CHAPTER 4

PROPOSED MEDICAL IMAGE MIXTURE DENSITY CLUSTERING SEGMENTATION WITH SPATIAL INFORMATION FUNCTION IN FREQUENCY DOMAIN

4.1 INTRODUCTION

A well known statistical approach of image segmentation based on pixel intensity is the Expectation-Maximization (EM) algorithm, which is used to estimate the parameters of different classes in the image (Moon 1996). A number of algorithms based on probability theory (McLachlan and Peel 2000, Bishop 1995) have been proposed. Amongst them, Gaussian Mixture Model (GMM) (Titterington et al 1985, Jain et al 2000) is a well-known method that has been widely used as a tool for image segmentation.

GMMs success is attributed to the fact that the model parameters can be efficiently estimated by adopting the Expectation Maximization (EM) algorithm (Dempster et al 1977, McLachlan and Peel 2000). Other advantages are its simplicity and ease of implementation. However, the major disadvantage of GMM (Thanh and Wu 2008, Thanh et al 2010) is that the model assumes that each pixel is independent of its neighbours. It is well known that pixels in an image are similar in some sense and cannot be classified consistently based on feature attributes alone. Thus, the segmentation result of GMM is extremely sensitive to noise. The application of EM-GMM to medical image segmentation faces certain difficulties such as
estimation of the number of components, slow speed of segmentation for large sized medical images and noise sensitivity (Yu-Qing Song et al 2010).

This thesis proposes three variations to Expectation Maximization (EM)- Gaussian Mixture Model (GMM) based segmentation methods; first one is based on Expectation Maximization (EM)- Gaussian Mixture Model (GMM) – Fast Fourier Transform (FFT) with spatial information function, the second one based on EM-GMM-Radix4-FFT with spatial information function and the third method based on EM-GMM-Mixed Radix-FFT with spatial information function particularly suitable for large sized medical images. The spatial information function has been proposed to modify the EM algorithm which works in frequency domain, so that it takes into consideration the effect of the neighbouring pixels when classifying the current pixel, and the use of FFT techniques also increases the speed of segmentation of large sized medical images.

4.2 THE GENERAL STATEMENT OF EXPECTATION AND MAXIMIZATION ALGORITHM

The EM algorithm consists of two major steps: an expectation step followed by the maximization step. Figure 4.1 shows an overview of the EM algorithm.

The expectation is with respect to the unknown underlying variables, using the current estimate of the parameters and conditioned upon the observations. The maximization step then provides a new estimate of the parameters. After initialization, the E-step and the M-step are alternated until the parameter estimate has converged (no more change in the estimate) (Moon 1996). This concept has been illustrated in Figure 4.1.
Figure 4.1 An overview of the EM algorithm

4.2.1 Description of EM Algorithm

1. The number of classes $K$ and image $I$ are provided to the system.

2. The initial estimation of parameters $\theta^{[0]}$ is based on the histogram of the image and the number of classes. In the case of brain MRI segmentation this requires a general guess for the means and variances for the three classes (WM, GM and CSF). White matter has highest mean and CSF has the lowest mean. Because of variation between different MRI acquisition systems, histogram of the image should be used. The priori class probabilities $p_k$ are assumed to be equally likely.
3. Performing the E-step and M-step iteratively until convergence, at each iteration the E-step computes the class probability of each pixel based on the current distribution of $i^{[k]}$. M step computes the new expectation of $i^{[k+1]}$ based on values computed in the previous E-step. After convergence the maximum estimator of $\Theta$ is produced.

4. Use Maximum likelihood estimation $\phi_{ML}$ in a classifier to generate the classification matrix $C$.

5. Assign color or label to each class based on the classification matrix $C$ and generate the segmented image.

Let $Y$ denote the sample space of the observations, and let $y \in \mathcal{Y}^m$ denote an observation from $Y$. Let $\chi$ denote the underlying space and let $x \in \mathcal{X}^m$ be an outcome from $\chi$, with $m < n$. The data $x$ is referred to as the complete data. The complete data $x$ is not observed directly, but only by means of $y$, where $y = y(x)$, and $y(x)$ is many-to-one mapping. An observation $y$ determines a subset of $\chi$, which is denoted as $\chi(y)$. Figure 4.2 illustrates the mapping (Moon 1996).

![Figure 4.2 Illustration of many-to-one mapping from X to Y](image)

The EM algorithm aims at parameterizing the different classes based on intensities. The initial phase of an iteration of the EM algorithm is followed by the mixture model parameters which are updated and classification has been performed. In order to understand the EM algorithm, it is important to understand the two main stages that make up the algorithm, shown in Figure 4.3. Since EM is an iterative process, the stages described below are at an intermediate iteration of the algorithm.

**Stage I: Classification (Expectation step)**: At this stage of the loop, an assumption is made so that an estimate of the parameters that describe the mixture of $K$ Gaussians have been obtained as shown in Figure 4.4, where $K$ is the number of classes (here GM, WM, and CSF).

This mixture model provides the likelihood for every voxel in the MRI, which allows for statistical classification of every voxel into one of these $K$ classes, achieving the classification required as part of the loop. Note that the intensity is the effective intensity after the current bias field estimate has been subtracted from the intensity at each voxel. Gaussian mixture models are used because the various classes of the brain can be represented as Gaussians with the mean equal to mean intensity of each class.
Stage I: Classification (Expectation Step)

Intensity

Stage II: Update the mixture model (Maximization Step)

Figure 4.3 Illustration of the two stages of the EM algorithm

Stage II - Maximization: Using the current classification of the MRI, the Gaussian distribution has been modeled as describing a class with mean intensity $\mu$ and variance $\sigma^2$. The voxel intensities corrected by the bias field are used to update the mixture model. In this approach, $K=3$, corresponding to WM, GM, and CSF. The intensity distributions of each class are modeled as a Gaussian distribution with the statistical mean being average intensity, and variance being variation around the average intensity. Once the mean and the variance of each tissue type are known, the voxels can be classified into each class based on voxel intensity. The EM algorithm ties together the classifications of stage I and class distribution parameter.
estimation of stage II. The algorithm fills in the missing data during stage I and then finds the parameters that increase the likelihood for the complete data during stage II.

Figure 4.4 Histogram of intensities modeled as a mixture of three Gaussians

4.2.2 Problem Formulation

In its general form, image segmentation may be formulated as follows: an image $I$ may be regarded as a mapping from a pixel lattice $L$ to a state space $E$. A segmentation $S$ of $I$ is a partition of $L$ into a set of nonoverlapping (not necessarily connected) regions \{${R}_i, i=1,\ldots,M$\} such that $L = \bigcup_i R_i$ and $I$ is uniform in some sense over every region. In particular, one may consider the case in which the values of the image $I$ in each region $R_i$ may be represented as the sum of some fixed deterministic functions plus noise. Considering these functions as parametric models whose parameters are constant for each region, one obtains

$$I(x) = \sum_{k=1}^{M} b_k(x)\phi(x; \theta_k) + n(x) \quad (4.1)$$
Where \( x \) denotes a pixel in \( L \); \( \phi(x; \theta_k) \) is a function (parametric model) \( \phi: L \rightarrow E \) that depends on the parameter vector \( \theta_k \); \( \{n(x) : x \in L\} \) is a set of independent random variables and \( b_k \) is the indicator function of region \( R_k \), i.e., \( b_k(x) = 1 \) iff \( x \in R_k \) and

\[
\sum_{k=1}^{M} b_k(x) = 1
\]  

(4.2)

For all \( x \in L \). In what follows, \( b(x) \) will denote the vector \( (b_1(x), \ldots, b_M(x)) \) and \( b \) will denote the set \( \{b(x) : x \in L\} \).

The appearance of the image \( I \) will depend on the nature of the functions \( \phi \); for example, if each function \( \phi(x; \theta_k) \) is smooth, \( I \) will be a piecewise smooth image, with discontinuities located at the boundaries between adjacent regions \( R_{i}, R_{j} \). A particular instance of this case is related to the segmentation of brain magnetic resonance images in terms of tissue type; in this case, \( \phi(x; \theta_k) \) represents the intensity associated with tissue type \( k \) and \( R_k \) represents the portion of the image classified as tissue \( k \) (Rivera et al. 2007).

4.2.3 Existing Solutions to Overcome the High Computational Complexity and High Sensitivity with Respect to Noise

The difficulties such as estimation of the number of components, speed of segmentation for large medical images and noise sensitivity have been approached from different perspectives: if the parameters \{\( \theta_k \)\} are known, the segmentation problem will be solved using the k-means algorithm and its variants (Pham and Prince 1999), region merging (Deng 2001) and active contour (Blake and Isard 2000) approaches, variational methods
(Weiss and Adelson 1996) and probabilistic (Bayesian) formulations (Geman and Geman 1984). Of these, the Bayesian formulations are one of the most powerful and general, since they allow for the inclusion of spatial coherence constraints that regularize the solution (via Markov Random Field (MRF) models) and makes it more robust with respect to noise, although they may be computationally expensive.

