CHAPTER 3

PROPOSED AN IMPROVED GRADIENT VECTOR FIELD SNAKE (VECTOR FIELD CONVOLUTION) BASED SEGMENTATION OF BRAIN MRI IN FREQUENCY DOMAIN

3.1 INTRODUCTION

Active deformable contours are based generally on the minimization of a two term energy function (internal and external) (Kass et al 1987). The internal energy encodes prior about the shape of the contour and it preserves and impose its smoothness. The external energy depends on image information and is designed to attract the solution to the desired boundary. Traditionally, the external energy is based only on edge information and is defined as \( f(\cdot) \), where \( f(\cdot) \) is an edge map of the image \( I(\cdot) \).

The edge map is usually defined as the gradient norm of a Gaussian filtered version of the original image \( I \), i.e \( \| \nabla G(\cdot) * I(\cdot) \| \). Gradient vectors of the edge map image \( \nabla f \) are normal to the edge, and pointing towards it. \( \nabla f \) is known as the external potential force field that pulls the contour towards the desired edges. These gradient vectors have a large magnitude in the immediate vicinity of the boundary only and are still nearly zero in image locations away from the boundary. Such model encounters several difficulties such as, its sensitivity to initialization due to noise and its low capture range, and more importantly its difficulties to converge to boundary concavities. Thus traditionally, close initialization to the desired contour is often required.
An external force for snakes, called gradient vector flow (GVF) (Xu and Prince 1998) and its improved version (Xu and Prince 1998) were introduced to overcome two key difficulties of snakes. The snake using the GVF field provides a large capture range and the ability to capture concavities by diffusing the gradient vectors of an edge map generated from the image. Although the GVF field has been widely used and improved in various models (Paragios et al 2004), there are some disadvantages, such as high computational cost, noise sensitivity, parameter sensitivity, and the ambiguous relationship between the capture range and parameters.

Efforts at improving the original GVF snake’s performance have been published. Xu et al (1999) combined GVF force with a constrained balloon force to segment gyri in the cortex. Although this combination works well on this case, its requirement of an a priori knowledge of the region of interest may restrict its application. At the meantime, GVF snakes will provide noisy boundaries of lesions. So a smooth lesion’s boundary should be provided.

This thesis proposes two variations to traditional GVF snakes, one based on novel external force for active models called as vector field convolution using Raxi4-FFT-Hamming Window method and the other on vector field convolution using Mixed Radix FFT-Hamming Window. Both methods are generalized algorithms suitable for all kinds of images.

3.2 DEFORMABLE MODELS

Deformable models are used extensively in image processing, computer vision, and medical imaging applications, particularly to delineate object boundaries. Problems associated with initialization and poor convergence to boundary concavities, however, has limited their utility. An external force for deformable models, largely solving both problems called as
Gradient Vector Flow (GVF), is computed as a diffusion of the gradient vectors of a gray-level or binary edge map derived from the image. It differs fundamentally from traditional deformable model external forces in that it cannot be written as the negative gradient of a potential function, and the corresponding deformable model is formulated directly from a dynamic force equation rather than an energy minimization formulation.

3.2.1 Traditional 2-D Parametric Deformable Models

A 2-D parametric deformable model or deformable contour is a curve \( v(s) = (x(s), y(s))^T, s \in [0,1] \) that moves through the spatial domain of an image to minimize the energy functional

\[
E = \frac{1}{2} (\alpha |v'(s)|^2 + \beta |v''(s)|^2) - E_{\text{ext}}(v(s))ds \quad (3.1)
\]

Where \( \alpha \) and \( \beta \) are weighting parameters that control the deformable contour's tension and rigidity, respectively and \( v'(s) \) and \( v''(s) \) the first and second derivatives of \( v(s) \) w.r.t \( s \). The external potential function \( E_{\text{ext}} \) is derived from the image so that it takes on its smaller values at the features of interest, such as boundaries. Typical external potential energies for a gray-level image \( I(x,y) \) for seeking the edges are given as (Kass et al 1987):

\[
E_{\text{ext}}^{(1)}(x,y) = -|\nabla I(x,y)|^2 \quad (3.2)
\]

\[
E_{\text{ext}}^{(2)}(x,y) = -|\nabla (G_{\sigma}(x,y) * I(x,y))|^2 \quad (3.3)
\]

where \( G_{\sigma}(x,y) \) is a two-dimensional Gaussian function with zero mean and standard deviation \( \sigma \), \( * \) denotes linear convolution, and operator
∇ is gradient operator. If the image is a line drawing (black on white), then appropriate external energies include (Cohen 1991).

\[ E_{\text{ext}}^{(3)}(x, y) = I(x, y) \]  
\[ \text{(or)} \]

\[ E_{\text{ext}}^{(4)}(x, y) = -G_\sigma(x, y)^*I(x, y) \]

It is easy to see from these definitions that larger \( \sigma \)'s will cause the boundaries to become blurry. Such large \( \sigma \)'s are often necessary, however, in order to increase the capture range of the deformable contour.

A deformable contour that minimizes \( E \) must satisfy the Euler equation (Kass et al 1987).

\[ \alpha v''(s) - \beta v'''(s) - \nabla E_{\text{ext}}(v) = 0 \]

This can be viewed as a force balance equation

\[ F_{\text{int}} - F_{\text{ext}}^{(p)} = 0 \]

Where

\[ F_{\text{int}} = \alpha v''(s) - \beta v'''(s) \text{ and } F_{\text{ext}}^{(v)}(p) = -\nabla E_{\text{ext}}(v). \]

To solve equation (3.6), \( v(s) \) is treated as a function of time \( t \). The solution is obtained when the steady state solution of the following gradient descent equation:
\[
\frac{\hat{v}(s,t)}{\hat{t}} = \alpha v'(s,t) - \beta v''(s,t) + f_{\text{ext}}(v(s,t))
\]  

(3.8)

is reached from an initial contour \(v(s,0)\). The internal force \(F_{\text{int}}\) discourages stretching and bending while the external potential force \(F_{\text{ext}}^{(p)}\) pulls the deformable contour toward the desired image edges.

### 3.2.2 External Forces

Instead of a standard energy minimization problem, the solution of the snake is formulated as a force balance equation. Different external forces have been proposed to improve the performance of snakes. The external forces can be generally classified as dynamic forces and static forces (Xu and Prince 1998). The dynamic forces are those that depend on the snake and, as a result, change as the snake deforms. The static forces are those that are calculated from the image, and remain unchanged as the snake deforms. The static forces can be further classified based on the force sources.

