechanical engineering). The total length of the vessels is equal to the length of the cylinder and twice the radius of the dome \((L+2R_c)\). Such vessels are subjected to membrane state of stress in both axial and hoop directions, with hoop direction component being double. In the junction between the cylinder and head, the stress field gets disturbed in the axial direction, inducing bending state of stress in addition to the membrane state of stress. This disturbance is due to the differential expansion of the cylinder and head parts. Dome, being a double curved surface, undergoes smaller deformation and cylinder, being a singly curved shell, undergoes larger deformation. However, the perturbed stress level in the axial direction of the junction is still lower than that of the hoop stress in the cylinder, about 30% increase over the membrane axial stress.

The torispherical heads are obtained by rotating a plane curve consisting of two circular arcs, giving rise to two regions in the dome: knuckle and crown. The radius of the knuckle is smaller than that of the crown. The knuckle is the transition region between the cylinder and the crown and also the crown radius is larger than that of the cylinder. The advantage in the pressure vessel with torispherical head is the reduction in overall lower length compared to that of the vessel with hemispherical head. This advantage is being utilized in the aerospace applications (solid propellant, liquid propellant and cryogenic rocket engines). Figs. 1.1(a-b) are examples of torispherical heads used in aerospace field. Fig.
1.1(a) is the Space Shuttle and Fig. 1.1(b) is ISRO’s GSLV. Both use solid propellant strap-on motors (cylindrical shell with torispherical heads). Fig. 1.2 is a longitudinal section of Liquid Metal Fast Breeder Reactor (LMFBR) under development in India, (Indira Gandhi Centre for Atomic Research, Kalpakkam). The important structural component in LMFBR is the main vessel, shown in red thick line whose bottom is of torispherical geometry. Shorter the overall length of the rocket, easier the control of stability of the vehicle in flight. However, there is a serious disadvantage with the design of pressure vessel with torispherical head. When internally pressurised, the differential deformation at the junction between cylinder and knuckle becomes very large, the cylinder expands outward and the knuckle deforms inward. The stress perturbation due to this differential deformation is very high and induces severe bending state of stress. The axial stress component becomes higher than the circumferential stress components for certain combination of geometrical parameters.

![Fig. 1.1 Use of SRBs in different missions, (a) Space Shuttle, USA (NASA website), (b) GSLV-MkIII Rocket, (ISRO, India).](image-url)
The structural behaviours of hemiellipsoidal and toriconical heads are also of this nature. As the stress level in the knuckle region is always high, it controls the design of pressure vessel. One way of managing/reducing the stress levels is to go for increased thickness in the knuckle region. The present study is focused on this issue. This thesis material has been presented in 9 chapters. Chapter 2 gives the literature survey on pressure vessel with torispherical heads that have been reported in the open literature in the last 50 years. The material has been divided into 3 sections: introduction, studies on torispherical
head (analytical, experimental and numerical) and lastly the objectives and scope of the present work. Chapter 3 presents pressure vessel with different geometrical heads. This chapter has been divided into 2 sections: first on description and analytical solution of the pressure vessel with hemispherical head and second on description of pressure vessel with torispherical head of uniform thickness and variable thickness at knuckle region obtained by spline and tangent methods. Chapter 4 presents the FE formulation of two types: thin shell axisymmetric element (two node and three node) and axisymmetric solid element (four node linear quadrilateral, four node with incompatible mode and eight node quadratic). The finite element modeling of hemispherical headed pressure vessel and torispherical head pressure vessel are presented in Chapter 5. Results of numerical investigations are presented in Chapter 6. The first and the second sections contain the introduction and numerical data, the third section is on relative performances of finite elements, the fourth on torispherical head with knuckle of uniform thickness, the fifth on knuckle of variable thickness obtained by spline method, and the sixth on knuckle of variable thickness obtained by tangent method. The last section presents a comparison of results on knuckle of variable thicknesses obtained by spline method and tangent method. Chapter 7 presents the conclusions arrived at. The scope of future work is presented in Chapter 8. The references are given at the end.