In the general case, when the parameters \( \{\theta_k\} \) are not known, some of these methods may be extended using two step algorithms, in which one is given an initial estimate \( \{\hat{\theta}_k\} \), and then one iterates the following steps.

- Estimate \( S \) given \( \hat{\theta} \).
- Estimate \( \hat{\theta} \) given \( S \).

Where \( \tilde{\theta} \) denotes the collection of all parameter vectors \( \theta_k \), until convergence (Rivera and Gee 2004). One particularly important class of these two-step methods was derived from the Expectation–Maximization (EM) algorithm, which was originally proposed for computing maximum likelihood estimators from incomplete data (Dempster et al 1977). This approach has also been used for computing estimators with respect to posterior distributions for segmentation tasks (classification) in image analysis and computer vision problems where the class model parameters are unknown (Geman and Geman 1984).

However, prior probability distributions based on MRF models introduce high correlation among the variables (labels in the segmentation task) that increments the computational complexity of the EM algorithm (Wells et al 1996). For this reason, instead of Monte-Carlo Markov chain methods, approximations such as mean field theory (Zhang 1992) or Gauss–
Markov measure fields (Marroquin et al 2003) have been used for computing the marginal probabilities in the expectation (E) step with relative success.

The problem with above said approaches is their high computational complexity, and their high sensitivity with respect to noise and to the choice of \( \{ \hat{\theta}_k^{(0)} \} \). The reason for this is that these two-step approaches can be guaranteed to converge only to a local maximum of the posterior distribution (Dempster et al 1997); since, in most cases, this distribution has multiple maxima, if one starts the iterations from a “bad” point \( \{ \hat{\theta}_k^{(0)} \} \), the local maximum to which the method converges may not be the global maximum, i.e., one may get suboptimal solutions (Marroquin et al 2003).

One may get more robust methods if one formulates the problem in such a way that \( S \) and \( \hat{\theta} \) are progressively refined at the same time, e.g., by the iterative minimization of a differentiable function that depends on \( \hat{\theta} \) and on the probability of assigning models to each pixel (i.e., on a “soft” version of the segmentation). These direct methods, such as (Marroquin et al 2003, Balafar 2011), do exhibit a better performance than that of two-step approaches; their computational complexity, however, is still relatively high, since the solution involves the minimization of a highly nonlinear function, and the hyperparameters of the corresponding algorithms are in general not easy to tune.

Hence this thesis proposes spatial information function which takes into account of neighbourhood information. The proposed methods are capable of overcoming the drawbacks of traditional EM algorithm, robust to noise and also increase the speed of segmentation of large sized medical images by means of making use of various FFT techniques along with EM-GMM.
4.3  STANDARD GAUSSIAN MIXTURE MODEL

The Gaussian mixture model assumes $M$ Mixed component densities (Gaussian distribution) for each pixel (voxel) with $M$ mixing coefficients. Each component is assigned to one target class and the goal is to obtain the class probabilities of each pixel (voxel).

The probability distribution of the $j$th component is denoted by $p_j(x_i | \theta_j)$, where $x_i$ is pixel $i$ in input image and $\theta_j$ is the parameter (mean $\mu_j$ and covariance matrix $\Sigma_j$) of component $j$. The probability distribution of each pixel (voxel) can be described as a mixture of probability distributions as follows:

$$p(x_i | \theta) = \sum_{j=1}^{M} \alpha_j p_j(x_i | \theta_j)$$ (4.3)

$$= \frac{1}{\sqrt{\det(2\pi\Sigma_j)}} \exp\left\{-\frac{1}{2}(x_i - \mu_j)^T \Sigma_j^{-1} (x_i - \mu_j)\right\}$$ (4.4)

Where $\alpha_j$ denotes the mixture coefficient with the constraint, $\sum_{j=1}^{M} \alpha_j = 1$. The probability distribution of component $j$ is modeled by a Gaussian distribution with mean $\mu_j$ and covariance matrix $\Sigma_j$.

$$p_j(x_i | \theta_j) = p_j(x_i | \mu_j, \Sigma_j)$$ (4.5)

Usually, Maximum Likelihood (ML) estimation is used to find the probability distribution of parameters. The log-likelihood expression for the parameter $\theta$ and the image $X$ is defined as follows:
\[
\log L(\theta | x) = \log \prod_{i=1}^{N} p(x_i | \theta) \\
= \sum_{i=1}^{N} \log \left( \sum_{j=1}^{M} (\alpha_j^i p_j(x_i | \theta_j^i)) \right) 
\] (4.7)

Finding the ML solution from this equation (4.7) is difficult. Usually, the expectation-maximization (EM) is used to obtain the parameters. EM steps are demonstrated in the following:

**E-step. Step a:** Bayes’ rule is used to obtain the probability of data \( x_i \) belonging to class \( \theta_j \) (E-step):

\[
p(j|x_i, \theta^i) = \frac{\alpha_j^i p_j(x_i | \theta_j^i)}{\sum_{i=1}^{M} \alpha_j^i p_j(x_i | \theta_j^i)} 
\] (4.8)

**M-step. Step b:** Probability obtained in E-step is used to obtain mixing coefficient, mean and covariance matrix (M-step):

\[
\alpha_j^{t+1} = \frac{\sum_{i=1}^{N} p(j|x_i, \theta^i)}{N} 
\] (4.9)

\[
\mu_j^{t+1} = \frac{\sum_{i=1}^{N} p(j|x_i, \theta^i)x_i}{\sum_{i=1}^{N} p(j|x_i, \theta^i)} 
\] (4.10)

\[
\sum_j^{t+1} = \frac{\sum_{i=1}^{N} p(j|x_i, \theta^i)(x_i - \mu_j^{t+1})(x_i - \mu_j^{t+1})^T}{\sum_{i=1}^{N} p(j|x_i, \theta^i)} 
\] (4.11)

**Step c:** EM steps are repeated until convergence.
4.4 K-MEANS ALGORITHM

K-means clustering is a partitioning method. The function K-means partitions data into k mutually exclusive clusters, and returns the index of the cluster to which it has assigned each observation. Unlike hierarchical clustering, K-means clustering operates on actual observations (rather than the larger set of dissimilarity measures), and creates a single level of clusters. The distinctions mean that K-means clustering is often more suitable than hierarchical clustering for large amounts of data. Because pixel-based methods based on K-means clustering are simple and the computational complexity is relatively low compared with other region-based or edge-based methods, the application is more practicable. Furthermore, K-means clustering is suitable for biomedical image segmentation as the number of clusters is usually known for images of particular regions of the human anatomy.

K-means treats each observation in the data as an object having a location in space. It finds a partition in which objects within each cluster are as close to each other as possible, and as far from objects in other clusters as possible. Any one of the different distance measure can be chosen depending on the kind of data to be clustered. Each cluster in the partition is defined by its member objects and by its centroid, or center. The centroid for each cluster is the point to which the sum of distances from all objects in that cluster is minimized. K-means computes cluster centroids differently for each distance measure, to minimize the sum with respect to the specified measure (Seber 1984).

K-means uses an iterative algorithm that minimizes the sum of distances from each object to its cluster centroid, over all clusters. This algorithm moves objects between clusters until the sum cannot be decreased further. The result is a set of clusters that are as compact and well-separated as
possible. The details of the minimization can be controlled using several optional input parameters to K-means, including ones for the initial values of the cluster centroids, and for the maximum number of iterations.

4.5 FFT-BASED IMAGE ENHANCEMENT AND SEGMENTATION

A Discrete Fourier transform (DFT) converts a signal in the time domain into its counterpart in frequency domain. The detailed explanation on FFT is given in Appendix A.

For any square image of size, N×N the two-dimensional DFT is given by

\[
F(k,l) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f(i,j)e^{-i2\pi\left(\frac{ki}{N} + \frac{lj}{N}\right)}
\]  

(4.12)

Where \(f(i,j)\) is in spatial domain and the exponential term is the basis function corresponding to each point \(F(k,l)\) in the Fourier space. Fourier image is re-transformed to the spatial domain.

The inverse Fourier transform is given as

\[
f(a,b) = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F(k,l)e^{i2\pi\left(\frac{ka}{N} + \frac{lb}{N}\right)}
\]

(4.13)

Using the Fast Fourier Transform (FFT) the complexity is reduced to \(N\log_2 N\). This is a significant improvement, in particular for large images. Input signal of the FFT in origin can be a complex and of arbitrary size. The result of the FFT contains the frequency data and the complex transformations.
4.6 FAST FOURIER TRANSFORM (FFT) FOR MR IMAGING

In MRI, samples are taken in frequency domain or k-space, and inverse FFT is applied to obtain the image. Principles of 2-D Fourier transform imaging are used on clinical MR scanners. Today, variants of this Fourier transform imaging technique is the most widespread MR method for obtaining structural and functional information from the living human body (Mark Haacke et al 1999).

The need of Fourier Transform arises in MRI, since the measurements are not voxel values, but the measurements are spatial frequencies. Image filtering (Smoothing) is computationally faster in frequency space (Rowe 2005). Speed of segmentation is faster since the solution via FFT is significantly faster – that is $O(N \log_2 N)$.

