CHAPTER 3

ANALYSIS OF TRANSMISSION HOUSING USING ENERGY METHOD

3.1 INTRODUCTION

In dynamic analysis energy method is used to determine only the fundamental natural frequency of the system by computing the strain and kinetic energies of the system at any point of time and make use of law of conservation of energy when there is no damping. It is also used to verify the efficacy of finite element method results for the fundamental natural frequency of the transmission housing. In this chapter, an attempt made to study the behavior of transmission housings by using the energy approach in two ways

(i) By assuming the transmission housing to behave like a beam 
(ii) By assuming the transmission housing to behave like plates in bending.

In the first approach, as per beam theory, a rectangular cross section of simple transmission housing remains rectangle after deformation about the neutral axis, but it was not realised when FEM and experimental results are obtained where it shows bulging of plates. The study considering the second approach gave satisfactory results with inclusion of modified shape function. This approach was also extended to transmission housing with ribs provided at the four corners.
3.2 NECESSITY OF USING ENERGY METHOD

In dynamics, the natural frequencies of longitudinal, torsional and lateral vibration are usually determined by drawing the free body diagram, identifying the forces on all the masses and formulating the equation of motion for the different masses in case of closed coupled systems. Essentially, the approach is concerned with the identification of forces and moments, which keep the masses in equilibrium, which is a vector approach. It is by the compatibility of deformation in case of far coupled systems. Another approach to compute natural frequency is the energy approach by computing the strain and kinetic energies of the system at any point of time. It is a scalar method. Quite often, the time involved in computing the energy is much less as compared to evaluating the vector method of a large size problem. Energy approach provides a quick check on the natural frequencies. This method is based on law of conservation of energy when there is no damping. Energy method is used as one of the tools to determine fundamental natural frequency for the problem presented in the thesis.

3.3 RAYLEIGH’S ENERGY METHOD

The kinetic energy is stored in the mass and is proportional to the square of the velocity. The potential energy is the strain energy of the elastically deformed member.

The method to determine the approximate value of the fundamental natural frequency of a discrete system based on Rayleigh’s principle, states that the frequency of a conservative system vibrating about an equilibrium position has a stationary value in the neighborhood of a natural mode. This stationary value is a minimum value in the neighborhood of the fundamental natural mode.
For a single degree of freedom system, the principle of conservation of energy, in the context of an undamped vibrating system can be stated us

\[ T_1 + U_1 = T_2 + U_2 \] (3.1)

where T and U represent kinetic and strain energy respectively. The subscripts 1 and 2 denote two different instants of time. The subscript 1 is used to denote the time when the mass is passing through its static equilibrium position and \( U_1 = 0 \) is chosen as reference for the potential energy. The subscript 2 indicate the time corresponding to the maximum displacement of the masses, and here \( T_2 = 0 \). Thus Equation (3.1) becomes

\[ T_1 + 0 = 0 + U_2 \] (3.2)

If the system is undergoing harmonic motion, then \( T_1 \) and \( U_2 \) denote the maximum values of T and U respectively and Equation (3.2) becomes

\[ T_{\text{max}} = U_{\text{max}} \] (3.3)

The Equation (3.3) states the Rayleighs energy method for solving dynamic problems.

### 3.4 SIMPLE MODEL OF TRANSMISSION HOUSING

A basic model of box type transmission housing carrying a motor is considered for the analysis. The foot dimensions of a standard DC Motor of 3.7 kW (5hp), speed range 0 to 1500 rpm, weighing of 750N are about 512x160mm. For analysis, the rectangular mild steel box to accommodate this motor is taken as having the dimensions of box as 1000 x 130 x 1000 mm. These were chosen by considering the functional requirements of box. The
thickness of the each side plates is taken as 4mm, which is minimum standard size available for such fabrication.

### 3.4.1 Beam Approach

Since the breadth of the box is only 130mm, the length and height is 1000mm, it is logical to assume that box bends as a beam, with every deformed section normal to the neutral axis remaining a box section.

![Figure 3.1 Assumed bent shape of box](image)

The two large rectangular faces of the box can be assumed to bend about Y axis with the shorter faces not participating in bending as shown in Figure 3.1. The two large faces can be assumed to contribute to strain energy due to flexure and short faces to kinetic energy only.

