CHAPTER 4

ANALYSIS OF TRANSMISSION HOUSING AND GEAR HOUSING USING FINITE ELEMENT METHOD

4.1 INTRODUCTION

The finite element method is a numerical analysis technique for obtaining approximate solutions to a wide variety of engineering problems. In several situations it becomes necessary to obtain approximate numerical solution to problems when determination of exact/closed-form solutions becomes impossible. A finite element model of a problem gives piecewise approximate solutions to the governing equations. The basic premise of the finite element method is that a solution region can be analytically modeled or approximated by replacing it with an assemblage of discrete elements. Since these elements can be put together in a variety of ways, they can be used to represent exceedingly complex shapes. In the present work, the finite element method has been used as the tool to perform the dynamic analysis of the drive housings. The analysis has been performed for two major types of drive housings viz. transmission housing and gear housing. In general the following types of finite elements have been used to perform the analysis.

4.2 ELEMENTS USED FOR THE ANALYSIS

- Shell element
- 3D-Beam element
- Brick element.
- Mass element
- Rigid link element

The description of each element is given below.

### 4.2.1 Shell Element

A shell is a structure that can be derived from a plate by initially forming the middle surface as a singly (or doubly) curved surface. The assumption used in thin plates for the transverse distribution of strains and stresses are valid for shells. The way in which the shell supports external loads is different from that of the flat plate. The stress resultants acting on the middle surface of the shell have both tangential and normal components which carry a major part of the load.

![Shell element diagram](image)

**Figure 4.1 Shell element**

Shell element has both bending and membrane capabilities. The element has six degrees of freedom at each node: translations in the nodal X, Y, and Z directions and rotations about the nodal X, Y, and Z-axis. Since in-plane and bending behavior are uncoupled for an isotropic plate, the individual stiffness matrices can be separately obtained and then combined to
get the element stiffness matrix. A linear variation for in-plane displacement \( u \) and \( v \) and a cubic variation for the bending displacement \( w \) have been assumed in the present work to represent the plates of both gear and transmission housing.

### 4.2.2 3D-Beam Element

This element is used to model beams subjected to load in all the three directions. Figure 4.2 shows a typical beam element stretching along its length and having lateral deformation along \( Y \) and \( Z \) axes. The two nodes have six degrees of freedom each, three translations and three rotations. In the current work the 3D beam element has been used for modeling shafts mounted in gear housing.

![3D-Beam element](image)

**Figure 4.2 3D-Beam element**

### 4.2.3 Brick Element

Simple 3D element is an eight-noded brick element. The eight nodes and local axes of this isoparametric element are given in Figure 4.3. It is advantageous to use isoparametric elements because transformation of the element stiffness matrix from the local to global co-ordinates is not needed and also these can take curved boundaries into account. It is used to model bearing blocks in gear housing.
4.2.4 Mass Element

The mass element is defined by a single node, concentrated mass components in the element coordinate directions, and rotary inertias about the element coordinate axes. The element coordinate system may be initially parallel to the global cartesian coordinate system or to the nodal coordinate system. It is possible to exclude the rotary inertia effects and to reduce the element to a 2-D element. The element has only one mass input, which is assumed to act in all appropriate coordinate directions. Mass element used in the present analysis is a point element having only three translatory degrees of freedom in X, Y, and Z directions. It is used to model the motor and gears as lumped masses.

4.2.5 Rigid Link Element

It is used to model a rigid constraint between two deformable bodies or as a rigid component used to transmit forces and moments in
engineering applications. The element has two nodes and three degrees of freedom at each node \((U_x, U_y, U_z)\). The cross-sectional area of the element is assumed to be one unit. In the present work it is used to connect mass element (motor) to the shell element (housing) in case of transmission housing. In gear housing, it is used to connect ends of shaft to the bearing blocks.

The modeling of transmission housing was drawn as per the Figures 4.4 for seven different cases using CAD software (Pro-Engineer). The modeling files are converted into IGES-files and are imported in analysis software (ANSYS) and free meshing is done. Similarly modeling and meshing for four different cases of gear housings as per the Figure 4.8 and Figure 4.9 are made.

Modal analysis (free vibration) is performed and the solution is obtained using various mode extraction solution procedures. Among various mode extract solution method, Block Lanczos method is used in the present work to solve the equation because computational time is relatively less. The Lanczos procedure is presented in Appendix II.