Edge-based static forces are calculated from the image edges, whereas region-based static forces are computed using the region intensity and/or texture information (Chakraborty et al 1996). The pressure force, also known as the inflation force, used in balloon models is a useful dynamic force that pushes the snake either outward (inflation) or inward (deflation) (McInerney and Terzopoulos 1995). Although the pressure force can avoid spurious edges, the pressure force causes leakage problem when there are significant gaps in the edges (Xu and Prince 1998). Another limitation of the snake using a pressure force is that it must be initialized either inside or outside the targeted object. A desirable static force should have an important property: a free particle placed in the force field should be able to move to the Features of Interest (FOI), such as edges.
The major drawback of standard external forces is that the force field has an initially zero magnitude in the homogeneous regions of the image. Therefore, the initial snake must be close to the FOI in order to converge. One way to alleviate this problem is to increase the standard deviation of the Gaussian filter used in the external energy, with the cost of distorting the FOI. The edge points are extracted from edge detector, such as the Canny edge detector (Canny 1986). The Gradient Vector Flow (GVF) field is another edge-based static force defined by the vector field that minimizes the energy functional,

$$E_{gvf} = \iint \left[ \mu \left( |\nabla u_{gvf}|^2 - |\nabla v_{gvf}|^2 \right) + |\nabla f_{gvf}^2| + |\nabla f|^2 \right] \, dx \, dy$$  \hspace{1cm} (3.9)$$

Where \( f \) is an edge map derived from the image, and \( \mu \) is a parameter controlling the degree of smoothness of the GVF field. The edge map \( f \) is typically the additive inverse of an external energy such as that given in Equations (3.2)–(3.5). The GVF field outperforms the distance forces by providing a large capture range and the ability to capture boundary concavities. The generalized GVF (GGVF) field, a generalization of the GVF formulation, improves the ability to capture narrow boundary concavities (Xu and Prince 1998). Although the GVF field has these desired properties, there are still several unsolved problems, such as the ambiguous relationship between the capture range and the parameters, the sensitivity to the parameters, noise, and expensive computational cost.

The calculation of the external force can be broken down to two independent steps: the formation of edge map from the image, and the computation of the external force from the edge map. Although the quality of the edge map is a critical factor in snake performance, this thesis focuses on how to obtain a desirable external force field given an edge map, which is likely to be corrupted by noise. A new class of edge-based static forces called
Vector Field Convolution (VFC) using Radix4-FFT has been proposed in this thesis. This new external force is calculated by convolving a vector field kernel with the edge map (Li and Acton 2007). The novel static external force has not only a large capture range and ability to capture concavities, but also reduced computational cost, superior robustness to noise and initialization, flexibility of changing the force field as compared to GVF snakes.

### 3.2.3 Vector Field Convolution (VFC) Snakes

Vector field convolution snakes are active contours using the VFC field as the external force. By replacing the standard external force $f_{ext}^E(v) = -\nabla E_{ext}(v)$ in Equation (3.8) by the VFC field $f_{vfc}(v)$, the iterative snakes solution is

$$\frac{\partial v(s,t)}{\partial t} = \alpha v''(s,t) - \beta v''''(s,t) + f_{vfc}(v(s,t))$$

(3.10)

This equation can be solved numerically using identical finite difference approach of standard snakes.

A new class of static external forces called Vector Field Convolution (VFC) using Radix4-FFT has been proposed. A vector field kernel has been defined as

$$k(x,y) = m(x,y)n(x,y)$$

(3.11)

$m(x,y)$ is the magnitude of the vector at $(x,y)$ and $n(x,y)$ is the unit vector pointing to the kernel origin $(0,0)$.

$$n(x,y) = [-x/r, -y/r]$$

(3.12)

Except that $n(0,0) = [0,0]$ at the origin, where
\[ r = \sqrt{x^2 + y^2} \] is the distance from the origin. If the origin is considered as the FOI, this vector field kernel has the desirable property that a free particle placed in the field is able to move to the FOI, such as edges.

The VFC external force \( f_{\text{vfc}}(x,y) = [u_{\text{vfc}}(x,y), v_{\text{vfc}}(x,y)] \) is given by calculating the convolution of the vector field kernel \( k(x,y) \) and the edge map \( f(x,y) \) generated from the image \( I(x,y) \).

\[
f_{\text{vfc}}(x,y) = f(x,y) * k(x,y) \tag{3.13}
\]

\[
= \begin{bmatrix} f(x,y) * u_k(x,y), f(x,y) * v_k(x,y) \end{bmatrix} \tag{3.14}
\]

Since the edge map is non-negative and larger near the image edges, edges contribute more to the VFC than homogeneous regions. Therefore, the VFC external force will attract free particles to the edges. If the vector field kernel has been represented using a complex-valued range, the VFC is just the filtering result of the edge map, which does not depend on the origin of the kernel. The VFC field highly depends on the magnitude of the vector field kernel \( m(x,y) \). By considering the fact that the influence from the FOI should decrease as the particles are further away, the magnitude should be a decreasing positive function of distance from the origin.

### 3.2.4 Numerical Implementation

The continuous vector field kernel \( k(x,y) \) is approximated by a discrete and finite matrix given as

\[
K = \{k(x,y); x, y = -R, \ldots, 1, 0, 1, \ldots R\} \tag{3.15}
\]
Where $R$ denotes the preferred kernel radius. An example discrete vector field kernel is demonstrated in Figure 3.1. To calculate the VFC field, each component of the discrete vector field kernel is convolved with the edge map. The discrete linear convolution has been well studied and can be accelerated by the Fast Fourier transform (FFT) and the inverse fast Fourier transform (IFFT) (Cooley and Tukey 1965).

![Figure 3.1 A discrete vector field kernel with $R = 4$](image)

Furthermore, if the vectors are treated as complex numbers instead of two separated real numbers, computational expense could be saved roughly by a factor of two without using a specialized FFT for real numbers (Sorensen et al 1987). The external forces $f_{\text{ext}}$ are normalized as unit vectors to encourage the contour evolve at a constant speed for a uniform time step $\tau$.