In beams, there is a constant exchange of kinetic energy acquired in the mean undisturbed position of the beam with the strain energy stored in bending of the beam at the two extremities of flexure. The kinetic energy contribution is accounted for from the distributed mass of the beam and also from the concentrated masses at the specified locations of the beam.
The general expression for the kinetic energy for a beam and a concentrated mass (motor) at the top of the beam is given by

\[ T_{\text{max}} = \frac{\omega^2}{2} \sum_{i=1}^n W_i w_i^2 \int_0^l \left( \frac{dW}{dx} \right) dx + \sum_{i=1}^n g W_i w_i^2 \]  

(3.4)

The strain energy stored in the beam is given by

\[ U_{\text{max}} = \frac{EI}{2} \int_0^l \left( \frac{d^2w}{dx^2} \right) dx \]

(3.5)

The amplitude of lateral deflection \( w \) is a function of \( x \), and the choice of this function very much depends on the boundary conditions of the beam.

The appropriate shape function for a cantilever beam (Figure 3.1) given by (Ramamurti 2008) is

\[ w(x) = C \left( 6 \frac{x^2}{H^2} - 4 \frac{x^3}{H^3} + \frac{x^4}{H^4} \right) \]

(3.6)

where \( C \) is arbitrary constant

(x measured from the base along the vertical and \( w \) the lateral deflection).

At fixed end of the neutral axis (\( x=0 \)), \( w=0, \frac{\partial w}{\partial x} = 0 \)  

(3.7)

and at the free end \( x=H \), \( \frac{\partial^2 w}{\partial x^2} = 0, \frac{\partial^3 w}{\partial x^3} = 0 \)  

(3.8)
The concentrated mass of the motor at the top contributes to kinetic energy. Using the expression for the kinetic energy and strain energy for the beam, and substituting the values of length, breadth, height, thickness of the box and material properties of mild steel in the above equations, fundamental natural frequency is computed. This gives a value of 62.8 cps for the model shown in Figure 3.1 with dimensions treating this as a cantilever with box section.

3.4.2 Closed form Solution with Hinged Ends

In general, when the length and breadth of a member are comparable and thickness is less than 10 times the other two dimensions, it is can be assumed as a plate. Since thickness of plate is 4mm and other two sides are 1000 mm, it is correct to assume as plate. In section 3.4.1, the box was assumed as behaving like a beam which did not give satisfactory result, when compared with results obtained using FEM and experimental to be discussed in next three chapters. Consider a simply supported rectangular plate as shown in Figure 3.2 of width a in X direction and length b in Y direction with thickness h.

Figure 3.2 Simply supported plate
The differential equation for a plate is given by

$$D \nabla^4 w + \rho h \ddot{w} = 0$$  \hspace{1cm} (3.9)

The hinged boundary condition at all edges of the plate is given by

and  \hspace{0.5cm} w_{,xx}=0 \text{ on the edge } x=0 \text{ and } x=a

w=0 \text{ on the edge } x=0 \text{ and } x=a

w_{,yy}=0 \text{ on the edge } y=0 \text{ and } y=b

w=0 \text{ on the edge } y=0 \text{ and } y=b  \hspace{1cm} (3.10)

The solution of the above harmonic motion is

$$w=W(x,y) \sin t$$  \hspace{1cm} (3.11)

Substituting the Equation (3.11) in Equation (3.9), it becomes

$$\nabla^4 W - \beta^2 W = 0$$  \hspace{1cm} (3.12)

Where  \hspace{0.5cm} \beta^2 = \frac{L^4 \omega^2}{D}  \hspace{1cm} (3.13)

The shape function to satisfy differential equation and boundary condition of plate is given by

$$W(x, y) = C \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}$$  \hspace{1cm} (3.14)
where \( C \) is an arbitrary constant

\[ m \text{ and } n \text{ are integers.} \]

Substituting the Equation (3.14) in Equation (3.12), it becomes

\[
\beta^2 = \pi^4 \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2
\]

Substituting the above in Equation (3.13)

\[
\frac{\rho h \omega^2}{D} = \pi^4 \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2
\]

Since the above equation is valid for any \( m \) and \( n \), the natural frequencies are given by

\[
\omega = \sqrt{\frac{D}{\rho h}} \pi^2 \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)
\]

With corresponding mode shapes

\[
W_{mn} = \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}
\]

Using Equation (3.16), fundamental natural frequency obtained for the box with dimensions 1000mm x 1000mm is 19.9 cps. This value is lower than the fundamental frequency obtained by finite element method as discussed in next chapter. This is due to the fact the top edges of the rectangular plates have zero lateral displacement as per Equation (3.14) used in this analysis, but in actual conditions, drive housing on the top cover may executes vibrations.
3.4.3 Improved Shape Function

![Figure 3.3 Deformation of vertical face with realistic boundary condition](image)

The assumption of hinged-hinged boundary conditions for vertical plates results in zero deflection at the top by using the shape function defined by Equation (3.14) which is found to be not agreeing with FEM and experimental results described in next chapters. To overcome this defect one has to choose an expression for mode shape which results in zero lateral deflection at the bottom edge and an expression which gives rise to non zero expression for the lateral deflection at the top edge.