### 4.3 RESPONSE OF TRANSMISSION HOUSING

#### 4.3.1 Various Models of Transmission Housing Considered for Study

The various models of transmission housing with inclusion of ribs, stiffeners, rectangular cutout, rectangular cutout with stiffener and circular cutout are shown in Figure 4.4.
Figure 4.4(a) Simple housing

Figure 4.4(b) Housing with ribs at four corners

Figure 4.4(c) Housing with ribs and stiffener

Figure 4.4(d) Housing with rectangular cutout

Figure 4.4(e) Housing with cutout and stiffener

Figure 4.4(f) Housing with circular cutout

Figure 4.4 Various models of transmission housing
The box shown in Figure 4.4(a) is analyzed by modeling the five faces except the bottom face of the box with shell elements and mass of the drive system on the top of the box, represented by mass elements. The number of degrees of freedom allowed for shell element is six at each node and the number of degrees of freedom for mass element is three. The box is modified subsequently by introducing ribs at the four corners in Figure 4.4(b). The box is assumed fixed at the base (all the six degrees of freedom zero) and free at the top.

![Figure 4.5(a) Simple housing](image)

For the box model, the mode shape of FEA model shown in Figure 4.5(a) indicates that the four vertical rectangular plates have approximately sinusoidal mode shape along any horizontal plane (laterally deflected plate) whereas along any vertical plane they behave like a cantilever. Fundamental natural frequency obtained for is 31.6 cps as against to 62.8 cps obtained treating the box as a beam as stated in section 3.4.1. The beam assumption implies the four rectangular side plates bend in such way that the rectangular cross section is preserved while bending. But the deformed shape obtained by finite element method indicates bulging of the plates which is true as shown in Figure 4.5(a). Flexibility of the individual plates is overlooked in the beam model. The beam approach is found to overestimate the natural frequency.
Hence use of energy approach for plate model with modified boundary condition as discussed in section 3.4.3 gives is validated.

4.3.2 Housing with Unequal Angles (ribs) at the Four Corners

In Figure 4.4(b), the rectangular box with unequal angles at the four corners of box is shown. Each corner is strengthened by welding two unequal angles of standard size (60x40x4mm). These are included in the finite element model by additional shell elements. The fundamental mode shape of housing with ribs at four corners is shown in Figure 4.5(b).

Figure 4.5 (Continued)
Figure 4.5 Fundamental mode shapes of transmission housing

It is seen from the Figure 4.5(b) that the effective length and breadth of the box section are reduced due to the angles welded at four corners. In the Figure 4.5(b) the original length and breadth are reduced by \((2 \times 40) = 80\) mm. Hence it will result in increase of natural frequency which is 35.2 cps. It is observed that the use of ribs increases the both weight of the box section and natural frequency by approximately 15%.

4.3.3 Housing with Unequal Angles (ribs) and Channels (stiffeners)

From Figure 4.6(a), 4.6(b) and 4.6(c) and Table 4.1 the following observations are made. The lowest natural frequency of housing without ribs is 31.6 cps for Figure 4.6(a) increases to 35.2 cps when ribs at the corners are
introduced as in Figure 4.6(b) and it further increases to 107.7 cps when ribs and stiffeners are introduced as shown in Figure 4.6(c). The effective length of the side plate which is 1000mm as in Figure 4.6a, decreases to 920 mm \((1000 - (2 \times 40))\) when the ribs are introduced. The length is further reduced to 260 mm when ribs and stiffeners are introduced. The height of the plate remains unchanged at 1000 mm. The corresponding mode shapes as shown in Figure 4.5(a) – 4.5(c) confirm this observation. It may also be observed from Figure 4.5(c), that the lateral deformation of the plate is pronounced between the ribs and the stiffeners and is almost absent in the stiffener zone. If the natural frequency of the deformed individual plate is assumed to be represented by Equation (3.16) (which is valid for hinged-hinged boundary conditions), the natural frequency of box without ribs, with ribs and with ribs and stiffeners should change in the ratio \([1+1]: [1 + (1/0.92^2)]: [1 + (1/0.26^2)]\). This ratio is found to be 1: 1.16: 7.7. For the lowest frequency of 31.6 cps obtained for plate with no ribs and stiffeners, the other two should be approximately 35 cps and 225 cps whereas the finite element method gives 35.2 cps and 178.2 cps. From the above discussions, it is concluded that the lowest natural frequency of transmission housing is very much dependent on the two vertical faces of the box which are parallel to XY plane. It is found that strengthening these two faces with unequal angles (ribs) and channels (stiffeners) increases the fundamental natural frequency from 31.6 cps to 35.2 cps and 178.2 cps.