### 3.2.5 Parameter Estimation

To estimate the parameter $\gamma$ of the vector field kernel for computing $m_c(x,y)$, a line $AB$ used to approximate the effective FOI $\tilde{AB}$ of $(x,y)$, as shown in Figure 3.2.
Figure 3.2 Using a line $\overline{AB}$ to approximate the effective features of interest (FOI) $\tilde{AB}$ of $(x, y)$

The effective FOI $(x, y)$ of is defined as the FOI within $R$ distance from $(x,y)$. In order to capture free particles placed at $(x,y)$, the VFC field should satisfy

$$f_{\text{vfc}}(x,y)d(x,y) > 0 \quad \text{for} d(x,y) = d \leq R$$

(3.16)

Where $d(x,y)$ is the vector pointing to the approximated effective FOI of $\overline{AB}$ of $(x,y)$, the magnitude of which is the distance from the FOI. The VFC force can be decomposed into the force $f_{\text{FOI}}(x,y)$ resultant from the FOI and the force $f_{\text{noise}}(x,y)$ from the noise. The projection of these two forces in the $d(x,y)$ direction leads to the scalar magnitudes $f_{\text{FOI}}(x,y)$ and $f_{\text{noise}}(x,y)$, respectively. The FOI force projection can be written as
\[
f_{\text{roi}}(x,y) = \sum_{i \in R^2} f_i R_i^{-\gamma} \cos \theta_i 
\]
(3.17)

\[
= \sum_{i \in R^2} f_i R_i^{-\gamma} \frac{d}{R_i} R^{\gamma+1} \frac{d}{R_i} \sum_{i \in R^2} f_i 
\]
(3.18)

### 3.3 CONNECTIONS BETWEEN VECTOR FIELD CONVOLUTION (VFC) AND GRADIENT VECTOR FIELD (GVF)

Note that there are two energy terms in the GVF energy functional of Equation (3.9), the first of which diffuses the initial vector field, and the second term guarantees that the GVF field maintains the initial vector flow field components given before diffusion. In homogeneous regions of the image, the second term can be ignored because the gradient of the edge map is zero (Xu and Prince 1998). The numerical solution involves computing as \( f_{\text{gvf}}(x,y) \) function of time as follows:

\[
\frac{f_{\text{gvf}}(x,y,t+\Delta t) - f_{\text{gvf}}(x,y,t)}{\Delta t} = \\
= \mu [\nabla^2 u_{\text{gvf}}(x,y,t), \nabla^2 v_{\text{gvf}}(x,y,t)] 
\]
(3.19)

Where \( \Delta t \) is the time step for each iteration. Equation (3.19) is simplified from equation (3.14) in (Xu and Prince 1998) by ignoring the second term and can be rewritten as

\[
f_{\text{gvf}}(x,y,t+\Delta t) = f_{\text{gvf}}(x,y,t) \\
+ \Delta t \mu [\nabla^2 u_{\text{gvf}}(x,y,t), \nabla^2 v_{\text{gvf}}(x,y,t)] 
\]
(3.20)

The steady-state solution is achieved iteratively from the initialization space \( f_{\text{gvf}}(x,y,0) = \nabla f(x,y) \). On a discrete grid, the Laplacian operator can be approximated by
\[\nabla^2 u_{\text{gvf}}(x, y, t) = u_{\text{gvf}}(x + 1, y, t) + u_{\text{gvf}}(x - 1, y, t) + u_{\text{gvf}}(x, y + 1, t) + u_{\text{gvf}}(x, y - 1, t) - 4u_{\text{gvf}}(x, y, t)\]
\[\nabla^2 v_{\text{gvf}}(x, y, t) = v_{\text{gvf}}(x + 1, y, t) + v_{\text{gvf}}(x - 1, y, t) + v_{\text{gvf}}(x, y + 1, t) + v_{\text{gvf}}(x, y - 1, t) - 4v_{\text{gvf}}(x, y, t)\]

Substituting Equation (3.21) into equation (3.20) gives an iterative solution in the form of the following convolution:

\[f_{\text{gvf}}(x, y, t+1) = \begin{bmatrix} 0 & \mu & 0 \\ \mu & 0 & 0 \\ 0 & \mu & 0 \end{bmatrix} * f_{\text{gvf}}(x, y, t)\]

\[= \land_{\mu} * f_{\text{gvf}}(x, y, t)\]

Where it can be assumed that \(\Delta t = 1\) without loss generality. According to the GVF stability restriction, \(\mu\) must be not larger than \(1/4\) by making \(\Delta x = \Delta y = 1\). Equation (3.23) can be expanded further, obtaining

\[f_{\text{gvf}}(x, y, t) = \land_{\mu} * \land_{\mu} * \land_{\mu} ...... * f_{\text{gvf}}(x, y, 0) = \land_{\mu}^t * f(x, y)\]

Where \(t\) is integer, \(\land_{\mu}^t\) denotes the sequential convolution of \(t \land_{\mu}\). The gradient of edge map \(f\) can be approximated by

\[\nabla f(x, y) = k \land_{\mu} f(x, y)\]
\[= \frac{1}{2} [f(x+1, y) - f(x-1, y) - f(x, y+1) + f(x, y-1)]\]

(3.25)
Where
\[
\begin{bmatrix}
[1,0] & [0,1] \\
[0,0] & [-1,0] \\
[0,-1] & [1,0]
\end{bmatrix}
\]

Using a different discrete gradient approximation, such as the Sobel operator, will result a slightly different $k_\nabla$. Therefore, Equation (3.24) can be rewritten as

\[
f_{\text{gvf}}(x, y, t) = (\wedge_{\mu} k_\nabla) * f(x, y) = k^l_{\mu} * f(x, y) \quad (3.26)
\]

By comparing Equation (3.26) with the corresponding VFC Equation (3.14), it can be seen that the GVF field in homogeneous regions of the image is a special case of VFC with kernel $k^l_{\nabla}$ the standard external force, i.e., the gradient of edge map, is also a special case of VFC, where the kernel is $k_\nabla$.

### 3.4 PROPOSED AN IMPROVED GVF (VECTOR FIELD CONVOLUTION) SNAKE USING RADIX4-FFT WITH HAMMING WINDOW TECHNIQUE FOR MRI BRAIN IMAGE SEGMENTATION

#### 3.4.1 System Overview

Figure 3.3 shows the overall system architecture diagram. A set of MR brain images of size $M \times N$ were given as input to the system. Canny operator has been chosen to detect the edges of brain MRI images, and the result was used as the edge map for GVF snake model. Then external force is calculated by convolving a vector field with the edge map derived from the image by using Radix 4-FFT.
After that, by applying a Hamming window with GVF snake-Radix4-FFT, the noise at the boundary of tumour has been eliminated. Mainly Hamming window has been introduced to alleviate the possible leakage problem caused by choosing inappropriate parameters. Hamming window with Radix4-FFT has been used to compute the spectral leakage of the MRI brain image which accurately defines the boundary region of tumour or boundaries of WM, GM and CSF tissues in the brain. The inverse Radix4FFT was taken then snake was initialized. The outputs of each stage were
obtained and the results of segmentation accuracy were calculated. The robustness of the algorithm with respect to noise elimination and speed of segmentation was compared with existing system.