4.7 PROPOSED BRAIN TISSUE CLASSIFICATION OF MRI USING FFT BASED EM-GMM MODEL WITH SPATIAL INFORMATION FUNCTION

4.7.1 System Overview

Figure 4.5 shows the block diagram of the proposed FFT based EM-GMM model with spatial information.

The proposed system starts with getting an MRI input image containing scanner introduced noise followed by the application of Fast Fourier Transform (FFT). The image has been smoothened and convolution is faster as compared to spatial domain by the application of FFT.

The output contains both real and imaginary parts. The absolute value of FFT (of input image) is calculated and applied as an input of modified EM-GMM algorithm with spatial information. The output is
classified into three classes as white matter, gray matter and cerebrospinal fluid (CSF). The solution via FFT is significantly faster as compared to the solution in spatial domain.

![Block diagram of the proposed FFT based EM-GMM model with spatial information](image)

**Figure 4.5** Block diagram of the proposed FFT based EM-GMM model with spatial information

### 4.7.2 The Proposed Algorithm for FFT Based- Modified EM-GMM with Spatial Information Function

**Input**

Input brain image of size $M \times N$.

**Output**

Segmented brain tissues.
Step 1: Read and display the Input medical Image X of size M×N.

Step 2: Apply FFT to each medical image containing noise and is given as input to the Expectation-Maximization algorithm.

Step 3: Assume the number of classes and set initial iteration as k=0 and log likelihood threshold value as 1e-10.

Step 4: Estimate the initial value of parameters $\mathbf{\Theta}^{[0]}$ by plotting the histogram of power spectrum.

Step 5: Perform E-step by computing the class probability of each pixel based on current estimation of $\mathbf{\Theta}^{[k]}$.

Step 6: Perform M-step and compute new expectation $\mathbf{\Theta}^{[k+1]}$ based on values computed from the previous step until convergence.

Step 7: Compute spatial information function as

$$h_{ij} = \frac{1}{n_i} \sum_{k \in \text{NB}(x_i)} P(j|\tilde{x}_i)$$

Step 8: Apply EM-GMM with spatial information to the output of absolute value of FFT.

Step 9: Apply inverse FFT to the output of EM-GMM-FFT with spatial information (i.e to Step 8).

Step 10: Apply Bayes Classifier to assign class membership to the classes.

Step 11: Display the classified gray matter, white matter and CSF of MRI brain images as segmented images.

Step 12: Compute MSE and PSNR of EM-GMM-FFT with spatial information function. Compute segmentation accuracies of WM, GM and CSF by the proposed method.
4.7.3 Proposed Spatial Information Function

The iteration formula described by Standard Gaussian Mixture Model in Section (4.3) did not involve any spatial information about current voxel. As discussed in Section (4.1), neighbourhood information is one of the most important spatial information. If the iteration procedure takes the neighbourhood effect into account, the classification results are more enhanced and accurate.

The original model calculates the class probabilities according to Bayes’ rule, which is described by Equation (4.8). This calculation is based on intensity distributions without any neighbourhood information. Usually the material is continuous, so that it is natural to have the idea that for each voxel, the probability of the \( j \) th class should be affected by the neighbours’ \( j \) th class probabilities. According to this belief, the neighbourhood effect can be integrated on the class distributions of the current voxel by modifying Equation (4.8). The neighbourhood information is calculated in clustering iteration.

A spatial information function is defined as

\[
    h_{ij} = \frac{1}{n} \sum_{k \in NB(x_i)} P(j|x_i)
\]

(4.14)

Where \( NB(x_i) \) is a squared window centered on pixel \( x_i \) in the spatial domain and \( n \) is the size of the neighbourhood that is set as 4. \( P(j|x_i) \) is the probability of the neighbouring pixels of \( x_i \) that is labeled as \( \tilde{x}_i \) and belongs to the \( j \)th class. When the neighbourhood size is increased, the Dice Similarity Metric of proposed EM-GMM-spatial information decreases sharply. This means blurring effect in depends on neighbourhood size. When the
neighbourhood size is set as 4, Dice Similarity Metric of the proposed EM-GMM-spatial information function is high. The size of the neighbourhood depends on image size.

4.7.4 Proposed EM-GMM-FFT with Spatial Information Function

The proposed new class distribution formula integrates the neighbourhood information to the current voxel’s class distribution during iteration. For each iteration step, the class distribution will be amended by the neighbours’ class distribution information. So that through this weighted formula, the neighbourhood information is taken into account to the classification process. The EM solution formula for the proposed neighbourhood weighted Gaussian mixture model is summarized as follows:

**Estep:**

\[
p(j|x_i, \theta^t) = \frac{\alpha^t_{h_{ij}} p_j(j|x_i, \theta_j^t)}{\sum_{j=1}^{M} \alpha^t_{h_{ij}} p_j(j|x_i, \theta_j^t)}
\]  \hspace{1cm} (4.15)

**M-step:**

\[
\alpha_j^{t+1} = \frac{1}{N} \sum_{i=1}^{N} p(j|x_i, \theta^t)
\]  \hspace{1cm} (4.16)

\[
\mu_j^{t+1} = \frac{\sum_{i=1}^{N} p_j(j|x_i, \theta_j^t) x_i}{\sum_{i=1}^{N} p_j(j|x_i, \theta_j^t)}
\]  \hspace{1cm} (4.17)

\[
\sum_{j}^{t+1} = \frac{\sum_{i=1}^{N} p_j(j|x_i, \theta_j^t)(x_i - \mu_j^{t+1})(x_i - \mu_j^{t+1})^T}{\sum_{i=1}^{N} p_j(j|x_i, \theta_j^t)}
\]  \hspace{1cm} (4.18)
\[ h_{ij}^{t+1} = \frac{1}{n} \sum_{k \in \text{NB}(x_i)} P(j|\tilde{x}_i) \]  

(4.19)

4.7.5 The Bayesian Rule

After the mixture identification, Bayesian rule was applied in order to classify pixels according to their gray levels.

\[ j(x_i) = \arg \max_{i \leq j \leq M} \{ p(j|x_i, i^T) \} \]  

(4.20)

where \( j(x_i) \) represents label of the class of the pixel \( x_i \).

4.8 EXPERIMENTS AND RESULTS OF PROPOSED EM-GMM-FFT WITH SPATIAL INFORMATION FUNCTION

In order to test the performance of proposed algorithm, data set containing both simulated images from BrainWeb and the manually segmented real images of MRI human brain has been used. The performance comparisons have been made with existing EM-GMM, proposed FFT based EM-GMM at different noise levels in terms of segmentation accuracy, speed of segmentation and signal-to-noise ratio. The similarity measures used to test are Dice similarity measure (DSM), MSE and PSNR have been used to measure the performance of proposed method on real clinical data and synthetic brain data with known ground truth. Peak Signal-to-Noise Ratio (PSNR) is defined as the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation (Huynh-Thu and Ghanbari 2008).

Mean Square Error (MSE) which for two M\( \times \)N monochrome images I and K, where one of the images is considered a noisy approximation of the other is defined as (Lehmann and George Casella 1998):
\[
\text{MSE} = \frac{1}{mn} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} [I(i,j) - K(i,j)]^2
\]  
(4.21)

\[
\text{PSNR} = 10 \log_{10} \left( \frac{\text{Max}_I^2}{\text{MSE}} \right)
\]  
(4.22)

The similarity measure used to test is Dice similarity measure (DSM) (Zijdenbos et al 1994). It is defined as

\[
\text{DSM}(r) = \frac{2N_{p \cap g}(r)}{N_p(r) + N_g(r)}
\]  
(4.23)

Where \(N_{p \cap g}(r)\) represents the number of pixels classified by both proposed method and ground truth as model \(r\), \(N_p(r)\) represent the number of pixels classified as model \(r\) by the proposed method and \(N_g(r)\) represent the number of pixels classified as model \(r\) by the ground truth.

### 4.8.1 Segmentation of Synthetic Brain MR Images

The simulated images are having the size as 181×217. The classes which are to be segmented are gray matter, white matter and CSF. The images in Figures 4.6 are original images taken from Brain web. The panels (a),(b),(c),(d) of Figure 4.6 illustrates the images under the noise levels at 3%, 5%, 7% and 9% respectively with gold standard from Brainweb. The segmentation of CSF, WM and GM for the input image of Figure 4.6 by the proposed method is shown in Figure 4.7(a)–(d), respectively.

The clustering results from proposed FFT based EM-GMM with spatial information are superior to those images obtained without FFT based EM-GMM method in terms of measured Signal-to-noise ratio. The noise
spots were largely reduced by using the proposed method even in low quality images at noise levels of more than 7%. It indicated that the proposed method with neighbouring spatial information has been resistant to noise whereas EM-GMM method is sensitive to noise. This is due to the fact that the proposed method is taking into account of both intensity distribution and spatial information. But EM-GMM method without using spatial information is taking into account of only intensity information.

