Chapter—VI
Chapter VI

VELOCITY AND TEMPERATURE VARIATIONS IN BOUNDARY LAYER FLOW REGIME

Introduction:

Boundary layer flow on a continuous moving solid surface in a Newtonian fluid have been studied by Sakiadis [106]. Jenge et al. [58] experimentally confirmed that the mathematically described boundary layer problem on a continuous moving surface is physically reasonable. Sahu et al. [104] studied the momentum and heat transfer in power-law fluid. Free convection laminar flow beneath a heated horizontal plate in a rapidly rotating system has been studied by Whittaker and Lister [129]. The flow that develops beneath a finite heated horizontal plate is a fundamental problem in natural convection. The heating creates a pool of buoyant fluid directly beneath the plate. Abedin et al. [2], Badruddin et al. [9], Basak et al. [11, 14], Borjini et al. [20, 21], Dalkilic et al. [43], Han [52], Kays and Crawford [61], Natarajan et al. [81] and Prud’homme and Jasmin [96] have studied free convection problems along bodies of different geometrical shapes.
In the present chapter, we have studied the momentum and heat transfers in laminar boundary layer flow past horizontal flat plate. Similar expressions for velocity and temperature fields have been obtained. Thicknesses of hydrodynamic and thermal boundary layers were obtained. The variation of their ratio with respect to different Prandtl numbers of the gases were obtained and shown in the Tables.

**Mathematical Analysis:**

Considered a two dimensional flow of a fluid of very small viscosity about a cylindrical body of slender cross-section. Excluding the immediate neighbourhood of the solid surface, velocities of the fluid are of the order of the free stream velocity \( V \) and the pattern of the stream lines and the velocity distribution deviate only slightly from those in frictionless (potential) flow. The transition of velocity from zero to the full magnitude takes place in a very thin layer, the so called boundary layer.

A very thin layer in the immediate neighbourhood of the body in which the velocity gradient normal to the wall, \( \partial u / \partial y \), is very large (called boundary layer). In this region the very small viscosity \( \mu \) of the fluid exerts an essential influence so as the shearing stress \( \tau = \mu (\partial u / \partial y) \) may assume large value.
Figure – 6.1: Boundary Layer Flow Along Cylindrical Body of Slender Cross-Section.
Outside the thin layer no such large velocity gradient occur and the influence of viscosity is unimportant. In this region flow is assumed to be frictionless and potential.

It is possible to state that the thickness of the boundary layer decreases with viscosity, or that it decreases with increasing Reynolds number.

It has been established that the boundary layer thickness is proportional to the square root of the kinematic viscosity, i.e.

\[ \delta \sim \sqrt{\nu}. \quad (6.1) \]

We further assume that this thickness is very small compared with linear dimension \( L \) of the body

\[ \delta \ll L. \quad (6.2) \]

**Dimensional Analysis of Navier – Stokes Equation in Boundary Layer Flow:**

In indicial notation, the Navier – Stokes equations red as

\[
\rho \left( \frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \right) = X_i - \frac{\partial p}{\partial x_i} \\
+ \frac{\partial}{\partial x_j} \left\{ \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial v_k}{\partial x_k} \right\} \]

\[ (6.3) \]
and continuity equation as

\[
\frac{\partial \rho}{\partial t} + \text{div} \left( \rho \mathbf{v} \right) = 0 \quad i, j, k = 1, 2, 3. \quad (6.3')
\]

The above system of equations for incompressible (\(\rho = \text{constant}\)) flows even if temperature is not constant reduce to

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = X - \frac{\partial p}{\partial x}
\]

\[
+ \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (6.4)
\]

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = Y - \frac{\partial p}{\partial x}
\]

\[
+ \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (6.5)
\]

\[
\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = Z - \frac{\partial p}{\partial x}
\]

\[
+ \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right). \quad (6.6)
\]

In the two-dimensional problem shown in Figure – 6.1, we assume that the wall of the flat plate is considered in the x-direction, y-axis being perpendicular to it. Free stream velocity is referred by \(V\) and all linear dimensions by \(L\) so as to insure that

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dimensionless derivative $\partial u/\partial x$ does not exceed unity. The pressure is
made dimensionless by $\rho V^2$ and time by $L/V$. The Reynolds number
\[ R = \frac{\rho V L}{\mu} = \frac{VL}{\nu} \]
is assumed very large.

