Chapter 6

Conclusion

In this thesis, nonclassicality of the quantum states in the Rabi and Kerr models are investigated. The chapters 2 and 3 are devoted to the study of the Rabi model. In the generalized rotating wave approximation scheme, the Hamiltonian is block diagonalized. This facilitates direct evaluation of the energy eigenvalues and the corresponding eigenstates satisfying the completeness property. The time evolution of the initial hybrid Bell state is explicitly computed. The ready construction of the time dependent density matrix of the bipartite pure state yields the reduced density matrices of the qubit and the oscillator via partial tracing of the complementary part of the respective degrees of freedom. The reduced qubit density matrix furnishes the expectation value of the spin variable and its von Neumann entropy. On the other hand the reduced density matrix of the oscillator provides the quasiprobabilities such as the diagonal $P$-representation, Wigner $W$-distribution and Husimi $Q$-function. Instances of rapidly oscillating derivatives of the $\delta$-function in the $P$-representation and the existence of the negative region of the $W$-distribution, which is quantified by the negativity measure $\delta_W$, indicate the nonclassicality of the states. The nonnegative Husimi $Q$-function yields the Wehrl entropy $S_Q$ that acts as a measure of delocalization on the phase space. The presence of multiple time scales observed in the evolution of the Wehrl entropy $S_Q$ is induced by the interaction, and it introduces a novel feature in the generation of kitten states. In the strong coupling limit $\lambda \lesssim 0.05\omega$ the $Q$-function evolves at rational submultiples of $T_{\text{long}}$ to a collection of uniformly separated Gaussian peaks representing the kitten states. The short-range time period $T_{\text{short}}$ is, however, associated with the splitting and rejoining of the peaks at a particular rational fraction leading to the scaling relation $T_{\text{short}}(T_{\text{long}}/q) \approx (1/q)T_{\text{short}}(T_{\text{long}})$. As the $S_Q$ approaches stochastic stabilization, the decoherence time is found to be proportional to $\sqrt{\lambda}$. A comparative study of the Wigner entropy $S_W$ and Wehrl entropy $S_Q$ is done on the following two possible cases. (i) In
the strong coupling regime $\lambda/\omega \gtrsim 0.1$ for a dominant value of $\delta_W$ the relation $S_W > S_Q$ is obeyed. On the other hand when $\delta_W \ll 1$ the inequality is reversed: $S_W < S_Q$. (ii) In the ultrastrong coupling regime $\lambda/\omega \gg 0.1$ the nonclassicality of the states remains prominent despite the stochastic stabilization. At relatively small value of the coherent state amplitude $\alpha \lesssim 1$ and for the strong coupling regime $\lambda \sim 0.1\omega$, the evolution to squeezing is observed for the large detuning and the resonant frequencies. The observed squeezed state represents almost pure state of the oscillator as the von Neumann entropy is much less than its maximal value. For the coupling strength $\lambda \lesssim 0.1\omega$ the Mandel parameter takes negative values during parts of its evolution indicating sub-Poissonian behavior of the photon statistics. The time averaged value of the Mandel parameter is plotted for the coupling regime $\lambda \gg 0.1\omega$ indicating the overall photon statistics to be Poissonian.

The chapters 4 and 5 constituting the second part of the thesis involve study of the nonclassicality of the multiple photon-added squeezed coherent Schrödinger cat state in a nonlinear Kerr medium. The unitary time evolution of the said pure state density matrix is obtained. The nonclassicality of the states is studied via the quasiprobability distributions. The presence of the anharmonic term in the Hamiltonian leads to periodicity of the Wehrl entropy $S_Q$ and the negativity $\delta_W$ of the $W$-distribution that develops a series of local minima at the rational submultiples of the said period. The transient kitten structures are formed at these local minima of the Wehrl entropy $S_Q$. The kitten states at these instants coincides with the superposition of finite number of photon-added squeezed coherent states. To validate the above transitory convergence of states, the fiducial marker state occurring as a superposition of multiple photon-added squeezed kittens is constructed and the Hilbert-Schmidt distance between the evolving state and the fiducial marker state is observed to vanish indicating their transient indistinguishability. This demonstrates that the time evolving state momentarily coincides with the Yurke-Stoler type of photon-added squeezed kitten states. The optical tomogram which provide an alternate description of the quantum state vis-à-vis the one provided by the quasiprobability distributions is utilized towards the study of the transient kitten-like
states, and its closed form expressions at the relevant times are obtained. Nonclassical depth is employed to study the extent of nonclassicality of the transient kitten states at the respective times. It is observed that at the times such as $\lambda t = \frac{\pi}{4\sqrt{2}}$ which are irrational multiples of the period, the kitten-like formations are not realized. These states, however, possesses large negativity measure $\delta_R(\sigma)$. The amplitude decay and the phase damping models are studied through the Lindblad master equation in the context of the evolution of the quantum state in a Kerr medium interacting with a reservoir at zero temperature. In the presence of amplitude damping, the asymptotic values of the corresponding Wehrl entropy and negativity read: $S_Q(t \to \infty) \to 1 + \log \pi$ and $\delta_W(t \to \infty) \to 0$, respectively. In the phase damping model, as the initial diagonal elements of the density matrix are retained in the dissipation process, the asymptotic limit of $S_Q$ achieves a steady state value corresponding to its universal lower bound, and the nonclassicality of the states are partially retained. In the asymptotic limit, the negativity decreases with increased squeezing.