The images in Figure 4.7 showed that the regions with different intensity are more homogeneous, indicating that the classification process is less affected by the noise levels. Because the membership of the correct cluster is enhanced by the cluster distribution of the neighbouring pixels, the better performance of the proposed FFT based EM-GMM with spatial information is ensured to the integration of the neighbourhood information into the iteration procedure. In contrast, because no similar cluster was present in the neighbourhood, the method of segmentation using EM-GMM caused misclassification of these noisy pixels. In addition, better performance of proposed method than the EM-GMM segmentation also was observed in the other 20 images of the simulated MR brain images from Brainweb.
Figure 4.6 Simulated MR brain images with gold standard from Brainweb (a), (b), (c) and (d) show the images obtained at 3%, 5%, 7%, and 9% noise levels, respectively.
Figure 4.7  Segmentation of WM, GM and CSF using the proposed EM-GMM-FFT with spatial information function of Figure 4.6 (a), (b), (c) and (d) respectively

The average DSM index and execution times for the 20 MRI brain images for Synthetic dataset of Brainweb with known ground truth are shown in Table 4.1. The proposed FFT based EM-GMM segmentation with spatial information is having better segmentation accuracy and speed of execution as compared to existing EM-GMM segmentation without FFT.
Table 4.1  The average DSM index and execution time for the 20 MRI brain images for Synthetic dataset of Brainweb with known ground truth

<table>
<thead>
<tr>
<th>Method</th>
<th>DSM For MR Images Without Noise</th>
<th>DSM For MR Images With 3% Noise</th>
<th>Time of Execution</th>
</tr>
</thead>
<tbody>
<tr>
<td>EM-GMM (Mayer and Greenspan 2006)</td>
<td>0.86 0.84 0.83</td>
<td>0.84 0.82 0.81</td>
<td>30 Minutes</td>
</tr>
<tr>
<td>Proposed FFT-EM-GMM with spatial information function</td>
<td>0.89 0.87 0.88</td>
<td>0.87 0.88 0.89</td>
<td>0.955 Seconds</td>
</tr>
</tbody>
</table>

4.8.2  Segmentation of Real Brain MR Images

Further comparison of the proposed method with EM-GMM without applying FFT has been done for real brain MRI images. The size of the image is of 512×512. A total 80 MRI brain images were analyzed, and one example has been presented here. The mid sagittal brain image of a patient is taken (Figure 4.8) and the proposed algorithm has been applied.

![Mid sagittal brain image](image1)

Figure 4.8  Mid sagittal brain image1
From the intensity histogram, image intensity can be modeled as a three component Gaussian mixture and is given by equation (4.4). This is shown in Figure 4.9 for the Midsagittal brain image1 of Figure 4.8. There are 19,660 values for intensity which is gray scale values between 50 and 250 of the magnetic resonance image (MRI) of the human brain has been found from the Figure 4.9. Figure 4.10 and 4.11 shows the Mid Sagittal brain image 2 and the corresponding fitted 3-component Gaussian Mixture Model respectively.

Figure 4.9  Histogram of image intensity of Mid sagittal brain image1
Figure 4.10 Mid Sagittal brain image 2

Figure 4.11 The fitted 3-component Gaussian Mixture Model for Mid Sagittal brain image 2
Figure 4.12, Figure 4.13(a) and 4.13(b) show the results of segmentation of WM, CSF and GM respectively by using proposed EM-GMM-FFT with spatial information function.

Figure 4.12  WM segmented mid sagittal brain image2

Figure 4.13  (a) CSF segmented image (b) GM segmented image of mid sagittal brain image2

A set of real images with variance of noise of 0.05 and 0.005 has been taken and FFT was applied. After that EM–GMM with spatial information has been calculated by initializing k-means algorithm. Figure
4.14(a) displays the original image and Figure 4.14(b) shows the image with noise of variance 0.005. Figure 4.15(a) shows the image with noise of variance 0.05. Figure 4.15(b) shows the result of Gaussian filtered image of Figure 4.15(a). Figure 4.16 (a) shows the magnitude and 4.16(b) shows the phase of Fourier transformed image obtained respectively by FFT.

Figure 4.14  (a) Original MRI Image1   (b) Image1 with noise of variance  0.005

Figure 4.15  (a) Image1 with noise of variance 0.05   (b) Output of Gaussian filtered image of Figure 4.15 (a)
Figure 4.16 (a) Magnitude of the original MR Image1 (b) Phase after applying Fourier Transform for the original MR Image1

Results of applying Circle filtering and Gaussian filtering is shown in Figure 4.17(a) and 4.17(b) respectively. As can be seen from the Figure 4.17(a) the Circular filter introduced ringing effects or artifacts where as the Gaussian filtered image reduces these artifacts and that is why the Gaussian filtered image is more suitable for MR images denoising. Figure 4.18(a) and 4.18(b) show the magnitude of Circle mask and Magnitude of Gaussian mask of original MR image1 respectively.

Figure 4.17 (a) Results of applying Circle filtering (b) Results of Gaussian filtering
As seen from Table 4.2, the proposed FFT based EM-GMM approach improves signal-to-noise ratio, reduces MSE and also reduces number of arithmetical computations by means of increase in segmentation speed. In the second experiment, another dataset (IBSR, 256x256x128) is obtained from the Center for Morphometric Analysis at Massachusetts General Hospital. The segmentation results obtained by employing GMM and by the proposed EM-GMM-FFT with spatial information function is shown in Figure 4.19. Figure 4.19 (a) shows the original image and Figure 4.19 (b) shows the expert-classified ground truth. Figure 4.19(c) and (d) show the classification of CSF, GM and WM obtained by the GMM and proposed method respectively. Qualitative results in Figure 4.19 (d) demonstrated very good resemblance between the segmentation results and the raw image, and between the expert-guided manual segmentation and the segmentation results based on the proposed method.
Figure 4.19 Segmentation results of the real brain image by the proposed EM-GMM-FFT with spatial information function.
Table 4.2  Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR) values after applying FFT to a set of input images containing noise variance of 0.05 and 0.005.

<table>
<thead>
<tr>
<th>Input Image</th>
<th>Noise Variance (V)</th>
<th>Without FFT</th>
<th>With FFT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSE</td>
<td>PSNR (db)</td>
<td>Time (seconds)</td>
</tr>
<tr>
<td>Image1</td>
<td>0.05</td>
<td>2.0400e+03</td>
<td>15.03</td>
</tr>
<tr>
<td>Image1</td>
<td>0.005</td>
<td>240.9571</td>
<td>24.31</td>
</tr>
<tr>
<td>Image2</td>
<td>0.05</td>
<td>1.9941e+03</td>
<td>15.13</td>
</tr>
<tr>
<td>Image2</td>
<td>0.005</td>
<td>216.8813</td>
<td>24.76</td>
</tr>
</tbody>
</table>

The Dice similarity coefficient for real brain images from IBSR data set (IBSR01, index 40) with the proposed and existing GMM algorithm is listed in Table 4.3. As evident in Table 4.3, the proposed EM-GMM-FFT with spatial information function demonstrates a better performance compared to GMM with a higher Dice similarity coefficient in WM, CSF and GM labels.

Table 4.3  The Average Dice Similarity Coefficient comparison for the real brain image (IBSR01, from index 1–128) by the proposed FFT-EM-GMM with spatial information function

<table>
<thead>
<tr>
<th>Method</th>
<th>DSM For MR Images</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CSF</td>
</tr>
<tr>
<td>GMM(Titterington et al 1985)</td>
<td>0.1811</td>
</tr>
<tr>
<td>Proposed FFT-EM-GMM with spatial information function</td>
<td>0.189</td>
</tr>
</tbody>
</table>
4.9 PROPOSED MEDICAL IMAGE MIXTURE DENSITY CLUSTERING SEGMENTATION USING RADIX4-FAST FOURIER TRANSFORM AND SPATIAL INFORMATION FUNCTION

4.9.1 System Overview

The block diagram of the proposed system is shown in Figure 4.20, in which the Radix 4-FFT is applied for input medical image of size M×N. When the number of data points unequal to a power of 4 (N \sim 4^v), pixels with a value of zero are added to make it to the correct size.

![Block diagram of the proposed system](image)

Figure 4.20 Block diagram of the proposed fast medical image mixture density clustering segmentation using Radix4-Fast Fourier Transform with spatial information
The radix 4FFT's output of input image contains both real and imaginary parts. The absolute value of radix4-FFT is calculated and after that EM algorithm is applied. This function learns the parameters of a Gaussian Mixture Model using a recursive Expectation-Maximization estimation of the parameters. The parameters of a Gaussian Mixture Model are initialized by using k-means clustering algorithm.

GMR performs Gaussian Mixture Regression (GMR), using the parameters of a Gaussian Mixture Model (GMM). Given partial input data; the algorithm computes the expected distribution for the resulting dimensions. This function plots a representation of the components (means and covariance matrices) of a Gaussian Mixture Model (GMM) or a Gaussian Mixture Regression (GMR).