3.4.2 Algorithm Description of an improved GVF Snake (VFC)-Radix-4FFT-Hamming Window

**Input:** An array of medical image of size MxN.

**Output:** Segmented tissues.

Step 1: Read the medical image of size MxN.

Step 2: Initialize the parameters as

\[
\mu = 0.2, \text{GVF-iter} = 100, \alpha = 0.5, \beta = 0 \quad \text{and} \quad \tau = 0.5
\]

After testing a range of values, it has been noted that the GVF computation may be unstable for \( \mu > 0.2 \) and, therefore, a maximal value of 0.2 has been used. Increasing \( \beta \) will increase the rigidity of the model and would affect the shape even if close to start with. It has been found that the rigidity weighting factor can be increased from 0 to 1 with almost the same results.

Step 3: Compute external force fields of traditional GVF using Gaussian filter with initial circle close to Features of Interest (FOI).

Step 4: Compute external force fields of traditional GVF snake using Gaussian filter with initial circle far away from Features of Interest (FOI).

Step 5: Compute external force fields by convolving edge map of input image with kernel using Radix4-FFT to speed up the process.
Step 6: Apply Hamming windowing of size [3×3] as a post processing step to VFC field of Radix4-FFT to eliminate the noise. The dominant frequency contents of the signal become more distinctive for normalized spectral energy distributions when the window size is of [3×3].

Step 7: Apply inverse Radix4-FFT to the result of VFC force field i.e. Step 6.

Step 8: Display the external force fields for traditional GVF snakes and vector field convolution force field -Radix4-FFT-Hamming window.

Step 9: Initialize the circle with radius R=28. When R=28, the GVF snake has been initialized with circle far away from FOI.

Step 10: Deform the VFC snake - Radix4-FFT-Hamming window.

Step 11: Display the segmented output of GVF snake-Radix4-FFT-Hamming window and compare the results with traditional GVF snake and compare the segmented output with the expert segmented boundary in terms of Dice Similarity Metric (DSM). Compute Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR) with the application of Radix4-FFT.

3.4.3 Data Preprocessing and Edge Map

The first step in processing a noise-affected image is to smooth the image for noise removal. This thesis uses a Gaussian function in the case of GVF snake for smoothening. Basic effects of a Gaussian filter are
smoothening the image and wiping off noisy pixels or voxels. In the proposed method of deformable surface model, a 3D edge map is required to form dynamic gradient vector flow. It is obvious that Canny operator is good at edge detection when compared with the results of other traditional edge detectors for brain MRI images. It shows the details of internal edges, while other detection operators like Roberts, Prewitt, Log and Sobel cannot show these features. As discussed by Jingyong Cheng et al (2008), Canny operator has preferable anti-noise ability.

3.4.4 Initialization of Parameters

The snake parameters for both GVF and vector filed convolution using Radix4-FFT are initialized as follows. The smoothness parameter $\mu = 0.2$, $\text{GVF-ITER} = 100$, normalize $= 1$, Active contour elasticity (alpha) $= 0.5$, Active contour rigidity (beta) $= 0.1$, the time step of each iteration (tau) $= 0.5$. Decreasing the tension weight causes the active contour to follow the influence of the external force and lose its smoothness. The acceptable range that it was found for tension was from 0 to 1.

3.4.5 Computation of External Force Fields using Radix4-FFT

The external force field has been calculated by using vector field convolution using Radix4-FFT and is given by

$$F_{\text{ext}} = \text{Inverse Radix 4FFT}(\text{Radix4FFT}(f * k))$$

(3.27)

Where $f$ is the edge map and $k$ is the user defined kernel. A detailed explanation on Radix-4 FFT is given in Appendix A.6.2.

3.4.6 Hamming Window

The FFT computation assumes that a signal is periodic in each data block. When the FFT of a non-periodic signal is computed then the resulting
frequency spectrum suffers from Leakage. If exactly one period or a multiple of periods of a signal cannot be captured exactly, the leakage (side lobe) will result. The windowing is a technique to reduce the leakage. The window function is selected for two characteristics: reducing the effects of side lobe and narrowing the main lobe of the window.

The shape of the Hamming window is similar to that of a cosine wave. The following equation defines the Hamming window.

\[ w(n) = 0.54 - 0.46 \cos\left(2 \pi \frac{n}{N - 1}\right) \]  \hspace{1cm} (3.28)

Where \( N \) is the number of input samples of FFT. A detailed explanation has been given in Appendix A.

### 3.4.7 Snake Deformation

After the calculation of external force field using VFC-Radix4-FFT, the snake has been initialized. The initial contour of the tumour can be obtained by manually selecting several points around the edges of the desired object. Then normalized Vector Field field with convolution using Radix4-FFT has been used to segment the tumour or brain tissues. Both the GVF field and VF field with convolution using Radix4-FFT is normalized to guarantee the snake converges to the desired boundaries.

### 3.5 RESULTS AND ANALYSIS

As the GVF snakes have gained tremendous popularity due to their ability to address a few difficulties appeared in previous snakes, VFC snake-Radix4-FFT-Hamming window results have been compared with the GVF snakes results. The magnitude function used in those experiments is with \( m_i(x, y) \) with \( \gamma = 1.7 \). The values used in the experiments for all snakes is \( \alpha = 0.5, \beta = 0 \) and \( \tau = 0.5 \). The performance measures used are Dice Similarity
Metric (DSM), Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR).

Peak Signal-to-Noise Ratio (PSNR) is defined as the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation (Huynh-Thu and Ghanbari 2008). Mean Square Error (MSE) which for two M×N monochrome images \( I \) and \( K \) where one of the images is considered a noisy approximation of the other is defined as (Lehmann and George Casella 1998):

\[
\text{MSE} = \frac{1}{mn} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} [I(i,j) - K(i,j)]^2
\]

\[(3.29)\]

Where \( \text{Max}_I \) is the maximum possible pixel value of the input image.

\[
\text{PSNR} = 10\log_{10} \left( \frac{\text{Max}_I^2}{\text{MSE}} \right)
\]

\[(3.30)\]

The similarity measure used to test is Dice similarity measure (DSM)(Zijdenbos et al 1994). It is defined as

\[
\text{DSM}(r) = \frac{2N_{p\sim g}(r)}{N_p(r) + N_g(r)}
\]

\[(3.31)\]

Where \( N_{p\sim g}(r) \) represents the number of pixels classified by both proposed method and ground truth as model \( r \), \( N_p(r) \) represent the number of pixels classified as model \( r \) by the proposed method and \( N_g(r) \) represent the number of pixels classified as model \( r \) by the ground truth. As shown in
Figure 3.4, both the GVF snake-Radix4-FFT-Hamming window and the GVF snake are capable of capturing the boundary concavity from a far-off initialization, which demonstrates that both snakes have a large capture range and concavity convergence property. In practical implementation, the capture range of the GVF field is influenced by two parameters: the smoothness parameter $\mu$ and the number of iterations. Although, the capture ranges increases as $\mu$ and the number of iterations increase, there is no specified relationship available to the user. If the user underestimates those parameters, the snakes may not converge correctly. In practice, over estimation of parameters results in wastage of computing time. In contrast, the capture range of the proposed VFC field using Radix4-FFT is determined by the capture range $R$ of the vector field kernel which results in computational time reduction.