4.3.4 Housing with Rectangular Cutout

In this model, the box is provided with rectangular cutouts of size 500 x 500 mm at center of two larger vertical plates. The provision of larger cut out brings down the fundamental natural frequency to 14.0 cps as seen in Figure 4.5(d) due to decrease in stiffness of the housing.
4.3.5 **Housing with Circular Cutout**

A circular cutout of diameter 500 mm at centre is now provided in the box. The fundamental natural frequency for this model is found to be 14.4 cps which is almost comparable with box of rectangular cutout.

4.3.6 **Housing with Rectangular Cutout and Stiffeners**

In this model, the box with cutout is stiffened by supporting channels of standard size (ISMC 400). The fundamental natural frequency is found to increase to 15.1 cps.

The summary of results is listed in Table 4.1.

**Table 4.1 Natural frequencies in cps – Transmission housing**

<table>
<thead>
<tr>
<th>Model</th>
<th>Housing without ribs</th>
<th>Housing with ribs</th>
<th>Housing with ribs and stiffeners</th>
<th>Housing with rectangular cutout</th>
<th>Housing with circular cutout</th>
<th>Housing with rectangular cutout and stiffeners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental natural frequency</td>
<td>31.6</td>
<td>35.2</td>
<td>178.2</td>
<td>14.0</td>
<td>14.4</td>
<td>15.1</td>
</tr>
<tr>
<td>Second natural frequency</td>
<td>44.7</td>
<td>70.2</td>
<td>256.0</td>
<td>53.4</td>
<td>47.1</td>
<td>54.6</td>
</tr>
</tbody>
</table>

4.4 **RESPONSE OF GEAR HOUSING**

Figure 1.2 shown in Chapter 1 gives the elevation and plan view of a housing of two stage fabricated spur gear box transmitting 1446 kW (1940 hp) with an input speed of 1500 rpm. It consists of six plates welded at the edges to form a box. The strength of the box is further enhanced by welding two unequal angles at each of the four corners to increase the rigidity
Similarly the bearing blocks are also supported by channels as shown in Figure 1.2. The details of the gear box are given in Table 4.2 and 4.3. Three antifriction bearings (SKF) are used to support the input, intermediate and output shafts respectively. The casing is modeled using 4 noded plate elements. Plate assumption is justified since the flat plates used in the manufacture is having thickness 10 times less than the other two dimensions viz length and width.

Table 4.2 Specification of gear box (Figure 1.2)

<table>
<thead>
<tr>
<th></th>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$z_3$</th>
<th>$z_4$</th>
<th>Module</th>
<th>face width</th>
</tr>
</thead>
<tbody>
<tr>
<td>High speed gears</td>
<td>45 120</td>
<td>---</td>
<td>----</td>
<td>----</td>
<td>7</td>
<td>240</td>
</tr>
<tr>
<td>Low speed gears</td>
<td>---</td>
<td>----</td>
<td>40 120</td>
<td>10</td>
<td>320</td>
<td></td>
</tr>
</tbody>
</table>

All the gears have 20° pressure angle

Weight of the shaft and gear at high speed end shaft = 0.24 tonnes

Weight of the shaft and gears at intermediate speed end shaft = 1.6 tonnes

Weight of the shaft and gear at low speed end shaft is = 3.3 tonnes

Table 4.3 Large size gear box dimensions (mm) (Figure 1.2)

<table>
<thead>
<tr>
<th>L</th>
<th>H1</th>
<th>H2</th>
<th>h</th>
<th>A</th>
<th>B</th>
<th>c</th>
<th>e1</th>
<th>e2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2530</td>
<td>1500</td>
<td>1000</td>
<td>800</td>
<td>830</td>
<td>820</td>
<td>578</td>
<td>400</td>
<td>200</td>
</tr>
</tbody>
</table>

d_1, d_2, d_3,- diameter of shaft 1, 2, 3 respectively.

D_1, D_2, D_3- diameter of bearing blocks 1, 2, and 3 respectively.

W- Width of the bearing block.
Since the gears in mesh are having line contact it is assumed that any shaft in the housing does not influence the lateral critical speed of any other shaft. Thus the shafts can be treated as separate entities when considering the housing natural frequencies. The transmission elements on each of the shafts are treated as lumped masses distributed at the bearing ends in the proportion of their distance from the bearing ends. The boundary conditions are as follows

1. All the degree of freedom are fixed at the base of gear box

2. The shaft does not allow the axial movement along the bearing ends, the degrees of freedom in translation along the axis are arrested at the periphery of the cutouts. The shaft is constrained from moving axially by the shoulders provided on the anti friction bearing housings as in Figure 4.7

3. The two halves of the housing are assumed integral for the purpose of analysis.

Figure 4.7 Gear drive – axial constraint
4.4.1 Analysis of Simple Gear Housing

For the gear casing (with no ribs and no strengthening channels) fundamental natural frequency obtained by finite element method is 35.9 cps. The mode shape is shown in Figure 4.8(a). This illustrates that the top cover plate is predominantly vibrating. Vertical plates holding the support for the horizontal shafts are not found to experience any modal deflection.