4.9.2 Radix-4 FFT and MR Imaging

In Magnetic Resonance Imaging (MRI)/functional magnetic resonance imaging (fMRI), magnetic field gradients $G_x$ and $G_y$ are applied to encode and then complex valued Fourier transformation (FT) of the effective Proton Spin Density (PSD) is measured in a real-valued physical object. In MRI/fMRI, these complex-valued measurements are acquired in spatial frequency space (usually two-dimensional), also called k-space from the use of the k variables for its axes ($k_x,k_y$). These measurements are transformed into a complex-valued image by an image reconstruction method to the Euclidian space. The most common image reconstruction method is the inverse Fourier transform. These discrete complex valued measurements, when placed at their proper spatial frequency location, are ideally the discrete FT of the PSD. Radix-4 FFT requires less number of multiplications than radix-2 FFT of the same size, and for the same throughput, it requires less hardware. Hence, quite often, radix-4 structure is preferred to radix-2
structure. In comparison with radix-2 the radix-4 FFT has got a marginally better error performance (Prakash and Rao 1981). Fast Fourier Transform (FFT) algorithms play a critical role in improving the feasibility of using the Discrete Fourier Transform (DFT) in a wide range of applications. It reduces number of arithmetical computations from $O(N^2)$ to $O(N\log_2 N)$ (Cooley and Tukey 1965).

The acquisition of MRI/ fMRI data in a finite subset of k-space produces ringing-artifacts and ‘side lobes’ that distort the image. In MRI/fMRI studies it is common practice to spatially smooth the acquired data prior to performing statistical analysis. Spatial smoothing involves blurring the functional MRI images by convolving the image data with a filter kernel, most frequently a Gaussian. Spatial smoothing with a large enough kernel can eliminate these artifacts, but at a cost in image resolution.

However, too little spatial smoothing leaves the ringing artifacts and side lobes caused by k-space truncation intact, leading to a decrease in signal-to-noise ratio and statistical power (Lindquist and Wager 2008). Hence, to make use of the high-resolution provided by MRI without introducing artifacts, Radix4-FFT is used instead of Gaussian smoothing. The advantages of Radix4-FFT as compared to Gaussian smoothing are

1. Low resolution of image results in artifacts. That is why accuracy is less if Gaussian filtering used. But Radix4-FFT reduces the side lobes (spectral leakage), decreases artifacts and increases accuracy of classification.

2. Spatial-domain filtering (Gaussian blurring) with convolution is computationally expensive operation.
For a $K \times K$ filter on an $M \times N$ image, convolution costs $O(MNK^2)$ additions and multiplications, or $O(N^4)$ supposing $M \sim N \sim K$. But the speed of segmentation using Radix4-FFT is significantly faster—it is $O(N \log_2 N)$. Using FFT convolution, the time depends only on the image size, and not the size of the kernel.

### 4.10 PROPOSED ALGORITHM FOR SEGMENTATION USING EM – GMM – GMR – RADIX 4 - FFT WITH SPATIAL INFORMATION FUNCTION

**Input:** Input medical images of Size $M \times N$.

**Output:** Segmented brain tissues.

**Step 1:** Read and display the Input Images.

**Step 2:** Create the signal and fix the FFT length, sample frequency and create the time vector.

**Step 3:** Compute the absolute value intensity of Radix4–FFT output.

**Step 4:** Estimate the values of MSE (Mean Square Error) and PSNR for without Radix 4-FFT and with Radix4-FFT.

**Step 5:** Display the Signal Spectrum of Radix4-FFT.

**Step 6:** Apply EM algorithm with spatial information by initializing k-means algorithm.

**Step 7:** Apply GMM-GMR algorithm to the output of Radix4-FFT-EM.

**Step 8:** After EM-GMM-GMR modeling, apply inverse Radix4-FFT.

$$g = \text{Inverse Radix4-FFT} [\text{EM-GMM-GMR} \{\text{abs} \{\text{Radix4-FFT} (f(i,j))\}\}]$$
**Step 9:** Apply Bayesian classifier in order to classify the brain tissues. Estimate MSE and PSNR. Compute DSM index for the segmented outputs.

### 4.10.1 Initialization Step of K-Means Algorithm by using Kaufman Approach (KA)

The disadvantages of K-Means algorithm such as especially sensitive to initial starting conditions and converges finitely to local minima can be eliminated by using Kaufman Approach (KA) proposed by Kaufman and Rousseeuw (1990). The initial cluster centers of K-means affect the convergence of EM-GMM and performance and KA approach is used to solve this problem. The initialization methods for the K-Means algorithm has been selected according to two criteria: quality of the final clustering performed by the K-Means algorithm when each concrete initialization method is used (effectiveness) and sensitivity of the K-Means algorithm with each initialization method to initial starting conditions (robustness).

The initial clustering is obtained by the successive selection of representative instances until K instances have been found. The first representative instance is the most centrally located instance in the database. The rest of the representative instances are selected according to the heuristic rule of choosing the instances that promise to have around them a higher number of the rest of instances.

\[
F_i = \frac{1}{n} \sum_{j=1}^{n} F((P_{ij}, O_{ij})), \quad i=1,\ldots,m, \tag{4.24}
\]

\( F_i \) is the mean of the square-error values when starting from the K seeds used to generate \( \{P_{i1}, \ldots, P_{in}\} \) independently on instance order and \( F((P_{ij}, O_{ij}) \) is the square-error value when starting from and the instance order
is $O_{ij}$. This proposed work uses KA approach to make the K-Means algorithm exhibit a robust behavior.

**4.10.2 The Pseudo-Code of the KA Initialization Method**

**Step 1**: Select the first seed as the most centrally located instance.

**Step 2**: For every non selected instance $ω_i$ do

- For every non selected instance $ω_j$ do
  
  Calculate $C_{ji} = \max(D_j - d_{ji}, 0)$ where $d_{ji} = \|ω_i - ω_j\|$ and $D_j = \min_s d_{sj}$ being $s$ one of the selected seeds.

- Calculate the gain of selecting $ω_i$ by $\sum_j C_{ji}$.

**Step 3**: Select the not yet selected instance $ω_i$ which maximizes $\sum_j C_{ji}$.

**Step 4**: If there are $K$ selected seeds THEN stop

ELSE go to Step 2.

**Step 5**: For having a clustering assign each non selected instance to the cluster represented by the nearest seed.

The parameters of mixture are initialized as: Initial values of mu and sigma are:

Mu: $\mu_1 = \begin{bmatrix} 10 \\ 29.8 \end{bmatrix}$, $\mu_2 = \begin{bmatrix} 5.2 \\ 7.2 \end{bmatrix}$, $\mu_3 = \begin{bmatrix} -0.1 \\ 31.9 \end{bmatrix}$

Sigma: $\Sigma_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 2.7 \end{bmatrix}$, $\Sigma_2 = \begin{bmatrix} 0.7 & 0.7 \\ 0.6 & 2.0 \end{bmatrix}$, $\Sigma_3 = \begin{bmatrix} 0.6 & 0.1 \\ 0.1 & 1.9 \end{bmatrix}$
4.11 SPATIAL INFORMATION FUNCTION

A spatial function is defined as

\[ s_{ij} = \frac{1}{n} \sum_{k \in NB(x_i)} P(\tilde{j} | x_i) \]  (4.25)

Where \( NB(x_i) \) is a squared window centered on pixel \( x_i \) in the spatial domain and \( n \) is the size of the neighbourhood that is set as 4. \( P(\tilde{j} | x_i) \) is the probability of the neighbouring pixels of \( x_i \) that is labeled as \( \tilde{x_i} \) and belongs to the \( j \)th class.

4.12 PROPOSED EM-GMM-GMR-RADIX4-FFT WITH SPATIAL INFORMATION FUNCTION

Gaussian Mixture Regressors (GMR) proposed by (Ghahramani and Jordan 1994) do not perform automatic feature selection. The GMR based algorithm is extended to perform sequential feature selection (Falk and Chan 2005). The details of GMM-GMR and details of feature selection extension has been given by (Tiago Falk et al 2006).

GMM based regression relay on modeling the joint density of the predictor variable vector (\( x \)) with target variable \( y \), i.e, \( u=[y, x] \).

