Chapter 4

Nonclassicality and decoherence: An overview

4.1 Introduction

As nonclassicality of the light states is a key feature in quantum communication protocols its amplification and control in physical systems are of much interest. One way of controlling the nonclassicality of a quantum light state is to add (subtract) a finite number of photons to (from) another suitable light state. The sub-Poissonian character of the fields in states produced by the addition of photons to a coherent state has been studied in [59]. The nonclassical properties of the single photon-subtracted squeezed vacuum as well as the coherent states have been observed [60, 61] specifically in terms of the sub-Poissonian statistics and the corresponding phase space quasiprobability distributions. Experimental generation of a single photon-added thermal state has been achieved [62] and its quantumness has been studied by constructing the negativity of the corresponding Wigner distribution. In this context the Kerr-type nonlinear dispersive medium exhibiting the third order light-matter interaction [63] is of importance.

4.2 Kerr Hamiltonian

In some medium, it has been observed that the refractive index changes with the intensity of the incident light. This effect is known as the optical Kerr effect and the medium is called as Kerr medium. While passing through such media the field undergoes an intensity dependent phase shift. It has been recently noticed [64] that a quantum mechanical resonator, whose tiny vibrations are controlled by the radiation pressure of the cavity field, mimics a Kerr oscillator when the cavity field is driven by a coherent light. Employing the Born-Oppenheimer approximation for the optomechanical system an effective Hamiltonian that leads to a nonlinear Kerr effect has been constructed [65]. Moreover, the optomechanical system has been found [65] to induce a squeezing effect in
the intensity spectrum of the cavity field. The optical Kerr effect has been employed [66] towards a quantum nondemolition measurement of the photon number, and its accuracy, which is linked with the imposed phase noise via the uncertainty principle, has been studied. It has also been observed [67] that under the condition of electromagnetically induced transparency the Kerr effect can make a very significant contribution to the group velocity of the slow light propagation through a four-level $N$-type atomic system.

The propagation of quantum states of light in a Kerr medium has been extensively studied. In particular, it has been observed [3–6] that an initial coherent state therein evolves, at certain specified times, to superposition of coherent states popularly known as Schrödinger kitten states that display macroscopic characteristics for a large value of the relevant complex amplitude in the phase space. Assuming a squeezed coherent state as input for a nonlinear Kerr medium it has been found [68] that its evolution provides a finite superposition of squeezed kitten states at certain instants. The squeezed states of the harmonic oscillator are, however, of much experimental focus as they involve the reduction of the fluctuations in one quadrature variable below the ground state uncertainty. These states play a key role in quantum metrology towards improving the sensitivity of the interferometers [69], and are of crucial importance in the recent gravitational wave detection via high-power laser interferometers [70]. The squeezed states are also important in the continuous variable quantum key distribution protocols [71], and they impart [72] an enhancement in comparison to their coherent analogs. Moreover, they have recently been used as sensitive detectors for photon scattering recoil events at the single photon level [73].

The Hamiltonian of a single field mode in a nonlinear Kerr medium [74] reads in natural units ($\hbar = 1$) as follows:

$$H = \omega a^\dagger a + \lambda a^\dagger^2 a^2,$$

(4.1)

where the oscillator of frequency $\omega$ is described by the annihilation and creation operators $(a, a^\dagger |\hat{n} \equiv a^\dagger a)$, and the coupling constant $\lambda$ corresponds to the third-order susceptibility
of the Kerr medium.

### 4.3 Optical tomogram

The optical tomography has been recently advanced as an important procedure towards measuring and reconstructing the quantum state of optical fields [75–77]. It is based on the one-to-one correspondence between the quasiprobability distributions and the relevant probability distribution of the arbitrarily rotated quadrature of the field variable [75]. The optical tomogram contains all informations on the quantum system and provides its alternate description vis-à-vis the conventional density matrix representation [75,76]. Experimentally, a series of measurements of the rotated quadrature variable is performed on an ensemble of identically prepared systems. The histogram obtained therefrom facilitates reconstruction [9] of the quasiprobability distributions.

The arbitrarily rotated quadrature variable and its eigenstate are defined as

\[
\hat{X} = \frac{1}{\sqrt{2}} \left( a \exp(-i\varphi) + a^\dagger \exp(i\varphi) \right), \quad \hat{X} |X,\varphi\rangle = X |X,\varphi\rangle. \tag{4.2}
\]

The construction [78] of the eigenstate (4.2) of the rotated position operator reads

\[
|X,\varphi\rangle = \frac{1}{\pi^{1/4}} \exp \left( -\frac{X^2}{2} - \frac{e^{2\varphi}}{2} a^\dagger a + \sqrt{2} e^{i\varphi} X a^\dagger \right) |0\rangle. \tag{4.3}
\]

The optical tomogram \(\Omega(X,\varphi)\) is defined [75] as the probability distribution of the rotated quadrature operator \(\hat{X}\) expressed via the expectation value of the density matrix in the basis of the eigenstates (4.3):

\[
\Omega(X,\varphi) \equiv \langle X,\varphi|\rho|X,\varphi\rangle, \quad \int_{-\infty}^{\infty} dX \Omega(X,\varphi) = 1. \tag{4.4}
\]

It has been observed [79] that the optical tomogram \(\Omega(X,\varphi)\) embodies signatures of revival and fractional revivals of a quantum system, and, therefore, may be suitably applied towards the study of the generation of kitten states.
4.4 Decoherence models

In realistic quantum system, the effect of the environment influences the dynamics of the system significantly. These systems are called as open quantum system and they play an important role in many applications of quantum systems as the perfect isolation of the system from the environment is not possible. Unlike the closed system, the dynamics of the open system cannot be given in terms of unitary evolution. In general, the open systems [80, 81] that admit interactions with suitable environmental degrees of freedom are characterized by the irreversible loss of information and the dissipative processes. The evolution of the states under decoherence is of interest as it explains the realistic scenario.

The decoherence of quantum states can happen by different ways based on what type of interaction that takes place between the field and the environment. In single mode field, the decoherence can occur either by photon decay induced by the reservoir modes or due to the phase damping. Considering the case of decoherence by photon decay to the environment, there is a loss of energy in the system. This type of decay model is called as the amplitude damping. In phase damping, the energy of the system does not change but the relative phase of the eigenenergy changes. In other words, in the phase damping only the off diagonal elements change whereas the diagonal elements remain invariant.

The Lindblad master equation [82, 83] for the time evolution of the density matrix provides a simple framework for a dynamical description of the decoherence mechanism. In the context of the evolution of a quantum state in a Kerr medium interacting with a reservoir at zero temperature the Lindblad master equation assumes [84] the form

\[ \frac{d\rho}{dt} = -i[H, \rho] + \gamma \left( [X\rho, X^\dagger] + [X, \rho X^\dagger] \right), \]  

(4.5)

where \( X \) is a suitable operator embodying the dissipation mechanism, and \( \gamma \) is the damping factor. Employing the algebraic structure of the superoperators an analytical solution to the master equation (4.5) has been provided in [85]. In our analysis we closely
follow the method prescribed therein.

The next chapter provides the study of nonclassicality of the photon-added squeezed coherent Schrödinger kitten states in a Kerr medium. In addition, nonclassicality in the presence of decoherence is also discussed.