In two-dimension velocity field $v$ ($u$, $v$, $t$) equations (6.3) and (6.3')
reduce to
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{R} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]
(6.7)
\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{R} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\]
(6.8)
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.
\]
(6.9)

The boundary conditions in absence of slip are

at \quad y = 0, \quad u = v = 0; \quad y \to \infty, \quad u = U

At the outer edge of the boundary layer $u = U(x, t)$.

In this region no large velocity gradient occurs, hence viscous force is
unimportant and so equation (6.7) becomes.
\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x}.
\]
(6.10)
For steady case this becomes

\[ U \frac{dU}{dx} = -\frac{1}{\rho} \frac{dp}{dx} \quad (6.11) \]

or \[ \frac{U^2}{2} = -\frac{1}{\rho} p + \text{constant} \]

or \[ p + \frac{1}{2} \rho U^2 = \text{constant}, \quad (6.12) \]

which is a Bernoulli equation.

In this way the Prandtl’s boundary layer equations take the form

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial y^2} \quad (6.13) \]

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6.14) \]

with the boundary conditions

\[ y = 0, \quad u = v = 0; \quad y \to \infty, \quad u = U(x, t). \quad (6.15) \]

The dimensionless boundary layer thickness \( \delta/L \) is estimated as

\[ \frac{\delta}{L} \sim \frac{1}{\sqrt{R}} = \sqrt{\frac{v}{VL}}. \quad (6.16) \]

The numerical coefficient missing in equation (6.16) is turn out to be equal to 5 for the case of flat plate at zero incidence when \( L \) measures the distance from the leading edge.
**Boundary Layer Separation:**

We observe that some of the retarded fluids in the boundary layer are transported to the main stream. When a region with an adverse pressure gradient exist along the wall, the retarded fluid particles can not in general penetrate too far into the region of increased pressure owing to their small kinetic energy.

**Hydrodynamic Boundary Layer:**

Let us consider the flat surface of a solid exposed to an infinite flow of fluid. The velocity and temperature of the free stream flow are constant and equal to $U_{\infty}$ and $T_{\infty}$. As the particles of the fluids come into contact with the solid surface they adhere to it. As a result, due to the action of viscous forces, a thin layer of stagnated fluid forms near the wall and within this layer of fluid flow velocity changes from zero at solid surface to the velocity of free stream. This layer of decelerated fluid particles is known as hydrodynamic boundary layer. The greater is the distance from the leading edge of the plate, the thicker the boundary layer, since the effect of viscosity penetrates deeper and deeper into the undisturbed flow of the fluid along the plate. The distribution of velocity at different values of $x$ is shown in Figure – 6.2. Thus as fluid flows past a solid body, the flow separates into two parts (a) the boundary layer and (b) the external flow.
Figure 6.2: Velocity Variation in Hydro-dynamic Boundary Layer
**Thermal Boundary Layer:**

The concept of thermal boundary layer as being the layer of fluid at solid wall within which the temperature changes from that of the wall temperature to the temperature of the fluid at a distance from the wall. The condition \( \frac{\partial T}{\partial y} \neq 0 \) within thermal boundary layer while outside this layer \( \frac{\partial T}{\partial y} = 0 \) and \( T = T_0 = \text{constant} \).