The mean vector and the covariance matrix of the \( i \)th GMM component become

\[ \mu_i = (\mu_{i,y}, \mu_{i,x}) \text{, and } \Sigma_i = \begin{pmatrix} \sum_{-1} y_{i} y_{i} & \sum_{-1} y_{i} x_{i} \\ \sum_{-1} x_{i} y_{i} & \sum_{-1} x_{i} x_{i} \end{pmatrix} \] respectively.
Given the GMM parameters, the minimum mean-square-error regression function $\hat{f}$ is the conditional expectation of the target variable, given the predictor variables (Ghahramani and Jordan 1994)

$$\hat{f}(x) = E[y|x] = \sum_{i=1}^{M} h_i(x) \left[ \mu_i^y + \sum_i y^x (\sum_i \Sigma_i^{-1})^{-1} \left( x - \mu_i^x \right) \right]$$  \hspace{1cm} (4.26)

The function $\hat{f}$ above is a weighted sum of linear models, with weights $h_i(x)$ representing the probability that the $i^{th}$ GMM component generated the vector $x$ and given by

$$h_i(x) = \frac{\alpha_i e^{-\frac{1}{2} x^T \Sigma_i^{-1} x}}{\sum_{k=1}^{M} \alpha_k e^{-\frac{1}{2} x^T \Sigma_k^{-1} x}}$$

If GMM covariance matrices are restricted to be the diagonal, Equation (4.25) simplifies to

$$\hat{f}(x) = \sum_{i=1}^{k} h_i(x) \mu_i^y$$  \hspace{1cm} (4.28)

The primary objective in feature selection and model optimization is to find, among $n$ candidate feature variables a subset of variables $x = \{x_1, x_2, ..., x_m\}$, $m < n$, and a mapping such that approximates the target variable $y$. The GMR based algorithm performs feature selection while progressively constructing $\hat{f}$ using Equation (4.25) or Equation (4.27) (Tzimiropoulos et al 2010, Mark et al 1999).
4.13 EXPERIMENTS AND RESULTS OF PROPOSED EM-GMM-GMR-Radix-4 FFT WITH SPATIAL INFORMATION METHOD

The performance of the proposed algorithm is compared with EM-GMM with radix4-FFT and EM-GMM without radix4-FFT. Although the algorithm should be validated on real MR data, a comprehensive validation is easier performed on simulated images, since the ground truth is not known for in vivo data. Furthermore, experiments with simulated data allow studying the influence of several imaging artifacts, such as noise etc. These tests were performed with the simulated MR images (Zijdenbos et al 1994) with gold standard from Brain web and with synthetic brain images available from McGill University, Montreal, Canada (www.bic.mni.mcgill.ca/brainweb.) with 3% noise and also with known ground truth. The results were also tested with real MRI images of brain.

4.13.1 The Segmentation of Simulated MR Brain Images

The simulated images are having the size as 181×217. The classes which are to be segmented are gray matter, white matter and CSF. The images in Figures 4.21, 4.22, 4.23 are original images, segmented images using EM-GMM-without radix4-FFT and EM-GMM-with radix4-FFT using spatial information segmentation method respectively. The panels (a),(b),(c),(d) of the above three Figures show the images under the noise levels at 3%, 5%, 7% and 9% respectively. Figure 4.24 shows some of the 20 images used in the proposed method, while Figure 4.25 show the images with 3% noise. The ground truth of CSF, WM and GM from the Brainweb images for axial slice no.66 in Figure 4.24(a) is shown in Figure 4.26 (a–c), respectively. The segmentation of CSF, WM and GM from Figure 4.24(a) by the proposed EM-GMM-FFT with spatial information segmentation method is shown in Figure 4.27 (a–c), respectively.
Figure 4.21 Simulated brain MR image with gold standard from Brainweb. Panels (a), (b), (c) and (d) shows the images obtained at 3%, 5%, 7% and 9% noise levels, respectively.

Figure 4.22 Segmentation of WM, GM, CSF using EM-GMM - without Radix4-FFT of Figure 4.21(a), (b), (c), (d) respectively. Images obtained at (a) 3% noise level (b) 5% noise level (c) 7% noise level (d) 9% noise level using EM-GMM - without Radix4-FFT

Figure 4.23 Segmentation of WM, GM, CSF using EM-GMM-GMR-Radix4-FFT with spatial information function of Figure 4.21(a), (b), (c), (d) respectively.
Figure 4.24 The BrainWeb images for different axial slices (a) Slice no.66 (b) Slice no.86 (c) Slice no. 116

Figure 4.25 The Brain Web images for different axial slices with 3% noise (a) Slice no. 66 (b) Slice no.86 (c) Slice no. 116

Figure 4.26 The ground truth of CSF, WM, GM of Figure 4.24(a) (a) CSF (b) WM (c)GM
The clustering results from the proposed radix4FFT-GMM-GMR are superior to those images obtained without radix4FFT-EM-GMM method in terms of measured Signal-to-noise ratio. The noise spots were largely reduced by using the proposed method even in low quality images at noise levels of more than 7%. Typically, a value DSM > 0.7 means that there is an excellent agreement between the two segmentation methods (Adelino 2007).

It indicated that the proposed method with neighbouring information is resistant to noise whereas EM-GMM method is sensitive to noise. This is due to the fact that proposed method is takes into account of both the intensity distribution and spatial information. But EM-GMM method-without radix4 FFT is takes into account of only intensity information only.

The images in Figure 4.23 and Figure 4.27 show that the regions with different intensity are more homogeneous, indicating that the classification process is less affected by the noise levels. Because the membership of the correct cluster is enhanced by the cluster distribution of the neighbouring pixels, the better performance of the proposed method in
image processing is ascribed to the integration of the neighbourhood information into the iteration procedure along with radix4-FFT-GMM-GMR.

The average DSM indices for those 20 images of EM-GMM, proposed FFT-EM-GMM with spatial information segmentation and by the proposed Radix4-FFT-EM-GMM-GMR with spatial information segmentation method are shown in Table 4.4. It can be inferred from the Table 4.4 that the average DSM of the 20 images by the proposed Radix4-FFT-EM-GMM-GMR with spatial information segmentation method is higher than the other proposed EM-GMM-FFT segmentation method and existing EM-GMM. The main reason for this improvement has been is the use of radix4-FFT along with spatial information function. The time of execution of the proposed Radix4-FFT-EM-GMM –GMR with spatial information function segmentation is obviously less than the other proposed method and EM-GMM. It mainly owes to the proposed radix4-FFT technique used with EM-GMM-GMR which operates in frequency domain instead of spatial domain.

### Table 4.4 The average DSM index and execution time for the 20 MRI brain images for Synthetic dataset of Brainweb with known ground truth

<table>
<thead>
<tr>
<th>Method</th>
<th>DSM For MR Images Without Noise</th>
<th>DSM For MR Images With 3% Noise</th>
<th>Time of Execution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CSF</td>
<td>WM</td>
<td>GM</td>
</tr>
<tr>
<td>EM-GMM (Mayer and Greenspan 2006)</td>
<td>0.86</td>
<td>0.84</td>
<td>0.83</td>
</tr>
<tr>
<td>Proposed EM-GMM-FFT with spatial info.</td>
<td>0.89</td>
<td>0.87</td>
<td>0.88</td>
</tr>
<tr>
<td>Proposed EM-GMM-GMR- Radix4-FFT</td>
<td>0.94</td>
<td>0.90</td>
<td>0.91</td>
</tr>
</tbody>
</table>
4.13.2 The Segmentation of Real Brain MR Images

Further comparison of the proposed method with EM-GMM without applying radix4 FFT is done for MRI brain images. This section shows validation results with real expert-classified MR data. The data set has been taken from the Internet Brain Segmentation Repository (IBSR) website. The data set comprises T1-weighted brain-MR images for 18 subjects. Figure 4.28(a) shows an example from the data set. The voxel size for this image is 0.9375x0.9375x1mm. It has been observed that the data has lower contrast and possesses certain acquisition-related artifacts that make the classification task more challenging than that for the BrainWeb dataset. A set of real images with variance of noise as 0.05 and 0.005 has been taken and radix4-FFT was applied.

The Radix4-FFT spectrum output of real brain MRI (Figure 4.28(a)) is shown in Figure 4.28(b). It shows that by applying Radix4 FFT the spectrum output contain less spectral leakage of energy. It contains many main lobes and represents the output of Radix4-FFT contains more signal than just to noise. Selected parts of the frequency spectrum can easily be subjected to piecewise mathematical manipulations (attenuated or completely removed). Therefore, signal smoothing can be easily performed with removing completely the frequency components from a certain frequency and up, while the useful (information bearing) low frequency components are retained. This useful (information bearing) main lobes then transformed into spatial domain by applying inverse Radix4-FFT after the application of EM-GMM-GMR modeling. This spectral manipulation is useful for clearly differentiating the boundary of desired tissue as compared to other tissues and that is why classification accuracy is increased.
After that EM–GMM-GMR algorithm was applied on a set of images containing high signal to noise ratio. The signal to noise ratio has been used in MRI to describe the relative contributions to a detected signal and random superimposed signals or noise - a criterion for image quality.

![Figure 4.28](image)

**Figure 4.28** (a) Real brain MRI input (b) Spectrum of Radix4-FFT of Figure 4.28(a)

Figure 4.29 shows Radix4-FFT-GMM-GMR regression plots of Figure 4.28(a). GMR performs regression using the parameters of a Gaussian Mixture Model. Given partial input data, the algorithm computes the expected distribution of WM, GM and CSF for the resulting dimensions. Figure 4.30 (a) shows the raw image and Figure 4.30 (b) shows the expert-classified ground truth. Figure 4.30(c) shows the classification of CSF, GM and WM obtained by the proposed method. Qualitative results in Figure 4.30(c) demonstrated very good resemblance between the segmentation results and the raw image, and between the expert-guided manual segmentation and the segmentation results based on the proposed method.
Figure 4.29  Radix4-FFT-GMM-GMR regression plots of Figure 4.28(a)

Figure 4.30  (a)Real brain MRI input (b) The expert-classified ground truth (dark gray region represents CSF, light gray represents GM and white region represents WM respectively) (c) The classification of CSF, WM and GM produced by the proposed method

In order to test the proposed algorithm, a real brain data set (IBSR01, 256x256x128) obtained from the Center for Morphometric Analysis
at Massachusetts General Hospital is used. As seen from Table 4.5, the use of radix-4 FFT approach improves Signal-to-Noise ratio and also reduces number of arithmetical computations.