The shape function $w$ for the vertical plate along height (X direction) and length (Y direction) is now expressed as

$$w(x, y) = C \sin \frac{\pi}{2} \left(\frac{x}{H^*}\right) \sin \frac{\pi y}{L}$$  

(3.18)

Where $H^*=1.1H$. The center line of the motor shaft is at height of 1.1H from the base. This ensures maximum kinetic energy contribution from motor weight as seen from Equation (3.23). Associated shape function $w$ for the vertical plate along height (X direction) and breadth (Z direction) (Figure 3.2) is expressed as
\[ w(x, z) = C \left( \frac{B}{L} \right) \sin \frac{\pi}{2} \left( \frac{x}{H} \right) \sin \frac{\pi z}{B} \]  

(3.19)

Equations (3.18) and (3.19) results in non zero deflection at \( x=H \) which agrees with actual conditions.

The strain energy for rectangular plate in bending is given by (Ramamurti 2008)

\[ U_{\text{max}} = \frac{D}{2} \int \int (w_{,xx} + w_{,yy})^2 - 2(1-\mu)(w_{,xx}w_{,yy} - w_{,xy}) \, dx \, dy \]  

(3.20)

and the kinetic energy for the same is

\[ T_{\text{max}} = \rho \int \int \frac{\omega^2 t}{2g} w^2 \, dx \, dy + \frac{\omega^2}{2} \sum_{i=1}^{n} W_i w^2 \]  

(3.21)

(X-vertical, Y-length, Z-breadth direction). For the four vertical faces of the box, this takes care of fixed-free ends vertically and hinged-hinged ends horizontally. The shape functions Equations (3.18) and (3.19) allow lateral deformation of the vertical faces. After substituting the expression given by Equations (3.18) and (3.19) in the Equations (3.20) and (3.21) the strain energy and kinetic energy are given by

\[ U_{\text{max}} = \frac{DC^2 \pi^4 LH}{8} \left[ \frac{0.62}{H^4} + \frac{0.9053}{L^4} + \frac{1.49636}{H^2L^2} \right] + \frac{D(1-\mu)C^2 \pi^4 LH}{4} \left[ \frac{0.156479}{H^2L^2} \right] \]

\[ + \frac{DC^2 \pi^4 BH}{8} \left( \frac{B}{L} \right)^2 \left[ \frac{0.62}{H^4} + \frac{0.9053}{B^4} + \frac{1.49636}{H^2B^2} \right] \]

\[ + \frac{D(1-\mu)C^2 \pi^4 BH}{4} \left( \frac{B}{L} \right)^2 \left[ \frac{0.156479}{H^2B^2} \right] \]  

(3.22)
\[ T_{\text{max}} = \frac{\omega^2 C^2 \rho t}{2} \left[ 0.22633 \cdot L + \left( \frac{B}{L} \right)^2 (0.4527 \cdot H \cdot B) \right] + \left( 0.045 \right)^2 \frac{W \omega^2 C^2}{2g} \left( 1 + \frac{B^2}{L^2} \right) \] (3.23)

Substituting the appropriate values for the box section considered, fundamental frequency is found to be 32.59 cps. This is found to be in good agreement with finite element and experimental solution.

### 3.4.4 Box with Unequal Angles (ribs) at the Four Corners

In this model, the dimensions of box are the same as in simple box as discussed above but having unequal angles welded at the four corners of box. Each corner is strengthened by taking two unequal angles of size 60x40x4 mm which are welded together as shown in Figure 3.4. It is seen from the Figure 3.4 that the effective length and effective breadth of the box section are reduced due to the welded angles. In the example presented, both the original length and breadth are reduced by \((2 \times 20) = 40\) mm. By substituting \(L=960\) mm, \(H=1000\) mm and \(B=130\) mm in Equation (3.22) and (3.33) the fundamental frequency obtained is 38.9 cps.

![Figure.3.4 View of box with welded ribs](image-url)
3.5 CONCLUDING REMARKS

In this chapter, a method for determining the fundamental natural frequency of transmission housing of box type without ribs by energy method was described. The fundamental natural frequency obtained initially by assuming the box structure to behave as a beam was found to be 62.8 cps. Subsequently by considering the box to behave as a plate under bending, the natural frequency was found to be 19.9 cps. In this case, the side plates were assumed to bend laterally with all edges hinged. Then boundary condition was modified such that it has a non zero displacement at the top of the box, this led to a fundamental natural frequency of 32.59 cps. It agreed well with FEM and experimental method as will be discussed in subsequent chapters. Addition of ribs along the edges, led to an increase in fundamental natural frequency to 38.9 cps which also agreed well with FEM procedure.