Thus, all variation in fluid temperature is concentrated in a comparatively thin layer which is in direct contact with the surface of the wall. The thickness \( k \) of thermal boundary layer in general does not coincide with thickness \( \delta \) of hydrodynamic boundary layer. The thickness \( k \) depends on certain characteristics and properties of the pattern of flow and heat transfer. Let us assume that both are of the same order of magnitude, i.e. \( k = O(\delta) \).

Due to the small thickness of thermal boundary layer, the heat transfer by conduction along the layer can be ignored compared with the transport of heat in the transverse direction, i.e. we can assume

\[
\frac{\partial^2 T}{\partial x^2} \ll \frac{\partial^2 T}{\partial y^2}, \quad \text{or} \quad \frac{\partial^2 T}{\partial x^2} = 0 \quad \text{(since } k^2 \ll l^2 \text{)}. \tag{6.17}
\]
Hence the equation of energy for incompressible flow appears as

\[ \rho \ C_v \ \frac{dT}{dt} = k \ \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2. \quad (6.18) \]

Hence, for incompressible steady flow and for constant viscosity, equations of hydrodynamic and thermal boundary layers appear as

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6.19 \text{ a}) \]

\[ \rho \left( u \ \frac{\partial u}{\partial x} + v \ \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \ \frac{\partial^2 u}{\partial y^2} \quad (6.19 \text{ b}) \]

\[ \rho \ C_v \left( u \ \frac{\partial T}{\partial x} + v \ \frac{\partial T}{\partial y} \right) = k \ \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2. \quad (6.19 \text{ c}) \]

**Integral Equations of the Boundary Layer:**

We write equation (6.19 c) in the form

\[ \rho \ C_v \left( u \ \frac{\partial T}{\partial x} + v \ \frac{\partial T}{\partial y} \right) = -\frac{\partial q_y}{\partial y} \quad (6.20) \]

where \[ q_y = -K \ \frac{\partial T}{\partial y}, \]

giving \[ K \ \frac{\partial^2 T}{\partial y^2} = -\frac{\partial q_y}{\partial y}. \]

Integrating equation (6.20) between the limit from \( y = 0 \) to \( y = \infty \).

Integral of right hand side of equation (6.20) gives
\[ - \int_0^\infty \frac{\partial q_y}{\partial y} \, dy = q_w = - \left[ (q_y)_\infty - (q_y)_0 \right] = q_w \]

as \( (q_y)_\infty = -\lambda \left( \frac{\partial T}{\partial y} \right)_\infty = 0 \).

From equation (6.19 a), we have

\[ dv = -\frac{\partial u}{\partial x} \cdot dy \]

\[ \int_0^y dv = -\int_0^y \frac{\partial u}{\partial x} \, dy \]

or \( (v)_y = (v)_0 = -\int_0^y \frac{\partial u}{\partial x} \, dy \)

\[ v = -\int_0^y \frac{\partial u}{\partial x} \, dy \]  

(6.21)

because \( (v)_y = 0 \) as wall is impermeable.

In view of equation (6.21) we have

\[ \int_0^y \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) \, dy = \int_0^y u \frac{\partial T}{\partial x} \, dy \]

\[ - \int_0^y \frac{\partial T}{\partial y} \left( \int_0^y \frac{\partial u}{\partial x} \, dy \right) \, dy. \]  

(6.22)
The second integral of right hand side is

\[ \int_0^\infty \frac{\partial T}{\partial y} \left( \int_0^\infty \frac{\partial u}{\partial x} \, dy \right) \, dy = \int_0^\infty \left( \int_0^\infty \frac{\partial u}{\partial x} \, dy \right) \, dT \]

\[ = \left( T \left[ \int_0^\infty \frac{\partial u}{\partial x} \, dy \right] \right) - \int_0^\infty T \frac{\partial u}{\partial x} \, dy \]

\[ = T_\infty \int_0^\infty \frac{\partial u}{\partial x} \, dy - \int_0^\infty T \frac{\partial u}{\partial x} \, dy \]

\[ = \int_0^\infty \left( T_\infty - T \right) \frac{\partial u}{\partial x} \, dy. \quad (6.23) \]