### Table 4.5 Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR) values after applying Radix4-FFT to a set of input images containing noise variance of 0.05 and 0.005

<table>
<thead>
<tr>
<th>Input Image</th>
<th>Noise variance (V)</th>
<th>Without Radix4 FFT</th>
<th>With Radix4 FFT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MSE (MSE)</td>
<td>PSNR (PSNR)</td>
</tr>
<tr>
<td>Image1</td>
<td>0.05</td>
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<td>Image2</td>
<td>0.005</td>
<td>216.8813</td>
<td>24.76</td>
</tr>
</tbody>
</table>

Figure 4.31(a) shows one example IBSR 01 data set slice (index=40) that is used to compare the proposed algorithm with existing method. The image shown in Figure 4.31(b) is the ground truth of the original image. Figure 4.31(c)–(e) shows the results obtained by implementing GMM, proposed FFT-EM-GMM with spatial information function and the proposed EM-GMM-GMR-Radix4-FFT with spatial information function. Qualitative results in Figure 4.31 demonstrated very good resemblance between the provided ground truth and the segmentation results based on the proposed method. As shown in Table 4.6, the Dice similarity coefficient obtained by employing GMM is very poor as compared to the proposed EM-GMM-Radix-4 FFT with spatial information function.
Figure 4.31  Segmentation results of the real brain image by the proposed EM-GMM-Radix-4 FFT with spatial information function
Table 4.6  The Average Dice Similarity Coefficient: Comparison for the real brain image (IBSR01, from index 1–128) by the proposed FFT-EM-GMM with spatial information function and the proposed Radix-4-FFT-EM-GMM with spatial information function

<table>
<thead>
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<th>Method</th>
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<tr>
<td>Proposed Radix-4FFT-EM-GMM with spatial information function</td>
<td>0.191</td>
</tr>
</tbody>
</table>

4.14 PROPOSED MEDICAL IMAGE SEGMENTATION USING MIXED-RADIX FAST FOURIER TRANSFORM BASED EM-GMM WITH SPATIAL INFORMATION FUNCTION

4.14.1 Proposed EM-GMM Algorithm Based on Mixed Radix-FFT with Spatial Information Function

In this proposed work, a new modification to GMM and EM has been introduced by incorporating neighbourhood information into likelihood function and EM steps. The average of neighbour pixels around each pixel (spatial information function) is calculated prior to GMM clustering and incorporated in GMM and EM functions beside the pixel value. The speed of segmentation got increased by the process of faster clustering iterations. By using the proposed spatial neighbourhood information, the regions with different intensity are more homogeneous, indicating that the classification process is less affected by the noise levels.
The Mixed radix FFT decreases the number of required computations and enhances the smoothing in terms of improvement in signal to noise ratio. Mixed-radix FFT algorithm does not require number of points \( N \) to be power of two. Higher radix decomposition is often preferred because it can decrease computational complexity by reducing complex multiplications which are required. The trade-off is that hardware complexity of the butterfly grows as the radix becomes high. However a fixed-radix algorithm sometimes is found to be deficient due to its limitation on FFT size (power of \( r \)). As higher radix algorithms have been prepared to reduce the computational complexity, the flexibility of the FFT size is also limited (Chi-Chen Lai and Wei Hwang 2011). Therefore, the Mixed-radix algorithm has been adopted in this thesis to keep the architecture flexible while using a high radix algorithm for decreasing the number of required computations and to enhance the smoothing in terms of improvement in signal to noise ratio.

4.14.2 Proposed System Overview

The following is an overview of the framework of the proposed method to classify the WM, GM and CSF of the brain MRI. The block diagram of the proposed Mixed-Radix EM-GMM with spatial information method is shown in Figure 4.32, in which the Mixed Radix 4-FFT is applied for input image medical of size \( M \times N \).
Figure 4.32  Block diagram of the proposed Mixed radix FFT-EM-GMM with spatial information segmentation method

The Mixed radix FFT's output of input image contains both real and imaginary parts. The absolute value of Mixed radix FFT output is calculated following which EM-GMM algorithm is applied. This function learns the parameters of a Gaussian Mixture Model (GMM) using a recursive Expectation-Maximization (EM) algorithm, starting from an initial estimation of the parameters. This function initializes the parameters of a Gaussian Mixture Model (GMM) by using k-means clustering algorithm.
4.14.3 Description of Algorithm with Proposed Neighbourhood Weighted Method

The proposed algorithm starts with application of Mixed Radix-FFT and spatial information function. First, a Mixed Radix-FFT had been applied to the input MRI of the brain. In the case of, Mixed Radix-FFT the general idea is to factor the length of the DFT, n, into factors that are efficiently handled by the routines. A number of short DFT's are implemented with a minimum of arithmetical operations and using (almost) straight line code resulting in very fast execution, when the factors of N belong to this set. Especially radix-10 is optimized by making use of Mixed Radix-FFT. Prime factors, that are not in the set of short DFT's are handled with direct evaluation of the Discrete Fourier Transform pair expression (Jens Joergen Nielsen 2000). These above mentioned advantages of Mixed Radix-FFT have been used in this thesis for speeding up the segmentation and higher classification accuracy of tissues. The details of Mixed Radix-FFT has been explained in appendix A.

The average of neighbour pixels around each pixel $\bar{x}_i$ is calculated prior to GMM clustering. In the likelihood function (Equation 4.7), distribution value of $\bar{x}_i$ is added to the distribution value of pixel $x_i$ as neighbourhood information.

$$\log(L(\theta \mid X)) = \log \prod_{i=1}^{N} p(x_i \mid \theta) =$$

$$\sum_{i=1}^{N} \log \left( \sum_{j=1}^{M} \alpha_j \left[ (1 - \beta) p_j(x_i \mid \theta_j) + \beta \cdot p_j(x_i \mid \theta_j) \right] \right)$$

(4.29)
The parameter $\beta$ determines the weight of neighbourhood information. Incorporating the neighbourhood information improves the performance of segmentation methods in high level of noise, but the blurring effect degrades the performance of them in low noise level. In order to overcome the degrading effect of algorithms in low level of noise, the variance of noise is used to specify the weight of neighbourhood information ($\beta$). Its value is set to $\sigma$, where $\sigma$ is the variance of noise. In previous neighbourhood based EM extensions, neighbourhood information is calculated in clustering iteration; but in this algorithm $\bar{x}_i$ is computed before iteration, thus, the clustering will be faster. The EM is modified as follows:

**Step a:** In Equation 4.8, distribution value of $\bar{x}_i$ is added to the distribution value of pixel $x_i$ as neighbourhood information:

$$A = (1-\beta)p_{ij}(x_i|\theta_j) + \beta P_{ij}(x_i|\theta_j)$$

$$p(j|x_i, \theta^1) = \frac{\sum_{j=1}^{N} A_{ij} \theta^1_j}{\sum_{j=1}^{N} A_{ij}}$$  

**Step b:** In Equation 4.10, $\bar{x}_i$ is added to $x_i$ as neighbourhood information:

$$\mu_{ij}^{t+1} = \frac{\sum_{i=1}^{N} (1-\beta) \cdot x_i + \beta \cdot \bar{x}_i \cdot P(j|x_i, \theta^1)}{\sum_{i=1}^{N} P(j|x_i, \theta^1)}$$
Step c: In Equation 4.11, the distance of $x_i$ from the component centre is added to the distance of $x_i$ from the component centre as neighbourhood information:

$$d(x) = (x - \mu_{j}^{t+1})(x - \mu_{j}^{t+1})^T$$  \hspace{1cm} (4.34)

$$= \sum_{j=1}^{N} \frac{p(j|x_i,\theta^i).(d(x_i) - \beta_i.d(x_i))}{\sum_{i=1}^{N} p(j|x_i,\theta^i)}$$  \hspace{1cm} (4.35)

### 4.15 RESULTS AND PERFORMANCE ANALYSIS

The proposed methods were validated with a three-class (WM, GM and CSF) segmentation on both simulated T1w brain MRI data and real brain MRI data. Segmentation was performed using the proposed methods with varying noise level and the results were compared with existing algorithms.