### 3.5.1 Initialization Sensitivity

As seen in Figure 3.4, (U-shaped example) both the GVF snake-Radix4-FFT-Hamming window and the GVF snake can be initialized far away outside the object. Figure 3.5 shows a set of initialization placed across and inside the boundary, with which the GVF snake-Radix4-FFT-Hamming window converge correctly. These examples demonstrate that the GVF snake-Radix4-FFT-Hamming window is insensitive to initialization and capable of converging into boundary concavities.
Figure 3.4  U-shape example (a) GVF snake with initial circle close to FOI (b) GVF snake with initial circle far away from FOI (c) Vector Field Convolution snake using Radix4-FFT-Hamming window with initial circle far away from FOI

Figure 3.5  Initial curves with the Vector Field Convolution snake using Radix4-FFT- Hamming window
As shown in Figure 3.6, for noisy U shaped object both the GVF snake-Radix4-FFT-Hamming window and the GVF snake are capable of capturing the boundary concavity from a far-off initialization, which demonstrates that both snakes have a large capture range and concavity convergence property.

Figure 3.6  (a) Noisy U-Shaped Object (b) Initial circle close to FOI for GVF (c) Initial circle far away from FOI for GVF (d) Initial circle far away from FOI for VFC-Radix4-FFT (e) Streamlines generated from the GVF field when initial circle close to FOI (f) Streamlines generated from the GVF field when initial circle faraway from FOI (g) Streamlines generated from the GVF snake-Radix4-FFT-Hamming window when initial circle far away from FOI
3.5.2 Complexity Analysis

In general, the complexity of the VFC field using Radix4-FFT technique is $O(N^2 \log_2(N))$, which compares favorably to the $O(N^3)$ complexity of the GVF computation. For a given edge map of $N \times N$ pixels and a vector field kernel $(2R+1) \times (2R+1)$, both of which are zero-padded $(N+2R) \times (N+2R)$ to avoid wrap-around effects, the proposed algorithm runs in $O(N^2 \log_2(N))$. The VFC field using Radix4-FFT can be calculated by the complexity of Radix4-FFT i.e of $O(N \log_2(N))$ which when multiplied by image size, it is of $O(N^2 \log_2(N))$. Typical values of $R$ range from $N/8$ to $N/4$. The expense of VFC using Radix4-FFT depends mainly on the size of the vector field kernel, whereas the computational cost of GVF depends by and large on the number of diffusion iterations. Figure 3.7 compares the computational cost of VFC using Radix4-FFT and GVF on different image size, from which it can be observed that GVF requires 3 to 10 times more computational expense than VFC using Radix4-FFT.

![Figure 3.7 Computational cost of GVF and VFC using Radix4-FFT for an NxN image](image)
GVF snake does not capture some boundary features precisely, such as concavity and the bottom left portion which are distorted by the Gaussian filter as shown in Figure 3.8. The application of VFC snake-Radix4-FFT-Hamming window converges to the desired features without using a Gaussian filter. Although the noise outnumbered the edges, the forces originated from the noise counteract each other and are overwhelmed by the forces generated from the true edges. The error of a point on the snake is defined by the minimum distance between the point and U-shape in the noise-free image. In Figure 3.9, the VFC snake using Radix4-FFT yields the smallest RMSE. These results reveal the superior robustness to noise afforded by the VFC field using Radix4-FFT.

Figure 3.8 Impulse noise corrupted U-shape images with GVF and VFC iterations

![Figure 3.8 Impulse noise corrupted U-shape images with GVF and VFC iterations](image)

Figure 3.9 Average RMSE of GVF and VFC at different SNR levels

![Figure 3.9 Average RMSE of GVF and VFC at different SNR levels](image)
The proposed method has been applied for the segmentation of both simulated and real clinical MRI scans, and has been demonstrated in the following sections. 1) the accuracy of the proposed segmentation method on simulated T1w brain MRIs; 2) the accuracy of the proposed method on real clinical brain MRI scans and Internet Brain Segmentation Repository (IBSR) data set.

3.5.3 Segmentation Validation using Simulated Brain MRI

The proposed method has been validated on 18 simulated T1-weighted BrainWeb MRI images with (0% noise and 0% inhomogeneity, 9% noise and 40% inhomogeneity, $181 \times 217 \times 181$ dimension and $1 \times 1 \times 1$ mm$^3$ spacing). For the Brainweb dataset, the ground truth is the phantom atlas used to generate the simulated scans. For all experiments, the parameters for GVF snake iterations are set $\alpha=0.5$, $\beta=0.1$ and $\tau=0.5$ to reinforce propagation, effectively remove boundary attraction and smoothness regularization. As seen from Table 3.1, Radix 4 FFT approach improves signal-to-noise ratio and also reduces number of arithmetical computations for the medical images with different noise variances.

Table 3.1 Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR) values after applying Radix 4 FFT to a set of input images containing noise variance of 0.05 and 0.005

<table>
<thead>
<tr>
<th>Input Image</th>
<th>Noise variance (V)</th>
<th>Without Radix4 FFT</th>
<th>With Radix4 FFT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSE</td>
<td>PSNR (db)</td>
<td>Time (Seconds)</td>
</tr>
<tr>
<td>Image1</td>
<td>0.05</td>
<td>2.0400e+03</td>
<td>15.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0313</td>
<td>0.0827</td>
</tr>
<tr>
<td>Image1</td>
<td>0.005</td>
<td>240.9571</td>
<td>24.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.8438</td>
<td>0.0523</td>
</tr>
<tr>
<td>Image2</td>
<td>0.05</td>
<td>1.9941e+03</td>
<td>15.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.563</td>
<td>0.0869</td>
</tr>
<tr>
<td>Image2</td>
<td>0.005</td>
<td>216.8813</td>
<td>24.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.8594</td>
<td>0.0591</td>
</tr>
</tbody>
</table>
A three-class (WM, GM, and CSF) segmentation using the proposed method on the simulated T1w brain MRI data was performed. Segmentation was performed using the traditional Gradient Vector Field and the proposed VFC snake-Radix4-FFT- Hamming window on all 18 datasets with varying noise and intensity inhomogeneity levels. Qualitative results in Figure 3.10 demonstrated very good resemblance between the provided phantom label and the segmentation results based on the proposed method for the best case (0% noise, 0% inhomogeneity). Figure 3.11 demonstrated very good resemblance between the provided phantom label and the segmentation results based on the proposed method for the worst case (9% noise, 40% inhomogeneity). Table 3.2 shows that on average, over the above mentioned noise and inhomogeneity levels, the proposed method achieved consistently accurate segmentation results for both WM and GM in terms of higher DSM accuracy as compared to GVF snake.