Put equation (6.23) in equation (6.22) and get

\[ \int_0^\infty u \frac{\partial T}{\partial x} \, dy - \int_0^\infty \left( T_\infty - T \right) \frac{\partial u}{\partial x} \, dy \]

\[ = \int_0^\infty u \frac{\partial T}{\partial x} \, dy - \int_0^\infty \left( T_\infty - T \right) \frac{\partial u}{\partial x} \, dy \]

\[ = -\int_0^\infty \left[ -u \frac{\partial T}{\partial x} \, dy + \left( T_\infty - T \right) \frac{\partial u}{\partial x} \, dy \right] \]

\[ = -\frac{d}{dx} \int_0^\infty u \left( T_\infty - T \right) \, dy. \quad (6.24) \]
Thus in view of equation (6.24), equation (6.20) becomes, after passing from the integration limit $\infty$ to $k$, we obtain the following integral differential equation

$$\frac{d}{dx} \int_0^k u \left( T_w - T \right) \, dy = -\frac{q_w}{\rho C_p}. \quad (6.25)$$

Equation (6.25) is the integral equation of heat flow. For the thermal boundary layer heat flow rate $q_w$ is the function of $x$ only.

**Integral Equation of Momentum for the Hydrodynamic Boundary Layer:**

For the case of flat gradient-less steady-state flow of a viscous fluid in the boundary layer near a flat surface, equation (6.19 b) becomes

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\partial s}{\partial y}, \quad (6.26)$$

where $s = \mu \frac{\partial u}{\partial y}$.

Comparing equation (6.26) with equation (6.20), we find that both are similar. Hence integrating equation (6.26) between the limits from $y = 0$ to $y = \infty$ (or $\delta$) and carrying out similar transformation we obtain the similar result of the form
\[
\frac{d}{dx} \left[ \int_0^y u \left( U_\infty - u \right) dy \right] = \frac{s_w}{\rho},
\]
(6.27)

where \( s_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0} \).

**Heat Transfer in a Laminar Boundary Layer:**

To find out the heat transfer in a laminar boundary layer with the help of equation (6.25), we must know the velocity distribution in the layer. It has been observed that distribution of velocity in the boundary layer is nearly parabolic, and it is convenient to describe the velocity distribution curve by the equation of a cubic parabola

\[
u = a + by + cy^2 + dy^2
\]
(6.28)

when \( y = 0, u = 0 \), also we assume \( \left( \frac{\partial^2 u}{\partial y^2} \right) = 0 \).

In addition, on the external border of boundary layer

\[
u = U_\infty \quad \text{and} \quad \frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = \delta.
\]

From equation (6.28), \( y = 0, u = 0 \) gives \( a = 0 \),

also \( 0 = \left( \frac{\partial^2 u}{\partial y^2} \right)_{y=0} = \left( 2C + 6d \right)_{y=0} \)

\[\Rightarrow C = 0,\]
at \( y = \delta \), \( u = U_\infty \) gives

\[
U_\infty = b \delta + \delta^3 d
\]

\[
\Rightarrow b = \frac{3}{2} \frac{U_\infty}{\delta}, \quad d = -\frac{1}{2} \frac{U_\infty}{\delta^3}
\]

\[
0 = \left( \frac{\partial u}{\partial y} \right)_{y=\delta} = b + 3 \delta^2 d.
\]

Hence the distribution of velocity acquires the form

\[
\frac{u}{U_\infty} = 3 \left( \frac{y}{\delta} \right) - \frac{1}{2} \left( \frac{y}{\delta} \right)^3.
\]  \hspace{1cm} (6.29)

From the momentum integral equation (6.27), the thickness of hydrodynamic boundary layer is determined by the expression

\[
\delta = \sqrt{\frac{280}{13}} \sqrt{\frac{\gamma x}{U_\infty}} \approx 4.64 \sqrt{\frac{\gamma x}{U_\infty}}.
\]  \hspace{1cm} (6.30)

Equation (6.30) shows that boundary layer thickness \( \delta \) varies proportionally the \( \sqrt{x} \) (\( x \) is the distance of considered point from the front edge of the plate).