Figure 4.33 shows the results of proposed EM-GMM-spatial information with different FFT algorithms on simulated MR images of Brainweb. Table 4.7 shows average Dice Similarity Measure (DSM) index of 20 images from the Brainweb and the time of execution of proposed Mixed Radix EM-GMM-spatial information function method. The proposed algorithms were tested with 3% noise level and the classification accuracy of WM, GM and CSF has been tabulated by making a comparison with ground truth as shown in Table 4.7. As evident from Table 4.8, the proposed Mixed Radix-FFT-EM-GMM with spatial information function method achieves the best accuracy and speed of execution as compared to traditional GMM, proposed FFT-EM-GMM with spatial information function, and proposed Radix4-FFT-EM-GMM with spatial information function.
<table>
<thead>
<tr>
<th>Brain Tissues</th>
<th>Synthetic Ground Truth Image</th>
<th>Proposed Algorithms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FFT-EM-GMM</td>
<td>Radix4-FFT-EM-GMM-GMR</td>
</tr>
<tr>
<td>WM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CSF</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.33  Results of various intensity based statistical segmentation of the brain tissues in frequency domain
Table 4.7  Average Dice Similarity Measure (DSM) index of 20 images from the Brainweb and Time of execution of proposed Mixed Radix FFT- EM-GMM-spatial information function method

<table>
<thead>
<tr>
<th>Method</th>
<th>DSM For MR Images Without Noise</th>
<th>DSM For MR Images With 3% Noise</th>
<th>Time of Execution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CSF</td>
<td>WM</td>
<td>GM</td>
</tr>
<tr>
<td>EM-GMM (Mayer and Greenspan 2006)</td>
<td>0.86</td>
<td>0.84</td>
<td>0.83</td>
</tr>
<tr>
<td>Proposed FFT-EM-GMM with spatial information function</td>
<td>0.89</td>
<td>0.87</td>
<td>0.88</td>
</tr>
<tr>
<td>Proposed Radix4-FFT-EM-GMM-GMR with spatial information function</td>
<td>0.94</td>
<td>0.90</td>
<td>0.91</td>
</tr>
<tr>
<td>Proposed Mixed Radix-FFT-EM-GMM with spatial information function</td>
<td>0.94</td>
<td>0.91</td>
<td>0.92</td>
</tr>
</tbody>
</table>

In comparison, the effect of noise on the final segmented image obtained by employing the proposed method is significantly small as shown in Table 4.8. The average Dice similarity coefficient for a real brain images from IBSR01 data set (from index 40) was calculated for the proposed and existing GMM algorithms is listed in Table 4.9. As seen from Table 4.9 the proposed methods exhibit higher segmentation accuracies for the classification of WM, GM and CSF as compared to traditional EM-GMM. Figure 4.34 shows the results of proposed EM-GMM-spatial information with different FFT algorithms on real brain MR images of IBSR data set. Qualitative results in Figure 4.34 illustrated very good resemblance between the provided ground truth and the proposed Mixed Radix-FFT-EM-GMM with spatial information function as compared to other methods.
Table 4.8 Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR) values after applying Mixed Radix-FFT to a set of input images containing noise variance of 0.05 and 0.005

<table>
<thead>
<tr>
<th>Input Image</th>
<th>Noise variance (V)</th>
<th>Without Mixed Radix-FFT</th>
<th>With Mixed Radix-FFT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSE</td>
<td>PSNR (db)</td>
<td>Time (seconds)</td>
</tr>
<tr>
<td>Image1</td>
<td>0.05</td>
<td>2.0400e+03</td>
<td>15.03</td>
</tr>
<tr>
<td>Image1</td>
<td>0.005</td>
<td>240.9571</td>
<td>24.31</td>
</tr>
<tr>
<td>Image2</td>
<td>0.05</td>
<td>1.9941e+03</td>
<td>15.13</td>
</tr>
<tr>
<td>Image2</td>
<td>0.005</td>
<td>216.8813</td>
<td>24.76</td>
</tr>
</tbody>
</table>

Table 4.9 The Average Dice Similarity Coefficient: Comparison for the real brain image (IBSR01, Index 1-128) by the proposed FFT-EM-GMM with spatial information function, proposed Radix-4FFT-EM-GMM with spatial information function and proposed Mixed Radix-FFT-EM-GMM with spatial information function

<table>
<thead>
<tr>
<th>Method</th>
<th>DSM For MR Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMM (Titterington et al 1985)</td>
<td>CSF</td>
</tr>
<tr>
<td>Proposed FFT-EM-GMM with spatial information function</td>
<td>0.189</td>
</tr>
<tr>
<td>Proposed Radix-4FFT-EM-GMM with spatial information function</td>
<td>0.191</td>
</tr>
<tr>
<td>Proposed Mixed Radix-FFT-EM-GMM with spatial information function</td>
<td>0.193</td>
</tr>
</tbody>
</table>
(a) Original image

(b) Ground Truth

(c) Segmentation by EM-GMM

(d) Segmentation by EM-GMM-FFT-spatial information
The speed of proposed algorithms and existing algorithms in segmenting a slice was also investigated. Table 4.10 represents the average time required to segment a slice using the different mentioned algorithms. It shows that the proposed various FFT based EM-GMM with spatial information function is faster than existing algorithms. The reason is that the proposed algorithms are making use of frequency domain. Table 4.11 shows the complexity of FFT based computation is lesser than DFT based computation. Therefore speed of segmentation of medical images by using FFT techniques is higher as compared to spatial domain segmentation.

Figure 4.35 shows the execution speed of FFT Versus Discrete time Fourier Transform. As seen from the Figure 4.35, that the speed of computation increases with the number of points N as it is proportional to $O(N\log_2 N)$. Spatial-domain filtering (Gaussian blurring) with convolution is computationally expensive operation since execution time depends on both kernel size and image size. Using FFT convolution, the time of execution depends on only the image size, and not the size of the kernel. As evident from the
Figure 4.36, the proposed algorithms in frequency domain require $M^2$ computation time when compared with conventional spatial domain filtering.

**Table 4.10** Comparison of Average times required to segment a slice using the proposed algorithms and existing algorithms

<table>
<thead>
<tr>
<th>Method</th>
<th>Average time of execution (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EM-GMM (Mayer and Greenspan 2006)</td>
<td>1800</td>
</tr>
<tr>
<td>EM-GMM (Tang et al 2009)</td>
<td>3.9</td>
</tr>
<tr>
<td>EM-GMM (Balafar 2011)</td>
<td>2.7</td>
</tr>
<tr>
<td>Proposed FFT-EM-GMM</td>
<td>0.945</td>
</tr>
<tr>
<td>Proposed Radix4-FFT-EM-GMM-GMR</td>
<td>0.95</td>
</tr>
<tr>
<td>Proposed Mixed Radix-FFT-EM-GMM</td>
<td>0.97</td>
</tr>
</tbody>
</table>

**Table 4.11** Time complexity of various algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Computational Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFT based computation</td>
<td>$O(N^2)$</td>
</tr>
<tr>
<td>FFT based computation</td>
<td>$O(N \log_2 N)$</td>
</tr>
</tbody>
</table>
CONCLUSION

This chapter has dealt with improved EM-GMM-spatial information function with FFT, Radix4-FFT, Mixed Radix FFT algorithms that were aimed at producing robust, improved segmentation accuracy and speed of segmentation as compared to other existing methods. The proposed methods has been validated first using simulated brain MRI sequence data and then on real clinical MRI datasets, methods also demonstrated improved accuracy, PSNR and speed of execution over another state-of-the-art approach, the EM-GMM method. The EM-GMM-FFT with spatial information function based method improves the DSM accuracy up to 3% for CSF and WM, 5% for GM, with improvement in speed of segmentation and improvement in average PSNR up to 39.95 db when compared with the other existing method (Mayer and Greenspan 2006).

Figure 4.35 Comparison of Execution Times of convolution in DFT versus FFT

Figure 4.36 Comparison of execution time of spatial filter with FFT based filtering
EM-GMM-spatial information function with FFT demonstrated significantly less computation time required for segmentation, higher segmentation accuracy and improved noise resiliency as compared to EM-GMM method. EM-GMM-GMR-Radix4-FFT with spatial information using K-means clustering (KA approach) is effective in segmentation of medical images for the diagnosis of abnormal tissues in medical images to solve the issues like initial clustering and instance order effects. Kaufman initialization method induces to the K-Means algorithm a more desirable behaviour with respect to the convergence speed. Therefore EM-GMM-Radix4-FFT with spatial information using K-means clustering has faster speed of segmentation, improved segmentation accuracy and reduce noise effectively but also has a prominent effect on the edge property of objects as compared to EM-GMM-FFT with spatial information function. This EM-GMM-Radix4-FFT with spatial information method gives an improvement in DSM accuracy of 4% for CSF, 2% for WM and 4% for GM with improvement in speed of segmentation up to 0.5% as compared with EM-GMM-FFT with spatial information function.

Results using EM-GMM-Mixed radix-FFT with spatial information function demonstrated similar accuracy performance as results using novel EM-GMM-Radix4-FFT with spatial information but with significantly less computation time required and improved noise resiliency. The time consumption is due to neighbourhood information is calculated in EM-GMM-Mixed Radix-FFT with spatial information before the clustering iteration. In EM-GMM-Mixed Radix-FFT with spatial information algorithm $\bar{x}_i$ is computed before iteration, thus, the clustering will be faster. This proposed EM-GMM-Mixed Radix-FFT with spatial information function method gives an improvement in DSM accuracy as 3% for WM, 2% for GM with speed of segmentation up to 0.97 seconds.