Figure 4.8(a) Model 1 (without ribs)    Figure 4.8(b) Model 2 (with ribs)    Figure 4.8 (c) Model 3 with (bearing blocks)    Figure 4.8(d) Model 4 (with ribs and stiffeners)

Figure 4.8 Fundamental mode shapes of the industrial gear housing
4.4.2 **Housing with Unequal Angles (ribs) at the Corners**

In this case the gear casing is added with ribs (with no strengthening channels) at the four corners of box. Each rib is fabricated by using two unequal angles of size 200x100x12 mm and is welded together. Ribs are included in the finite element model by additional plate elements. From the modal analysis, it is found that its fundamental natural frequency is around 36.6 cps. The mode shape as shown in Figure 4.8(b) illustrates that the top plate is again subjected to deformation.

4.4.3 **Housing with Boss at the Bearing Ends**

The bearing ends are projected along the shaft axis by 40 mm and along the radial direction by 120mm as shown in Figure 4.8 (c). These ends are meshed with eight noded brick elements. From the modal analysis, the fundamental natural frequency was found to be 36.8 cps. The mode shape as shown in Figure 4.8(c) illustrates the top plate is alone experiences deformation.

4.4.4 **Housing with Bearing Blocks Supported by Channels**

The bearing blocks are now supported by C-channels. High speed end, intermediate and low speed end bearing blocks are supported by standard channel sections (ISMC 200, 300, 400 respectively). The three sides of the channels are modeled as plates and meshed with four noded shell element as shown in Figure 4.8(d). From the modal analysis the fundamental natural frequency was found to be 43.7 cps. The mode shape as shown in Figure 4.8(d) illustrates the top plate is again experiences deformation. In all these cases the top plate is stiffened along its ends and its natural frequency is found to increase. If the top plate is considered as fixed at all its edges, and finite element analysis is performed on the plate alone fundamental natural frequency obtained is 46.5 cps. It agrees very well with the values in Model 4.
4.4.5 Validating the Assumed Boundary Condition

The difference between the transmission housing and the gear housing is the presence of driving shafts connecting the two longer vertical faces of the box and providing axial constraint along the axis of the shaft. The shaft is modeled as a 3D-Beam finite element. The gears on each of the shafts are treated as lumped masses distributed along the beam elements. The ends of the shafts are connected to the face of housing by rigid link elements.

![Figure 4.9 FE model of gear housing](image)

The dynamic analysis is performed for the above model and the fundamental natural frequency is observed to be 43.623 cps. It agrees with the value as discussed in section 4.4.4. In this model also top plate is vibrating. The mode shape is as shown in Figure 4.10.
4.4.6 Housing with provision of stiffeners on the top plate

The flexible top plate of gear housing is stiffened by standard channel section (ISMC 300) across the length of the plate. From the modal analysis the fundamental natural frequency was found to be 67.26 cps. The mode shape shown in Figure 4.11 (a) indicates the top plate again experiences deformation, but the fundamental natural frequency is found to increase by 35% in comparison with the case of top plate without stiffeners.
The flexible top plate of gear housing is also stiffened by standard channel section (ISMC 300) across the breadth of the plate. The modal analysis indicates that the fundamental natural frequency is found to be 57.12 cps. The mode shape shown in Figure 4.11 (b) shows that the top plate again experiences deformation, but the fundamental natural frequency is found to increase by 23.6%.

From the results, it is observed that stiffening of the top plate by standard channels increases the fundamental natural frequency of the gear housing. Depending upon the value of exciting frequency, top plate can be stiffened by either along the length of the plate or along the breadth of the plate.

4.5 CONCLUDING REMARKS

In this chapter, a comprehensive method for performing dynamic analysis on the drive housing by FEM was described. The fundamental natural frequency and mode shapes were extracted from the finite element model in drive housings. In case of transmission housings, the fundamental natural frequency was found to increase upon addition of ribs and stiffeners. The cutout on transmission housing reduces the fundamental natural frequency and it was found to increase by stiffening along the cutout using stiffeners. In case of gear housing, provision of axial constraint along the axis of the shaft on the side plates stiffened it and made the top plate more flexible. Introducing of ribs, stiffeners and bearing blocks marginally increased the flexible top plate fundamental natural frequency. To validate the FEM results of drive housing, experiments were conducted and the same has been discussed in the next chapter.