In non-dimensional form

\[
\frac{\delta}{x} = \frac{4.64}{\sqrt{\frac{U_\infty}{x}}} = \frac{4.64}{\sqrt{\text{Re}_x}}.
\]  \hspace{1cm} (6.31)
Assume, that the surface temperature of the solid wall $T_w$ is independent of $x$, i.e. $T_w = \text{Constant}$.

Let $T - T_w = \theta$ and $\theta_x = T_x - T_w$

where $T_x$ is fluid temperature beyond the thermal boundary layer.

Boundary conditions are similar to that of hydrodynamic layer

at $y = 0$, $\theta = 0$, $\left( \frac{\partial \theta}{\partial y} \right)_{y=0} = 0$ and $\left( \frac{\partial^2 \theta}{\partial y^2} \right)_{y=0} = 0$. \hfill (6.32)

If we take into account that heat is transported only by conduction in the liquid closely adjoining the wall. The condition for the outer border of thermal layer ($y = k$) becomes

$\theta = \theta_x = \text{Constant}$ and $\left( \frac{\partial \theta}{\partial y} \right)_{y=k} = 0$. \hfill (6.33)

Temperature distribution in the medium is described by the equation similar to that of velocity distribution. Thus in analogy to equation (6.29) we have

$$\frac{\theta}{\theta_x} = 1.5 \left( \frac{y}{k} \right) - 0.5 \left( \frac{y}{k} \right)^3$$ \hfill (6.34)

which gives $\frac{d\theta}{dy} = 1.5 \frac{\theta_x}{k} - 1.5 \frac{\theta_x}{k^3} y^2$
and \( \left( \frac{d\theta}{dy} \right)_{y=0} = 1.5 \frac{\theta}{k} \). \hspace{1cm} (6.35)

Equation (6.35) shows that heat transfer from the plate is linearly proportional to free stream temperature and inversely proportional to thermal boundary layer thickness.

Equation (6.25) gives heat balance equation provided \( k \leq \delta \). It the integration was spread to \( \delta > k \), then it would mean that in thermal boundary layer velocity distribution was governed by two laws. With \( y < \delta \), velocity distribution was given by equation (6.29) and with \( \delta \leq y \leq k \), the condition \( u = U_{\infty} = \text{constant} \) prevails.

In equation (6.25)

\[
\int_{0}^{k} (T_{\infty} - T) u \, dy = \int_{0}^{k} (\theta_{\infty} + T_{w} - \theta - T_{w}) u \, dy
\]

\[
= \int_{0}^{k} (\theta_{\infty} - \theta) u \, dy
\]

\[
= U_{\infty} \theta_{\infty} \int_{0}^{k} \left( 1 - \frac{\theta}{\theta_{\infty}} \right) \frac{u}{U_{\infty}} \, dy
\]

\[
= \theta_{\infty} U_{\infty} \int_{0}^{k} \left[ 1 - 1.5 \left( \frac{y}{\delta} \right) - 0.5 \left( \frac{y}{k} \right)^3 \right] \left[ 1.5 \left( \frac{y}{\delta} \right) - 0.5 \left( \frac{y}{\delta} \right)^3 \right] \, dy
\]
\[= \theta_\infty U_\infty \delta \left[ \frac{3}{20} \left( \frac{k}{\delta} \right)^2 - \frac{3}{280} \left( \frac{k}{\delta} \right)^4 \right].\]

As \(k < \delta\), therefore \(\left( \frac{k}{\delta} \right)^4\) is negligibly small.