![Figure 3.10](image.png)

**Figure 3.10** (a) Phantom labels (b) T1-w for best case (0% noise, 0% inhomogeneity) (c) Segmentation results of proposed VFC snake-Radix4-FFT-Hamming for best case
Figure 3.11 (a) Phantom labels (b) T1-w input (c) Segmentation by the proposed GVF snake-Radix4-FFT

Table 3.2 The Average Dice Similarity Coefficient comparison for Brainweb images by the proposed VFC snake-Radix4-FFT-Hamming window method

<table>
<thead>
<tr>
<th>Method</th>
<th>CSF</th>
<th>WM</th>
<th>GM</th>
</tr>
</thead>
<tbody>
<tr>
<td>GVF snake</td>
<td>0.76</td>
<td>0.77</td>
<td>0.79</td>
</tr>
<tr>
<td>Proposed VFC snake-Radix4-FFT-Hamming Window</td>
<td>0.77</td>
<td>0.79</td>
<td>0.81</td>
</tr>
</tbody>
</table>

3.5.4 Segmentation Validation using Real Brain MRI

In validating the performance of the proposed method, the database consisting of magnetic resonance images of several anonymous brain tumour
patients, as well as segmentations of the brain tissues and tumour from IBSR dataset (MR T1-weighted image in the sagittal plane, 256 x 256 x 124, 0.9375 x 0.9375x1.5 mm³) were taken. Manual segmentations done by neurosurgeons and automated segmentation done by the proposed method were obtained. In Figure 3.12, the manual segmentation of a glioma on the MRI of the patient number 4 in the database, the slice 35 is presented. To investigate the performance of proposed segmentation method and compare it with original GVF snake, experiments has been designed. For each experiment, both original GVF snake and improved GVF snake-Radix4-FFT-Hamming window have the same values of $\alpha$, $\beta$ and $\tau$ and using the same initial position for the curve. Figure 3.12 shows the original image and the initial snake position drawn in white curve.

Figure 3.13 (a) shows the result of Prewitt edge detection and (b) shows the result of Sobel edge detection. Figure 3.14(a) shows the result of Robert and (b) shows the result of LOG edge detection. Figure 3.15 shows the result of Canny edge detection. The output of Canny edge detection provides clear boundaries as compared to Prewitt, Sobel, Robert and LOG edge detectors.

![Figure 3.12](image)

**Figure 3.12** The original MRI slice number 35 of patient 4 and the initial snake position drawn as white
Figure 3.13 (a) Result of Prewitt edge detection (b) Result of Sobel edge detection

Figure 3.14 (a) Result of Robert (b) Result of LOG edge detection

Figure 3.15 Result of Canny edge detection

Figure 3.16 shows manual segmentation by four different experts of a glioma on the MRI. Figure 3.17 shows the automatic segmentation of the glioma, using original GVF snake. Figure 3.18 shows the segmentation of the
glioma by the proposed VFC Snake-Radix 4-FFT-hamming window. As compared with the automatic segmentation of the glioma using original GVF snake, the proposed VFC Snake-Radix 4-FFT-hamming window method demonstrated improved segmentation accuracy qualitatively. As seen from Figure 3.18, segmentation of the glioma by the proposed method converges to the concave part of the boundary accurately as compared to traditional GVF snake (Figure 3.17).

**Figure 3.16** Manual segmentation by four different experts of a glioma on the MRI

**Figure 3.17** Segmentation of the glioma using original GVF snake

**Figure 3.18** Segmentation of the glioma by the proposed improved Snake (Vector Field Convolution)-Radix 4-FFT-Hamming window
The second dataset was taken from Internet Brain Segmentation Repository (IBSR). The 20 normal MRI brain data sets and their manual segmentations were provided by the Center for Morphometric Analysis (CMA) at Massachusetts General Hospital and are available at http://neurowww.mgh.harvard.edu/cma/ibsr/. Figure 3.19 demonstrates the segmentation result of the GVF snake and by the proposed method on an image of the IBSR dataset. Figure 3.19(a) shows the original image and (b) shows the ground truth. Figure 3.19(c) shows results of segmentation by GVF snake and (d) displays the segmentation obtained by the proposed VFC Snake-Radix 4-FFT-Hamming window segmentation. Qualitative results in Figure 3.19 demonstrated very good resemblance between the provided ground truth (Figure 3.19(b)) and the segmentation results based on the proposed Vector Field Convolution-Radix 4-FFT-Hamming Window method. Table 3.3 compares the proposed method quantitatively on the IBSR dataset. It shows that the proposed method achieved consistently accurate segmentation results for both WM and GM.
Figure 3.19  (a) Original Image (b) Ground Truth (c) Segmentation by GVF snake(d) Segmentation by the proposed VFC Snake-Radix 4-FFT-Hamming window segmentation
Table 3.3 The Average Dice Similarity Coefficient comparison for IBSR dataset images by the proposed VFC snake-Radix4-FFT-Hamming window method

<table>
<thead>
<tr>
<th>Method</th>
<th>DSM For MR Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>GVF snake</td>
<td>0.76 0.78 0.79</td>
</tr>
<tr>
<td>Proposed VFC snake-Radix-4FFT-Hamming Window</td>
<td>0.77 0.823 0.874</td>
</tr>
</tbody>
</table>

3.6 PROPOSED SEGMENTATION OF BRAIN MRI USING AN IMPROVED GVF (VFC) SNAKE-MIXED RADIX FFT TECHNIQUE-HAMMING WINDOW

3.6.1 Problem Formulation

The boundaries of brain tissues or lesions are extracted using Gradient Vector Flow snakes (Xu and Prince 1998). GVF snakes can converge to the concave parts of the boundaries, thus capture the detailed information of the boundaries. At the meantime, GVF snakes will provide noisy boundaries of lesions or brain tissues. So smooth lesion’s boundaries should be provided. Least square estimation (LSE) can be used to solve this problem. One problem with LSE is that LSE is very computational inefficient, especially for those high frequency signals. For medical images, the data can be extremely large, which means the design matrices can be too large for computers’ memory to load and it is impossible to operate these large matrices. For LSE method, one also has to calculate the inverse matrices. General Personal Computers cannot handle the computation of the inverse of very large matrices. To handle this problem, Iterative Residual Fitting (IRF) was proposed by Chung et al (2007). But the limitations of IRF were no model selection procedure and no stopping rules. To overcome these problems an improved GVF snakes-Mixed Radix FFT–Hamming window technique has been proposed.
3.6.2 System Architecture

