Chapter 1

Preface

Quantum mechanics is one of the cornerstones of our description of physical reality in general, and the microscopic realm of fields and elementary particles in particular. Characteristically the quantum mechanical states exhibit nonclassical features such as uncertainty and entanglement [1,2]. For a harmonic oscillator the coherent states carry the minimum uncertainty measure, and its phase space distributions display Gaussian features. For a large expectation value of the photon number operator the coherent state may be regarded as macroscopic in nature. The superposition of coherent states such as the so-called cat states, however, maintain non-Gaussian properties in their phase space quasiprobability distributions. The states endowed with non-Gaussian attributes are known to play important roles in elucidating the fundamental principles of quantum mechanics, and are expected to provide advantages in various quantum information protocols. In this work we will use the word ‘nonclassical’ and ‘non-Gaussian’ somewhat interchangeably.

A two-level system (qubit) interacting with an oscillator can exhibit very rich nonclassical features and has been studied widely. This involves the production of the cat-like states and atom-field entanglement via the qubit-oscillator interaction. The appearance of the cat (and, additionally; kitten) states in such framework can be compared with the effect of the Kerr-like nonlinear self-interacting photonic models, where it has been observed [3–6] that an initial coherent state therein evolves, at rational fractions of the time period, to the superposition of certain macroscopic coherent states popularly known in general as kitten states showing nonclassical behavior. This phenomenon is replicated in the qubit-oscillator interacting model with, however, an important distinction that sets it apart from the Kerr-like evolution. Namely, the existence of interaction-generated multiple time scales gives rise to a periodic doubling of the number of lobes in the phase space description of the kitten states. In the strong coupling limit, in particular, the
study of the nonclassicality of the qubit-oscillator interacting system is of much interest. On the other hand, the nonclassicality of a quantum light state can be enhanced by adding (subtracting) a finite number of photons to (from) another suitable light state. Investigating the evolution of an arbitrary number of photon-added states in a nonlinear medium may lead to better understanding of nonclassicality. In a realistic physical system, the influence of environment to the quantum system should be considered. This leads to decoherence which leads to the emergence of classical properties from a quantum system.

1.1 Objective of the thesis:

The main objectives of the thesis are listed below:

1. To analytically study the evolution of an initial hybrid entangled state in a coupled qubit-oscillator system within the generalized rotating wave approximation scheme: The transitory generation of kitten states and the nonclassicality of the time evolved state in the bipartite system is discussed. The quasiprobability distributions allow us to study the underlying differences between the information theoretic measures such as the Wehrl entropy and the Wigner entropy.

2. To study the nonclassicality of the evolution of a superposition of an arbitrary number of photon-added squeezed coherent Schrödinger kitten states in a nonlinear Kerr medium: The nonclassicality of the kitten states is studied in the presence of decoherence.

1.2 Thesis organization:

The aim of Chapter 2 is to describe the qubit-oscillator model and its importance. Quoting the new experimental advancements an overview of the necessity for considering the ultrastrong coupling regime in the qubit-oscillator model is provided. This chapter gives a brief review of various approximation schemes and their validity regimes. In
particular, the generalized rotating wave approximation [7] is discussed. We also indicate the hybrid entangled states of the bipartite coupled qubit-oscillator system and their importance in various areas.

In Chapter 3, we analytically study the evolution of an initial hybrid entangled state in a coupled qubit-oscillator system within the generalized rotating wave approximation scheme. Our study includes both the strong and ultrastrong coupling domains. The qubit reduced density matrix provides the von Neumann entropy of the system that measures the entanglement and the mixedness of the state. The oscillator reduced density matrix, in turn, yields the phase space quasiprobability distributions [8] such as the $P$-representation, the Wigner $W$-distribution, and the Husimi $Q$-function. The negativity of the $W$-distribution reflects the departure of the oscillator from the classical states, and allows us to study the underlying differences between various information-theoretic measures such as the Wehrl entropy and the Wigner entropy.

The Chapter 4 gives a brief review about the effect of photon addition to the quantum states and also about the importance of squeezed states in various applications. This chapter also discusses a brief mathematical description of optical tomogram which facilitates reconstruction [9] of the quasiprobability distributions.

In Chapter 5 we investigate the evolution of an arbitrary number of photon-added squeezed coherent Schrödinger cat states in a nonlinear medium. The nonclassical depth [10, 11] that utilizes a highly singular $P$-representation to estimate the degree of nonclassicality is introduced. The nonlinearity of the medium gives rise to the periodicities of the quantities such as the Wehrl entropy and the negativity of the $W$-distribution, and a series of local minima of these quantities arise at the rational submultiples of the said period. By using the Hilbert-Schmidt distance between the quantum states we demonstrate that our evolving state transitorily coincides with, in general, the Yurke-Stoler type of photon-added squeezed kitten states, which maintain a uniform rotation of the phase space variables on the complex plane. Employing the Lindblad master equation approach we study the amplitude and the phase damping models for the initial state considered here.
In Chapter 6 we make concluding remarks. The main results of this thesis are briefly summarized in this chapter.