Now
\[
\frac{d}{dx} \int_0^k (T^* - T)^4 \, dy = \frac{d}{dx}\left[ \theta_\infty U_\infty \delta \frac{3}{20} \left( \frac{k}{\delta} \right)^2 \right]
\]
\[= \frac{3}{20} \theta_\infty U_\infty \frac{d}{dx} \left( \beta^2 \delta \right)
\]

where \(\beta = \frac{k}{\delta}\)

and
\[-\frac{q_w}{\rho C_p} = + \frac{K}{\rho C_p} \left[ \frac{d}{dy} \left( \theta + T_* \right) \right]_{y=0} = \frac{K}{\rho C_p} \left( \frac{d\theta}{dy} \right)_{y=0}
\]
\[= a \frac{3}{2} \frac{\theta_\infty}{k}, \quad \text{from equation (6.35)}
\]
\[= \frac{3}{2} a \frac{\theta_\infty}{\beta \delta}.
\]

Hence
\[\frac{3}{20} \theta_\infty U_\infty \left[ \beta^2 \frac{d\delta}{dx} + 2\delta \beta \frac{d\beta}{dx} \right] = \frac{3}{2} \frac{a \theta_\infty}{\beta \delta}
\]

or
\[\frac{1}{10} U_\infty \left[ \beta^3 \delta \frac{ds}{dx} + 2\beta^2 \delta^2 \frac{d\beta}{dx} \right] = a. \quad (6.36)
\]

If we assume that \(k/\delta\) is constant, that is there is no unheated section of the plate at the beginning, i.e.
\[ \frac{k}{\delta} = \beta = \beta(x), \]

then \(d\beta/dx = 0\) and equation (6.36) takes the form

\[ \frac{1}{10} \quad U_\infty \beta^3 \delta \frac{d\delta}{dx} = a. \quad (6.37) \]

From equation (6.30), \(\delta^2 = \frac{280}{13} \frac{v}{U_\infty} \times \)

\[ \delta \frac{d\delta}{dx} = \frac{140}{13} \frac{v}{U_\infty}. \]

Hence equation (6.37) gives

\[ \frac{1}{10} \beta^3 \frac{140}{13} v = a \]

\[ \beta^3 = \frac{130}{140} \frac{a}{v} = \frac{130}{140} \frac{1}{P_r} \]

\[ \frac{k}{\delta} = \beta = \left(\frac{130}{140} \frac{1}{P_r}\right)^{1/3} \approx \left(\frac{1}{P_r}\right)^{1/2} \quad (6.38) \]

as \(\left(\frac{130}{140}\right)^{1/3} \approx 0.98 \approx 1.\)

Substituting value of \(\delta\) from equation (6.34), we get

\[ k = \frac{4.64}{\sqrt{Re_x} 3 \sqrt{P_r}} \quad (6.39) \]
Table – 6.1: Variation of Velocity Field in Hydrodynamic Boundary Layer.

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<tr>
<th>$y/\delta$</th>
<th>$u/u_\infty$</th>
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<tr>
<td>0.1</td>
<td>0.1495</td>
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<tr>
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</tr>
<tr>
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Table – 6.2: Variation of Thermal and Hydrodynamic Boundary Layer Ratio $\beta$ with Thermal Diffusivity, Kinematic Viscosity and Prandtl Number for Gases.

<table>
<thead>
<tr>
<th>A</th>
<th>$\nu$</th>
<th>$P_r$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1.006</td>
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<td>0.159</td>
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<td>0.169</td>
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<td>0.8298</td>
</tr>
</tbody>
</table>
Graph – 6.1: Variation of Velocity Field in Hydrodynamic Boundary Layer.
Graph 6.2a: Variation of $\beta$ with respect to Thermal Diffusivity.
Graph 6.2b: Variation of $\beta$ with respect to Kinematic Viscosity.
Graph - 6.2c: Variation of $\beta$ with respect to Prandtl Number.