The proposed system starts with applying MRI brain image as input. Improved GVF (Vector Field Convolution) snakes using Mixed Radix-FFT has been applied for the segmentation of boundaries of tumour and classification of brain tissues. Hamming window has been applied to alleviate the possible leakage problem caused by choosing inappropriate parameters. Figure 3.20 shows the system architecture diagram. Then the outputs of GVF snake and Vector Field Convolution using Mixed Radix-FFT-Hamming Window were compared in terms of DSM.

Figure 3.20 Overall system flow diagram of proposed VFC snakes with Mixed- Radix-FFT-Hamming Window

3.6.3 Algorithm Description of VFC-Snake-Radix-4FFT-Hamming Window
**Input:** An array of Brain MR medical image of size MxN.

**Output:** Segmented brain tissues.

Step 1: Read the medical image of size MxN.

Step 2: Initialize the parameters as

\[ \mu = 0.2, \text{GVF-iter} = 100, \alpha = 0.5, \beta = 0 \text{ and } \tau = 0.5 \]

Step 3: Compute external force fields of traditional GVF using Gaussian filter with initial circle close to Features of Interest (FOI).

Step 4: Compute external force fields of traditional GVF snake using Gaussian filter with initial circle far away from Features of Interest (FOI).

Step 5: Compute external force fields by convolving edge map of input image with kernel using Mixed radix-FFT to speed up the process.

Step 6: Apply Hamming windowing of size [3x3] as a post processing step to VFC field of Mixed Radix FFT to eliminate the noise.

Step 7: Apply inverse Mixed Radix FFT to the result of VFC force field i.e. Step 6.

\[ F_{\text{ext}} = \text{Inverse Mixed-Radix FFT}(\text{Mixed-Radix FFT}(f \ast k)) \]

Step 8: Display the external force fields for traditional GVF snakes and vector field convolution force field–Mixed Radix-FFT-Hamming Window.

Step 9: Initialize the circle with radius R=28.
Step 10: Deform the VFC snake-Mixed Radix FFT- Hamming Window.

Step 11: Display the segmented output of GVF snake-Mixed Radix-FFT-Hamming window and compare the results with traditional GVF snake. Compare the segmented output with the expert segmented boundary in terms of Dice Similarity Metric (DSM). Compare Mean Square Error (MSE) of the GVF and the proposed VFC snake-Mixed Radix FFT- Hamming Window.

3.7 RESULTS AND DISCUSSION

The proposed method has been applied for the segmentation of both simulated and real brain MRI images, and demonstrated. In order to test the proposed algorithm, two datasets were used. The first dataset was the simulated MR images provided by the BrainWeb Simulated Brain Dataset from the McGill University and is available at http://www.bic.mni.mcgill.ca/brainweb.

The second dataset is Internet Brain Segmentation Repository (IBSR). The 20 normal MRI brain data sets and their manual segmentations were provided by the Center for Morphometric Analysis (CMA) at Massachusetts General Hospital and are available at http://neurowww.mgh.harvard.edu/cma/ibsr/. Figure 3.21 (a) shows Phontom Label, Figure 3.21 (b) shows the T1w-image for the best case(0% noise,0% inhomogeneity). Qualitative results in Figure 3.21(c) and in Figure 3.21(d) demonstrated very good resemblance between the provided Phontom Label and the segmentation results based on the proposed VFC Snake- Radix4-FFT-Hamming Window and VFC Snake-Mixed-Radix-FFT-Hamming Window segmentation. Figure 3.22 (a) shows Phontom Label. Figure 3.22 (b) shows
the T1w-image for the worst case (9% noise, 40% inhomogeneity). Qualitative results in Figure 3.22(c) and in Figure 3.22(d) demonstrated very good resemblance between the provided Phantom Label and the segmentation results based on the proposed VFC Snake- Radix4-FFT-Hamming Window and VFC Snake-Mixed-Radix-FFT-Hamming Window segmentation.

Figure 3.21 (a) Phantom label (b) T1w-image for the best case (0% noise, 0% inhomogeneity) (c) Segmentation results obtained by the proposed VFC snake-Radix-4 FFT-Hamming Window for the best case (d) Segmentation results obtained by the proposed VFC snake-Mixed Radix-FFT-Hamming Window for the best case
Figure 3.22 (a) Phontom label (b) T1-w image for the worst case (9% noise, 40% inhomogeneity) (c) Segmentation results obtained by the proposed VFC snake - Radix 4 FFT - Hamming Window for the worst case (d) Segmentation results obtained by the proposed VFC snake - Mixed Radix FFT - Hamming Window for the worst case

As seen from Table 3.4 the average DSM index is higher for the proposed GVF (vector field convolution) Snake-Mixed Radix-FFT-Hamming Window segmentation as compared to segmentation results of the proposed GVF (vector field convolution) Snake-Radix-4FFT-Hamming Window. The proposed GVF (vector field convolution) Snake-Mixed Radix-FFT-Hamming Window segmentation method achieved consistently accurate segmentation results for both WM and GM. As observed from Figure 3.23, the proposed segmentation results matched closely with what can be visually observed from the raw images and correspond well to the provided expert-guided manual segmentation as compared to other methods.
Table 3.4  The Average Dice Similarity Coefficient comparison for Brainweb images by the proposed VFC snake-Radix-4FFT-Hamming Window and proposed VFC snake- Mixed Radix FFT-Hamming Window methods

<table>
<thead>
<tr>
<th>Method</th>
<th>DSM For MR Images</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CSF</td>
</tr>
<tr>
<td>GVF snake</td>
<td>0.76</td>
</tr>
<tr>
<td>Proposed VFC snake-Radix-4FFT-Hamming Window</td>
<td>0.77</td>
</tr>
<tr>
<td>Proposed VFC snake- Mixed Radix FFT-Hamming Window</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Figure 3.23 (a) Original Image (b) Ground Truth (c) Segmentation by the GVF snake (d) Segmentation by the proposed VFC Snake—Radix4-FFT-Hamming window segmentation (e) Segmentation by the proposed VFC snake-Mixed Radix FFT-Hamming Window segmentation
Table 3.5 shows the average DSM index of IBSR dataset for the proposed methods with the existing GVF snake method. The proposed VFC snake- Mixed Radix FFT-Hamming Window outperforms the classical GVF snake and the proposed VFC snake-Radix-4FFT-Hamming Window in terms of higher segmentation accuracy.

### Table 3.5 The Average Dice Similarity Coefficient comparison for IBSR dataset images by the proposed VFC snake-Radix-4FFT-Hamming Window and proposed VFC snake- Mixed Radix FFT-Hamming Window methods

<table>
<thead>
<tr>
<th>Method</th>
<th>DSM For MR Images</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CSF</td>
</tr>
<tr>
<td>GVF snake</td>
<td>0.177</td>
</tr>
<tr>
<td>Proposed VFC snake-Radix-4FFT-Hamming Window</td>
<td>0.180</td>
</tr>
<tr>
<td>Proposed VFC snake- Mixed Radix FFT-Hamming Window</td>
<td>0.183</td>
</tr>
</tbody>
</table>

The TM(Tanimoto Metric) values of GVF, the proposed VFC-Radix4-FFT-Hamming Window and the proposed VFC-Mixed Radix-FFT-Hamming Window on real brain image #4 when the initial contour is inside, outside the brain tumour and the overlap is shown in Table 3.6 from the IBSR dataset. Also a perfect TM value of 1.0 is difficult to achieve especially when the target object is small. For example, the TM value is only 0.891 even though the method generated contour and the ground-truth are similar as shown in Figure 3.18. This is because the brain tumors are small, composed of only 50 to 200 pixels in the MR images; a few pixels of deviation from the ground-truth can result in a less than perfect TM value. A comparison of time of execution and accuracy of segmentation of MRI brain tumours by the existing and proposed methods are provided in Table 3.7. As seen from the Table 3.7, the proposed methods are showing higher speed of segmentation.
and accuracy for the segmentation of Brain MRI tumour as compared to existing methods reported in the literature.

Table 3.6  Quantitative analysis of GVF, proposed VFC-Radix4-FFT-Hamming Window and the proposed VFC-Mixed Radix-FFT-Hamming Window based on IBSR brain tumour MR images

<table>
<thead>
<tr>
<th>Image</th>
<th>Method</th>
<th>TM (inside)</th>
<th>TM (outside)</th>
<th>TM (overlap)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GVF</td>
<td>0.005</td>
<td>0.092</td>
<td>0.009</td>
</tr>
<tr>
<td>#4</td>
<td>Proposed VFC snake-Radix-4FFT Hamming Window</td>
<td>0.876</td>
<td>0.080</td>
<td>0.842</td>
</tr>
<tr>
<td></td>
<td>Proposed VFC snake-Mixed Radix FFT-Hamming Window</td>
<td>0.891</td>
<td>0.162</td>
<td>0.860</td>
</tr>
</tbody>
</table>

TM- Tanimoto Metric

Table 3.7  Comparison of time of execution and accuracy for the segmentation of MRI brain tumours by the existing and proposed methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Computation Time (Seconds)</th>
<th>Tanimoto Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Existing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gibbs (1996)</td>
<td>3,60,000</td>
<td>0.203</td>
</tr>
<tr>
<td>Kaus(2001)</td>
<td>2,70,000</td>
<td>0.506</td>
</tr>
<tr>
<td>Prastawa(2004)</td>
<td>3,24,000</td>
<td>0.604</td>
</tr>
<tr>
<td>Proposed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VFC snake-Radix-4FFT-Hamming Window</td>
<td>180</td>
<td>0.886</td>
</tr>
<tr>
<td>VFC snake-Mixed Radix FFT-Hamming Window</td>
<td>120</td>
<td>0.906</td>
</tr>
</tbody>
</table>

As seen from Figure 3.24, the VFC snake using Mixed Radix-FFT yields the smallest RMSE. These results reveal the superior robustness to noise afforded by the VFC field using Mixed Radix-FFT as compared to
The results in Figure 3.25 demonstrated that the proposed algorithm outperforms the traditional GVF snake and the proposed VFC snake using Radix-4 FFT–Hamming Window in speed on different image size.

**Figure 3.24** Average RMSE of GVF and the proposed methods at different SNR levels

**Figure 3.25** Computational cost of GVF and proposed methods for an NxN image
3.8 CONCLUSION

In this chapter, two novel improved GVF snake (vector field convolution based GVF snakes) segmentation techniques have been proposed and discussed. The first proposed GVF snake with Radix4-FFT–Hamming window uses an external force which is calculated by convolving a vector field kernel with the edge map derived from the image using Radix4-FFT technique to segment brain tissues and lesions. VFC snakes using Radix4-FFT were constructed by way of a force balance condition which has the flexibility of changing the force field.

This proposed VFC snakes effectively segmented the MRI brain and solves ambiguous relationship between the capture range and parameters. Hamming window has been introduced to alleviate the possible leakage problem caused by choosing inappropriate parameters. It can be inferred from the results that VFC snakes using Radix4-FFT coupled with Hamming Window produced good segmentation results, superior noise robustness, reduced computational cost. Experimental results show that the novel VFC snakes using Radix4-FFT is highly robust to noise and has smallest Mean Square Error. The proposed VFC snake with Radix4-FFT–Hamming window has improved segmentation accuracy of 0.699 to 0.723 for GM and 0.699 to 0.723 for WM, PSNR values of 58.0920 to 59.7726 db and speed of execution of 0.7322 to 1.272 seconds.

The second proposed VFC using Mixed Radix-FFT has a unique way of increasing the quality of edge map which is a critical factor in snake performance. VFC snakes have large capture ranges, and converge to boundary concavities, similar to the GVF snakes but has greater segmentation accuracy, improved segmentation speed and robust to noise.
The novel GVF snake-Mixed Radix-FFT-Hamming window method overcomes the problems of computational cost, noise sensitivity which are existing in traditional GVF snake, by the forces originated from the noise counteract each other and are overwhelmed by the forces generated from the true edges. VFC using Radix4-FFT-Hamming window and Mixed Radix-FFT can also be easily customized and enhanced for different applications. When compared with other traditional GVF snake methods the proposed algorithms were less computationally expensive, more robust to noise and initialization. Experimental results proved that proposed VFC segmentation using Radix4-FFT and VFC segmentation-Mixed radix FFT is adaptable to brain MRI images and produces good segmentation results with segmentation accuracy of 92% and the complexity of the VFC field is reduced to $O(N^2 \log_2(N))$, from $O(N^3)$ complexity of the GVF computation.