CHAPTER 5

EFFECTS OF CHEMICAL REACTION AND MHD ON FREE CONVECTIVE FLOW OVER A VERTICAL CYLINDER IN THE PRESENCE OF POROUS MEDIUM

5.1 INTRODUCTION

Transport phenomena are very essential in most of the fields like chemical, biological, mechanical and environmental sciences. It helps us to understand the fluid flow, heat and mass transfer in many practical applications. The study of natural convection and heat transfer of fluid flow in a porous medium has wide applications in mechanical, chemical and civil engineering: for storage and preservation of grains and cereals; solar energy collectors; filter systems; transport of radioactive wastes through the soil; and post accident heat removal in nuclear reactors.

Chemical kinetics and reactor design are at the heart of producing almost all industrial chemicals. The selection of a reaction system that operates in the safest and most efficient manner can be the key to the economic success or failure of a chemical plant. For example, if a reaction system produced a large amount of undesirable product, subsequent purification and separation of the desired product could make the entire process economically unfeasible.

The chemical kinetic principles, in addition to the production of chemicals, can be applied in areas such as living systems, waste treatment, and air and water pollution. Some of the examples and problems used to illustrate the principles of chemical reaction engineering are: the use of wetlands to remove toxic chemicals from rivers, increasing the octane number of gasoline,
the production of antifreeze starting from ethane, the manufacture of computer chips, and the application of enzyme kinetics to improve an artificial kidney.

Brinkman calculated the viscous force, exerted by a flowing fluid on a dense swarm of particles (Brinkman 1949). The author made a modification of the Darcy’s equation in order to obtain consistent boundary conditions and obtained a relation between permeability, particle size and density. Whitaker formulated the macroscopic equations of continuity and the equations of motion for flow in anisotropic porous medium using finite volume local surface averaging and established the conditions under which Darcy’s laws hold (Whitaker 1996).

Slattery considered local volume averaging of the equations of continuity and equations of motion for the flow of viscoelastic fluids through a porous medium which is an extension of Darcy’s law appropriate to viscoelastic fluids. In this case, Inertia effects have been neglected for steady state flow (Slattery 1967).

Boundary and inertia effects upon flow and heat transfer in porous media were formulated by Vafai and Tien and they applied the formulation to the two dimensional flow through a porous medium confined by an external impermeable boundary considering the following flow resistances: the bulk damping resistance, the viscous resistance and the resistance due to the inertial forces (Vafai & Tien 1981). The local volume averaging technique has been used to solve the governing equations with certain physical limitations and assumptions.

Joseph et al formulated a non-linear theory for appropriate generalization of Darcy’s law when inertia effects are significant. They estimated the ‘form drag coefficient’ from the experimental drag law for a fixed solid sphere placed in a uniform stream (Joseph et al 1982).
Stangle and Aksay proposed a theoretical model to describe simultaneous momentum, heat and mass transfer phenomena in disordered porous materials which has potential applications to many engineering problems (Stangle & Aksay 1990). These considerations are important in the design of a successful ceramic fabrication process. This paper gives the important factors involved and establishes some guidelines for the rational design of an improved binder removal process.

Shenoy discussed the non-Newtonian fluid behaviour with porous media which has wide range of applications in various scientific and engineering discipline: Biomechanics, Ceramic Engineering, Chemical Engineering, Food technology, Geophysics, Groundwater hydrology, Industrial Engineering, Mechanical Engineering, Petroleum Engineering and Soil Mechanics (Shenoy 1994).

Nakayama and Hossain proposed a simple integral treatment for combined heat and mass transfer by natural convection in a porous medium by examining two distinct regime maps for heat transfer and mass transfer respectively, which lead to pure heat transfer formula and pure mass transfer formula that holds even when the flow close to the wall is driven by heat transfer and mass transfer (Nakayama & Hossain 1995).

Nield considered a porosity factor for modelling the effect of a magnetic field or rotation in the momentum equation for flow in a porous medium discussed the analogy between the MHD flow in an isotropic porous medium and hydrodynamic flow in a medium with anisotropic permeability (Nield 1999). Nield has reviewed the effects of inertia, boundary friction, viscous dissipation and also effects due to rotation or a magnetic field for modelling fluid flow and heat transfer in a saturated porous medium (Nield 2000).
An analysis has been carried out by Cortell to investigate the flow and mass transfer of a chemical species with first and higher order reactions for a steady laminar boundary layer flow of an electrically conducting fluid of second grade in a porous medium over a stretching surface with an applied magnetic field which is applied normal to the surface (Cortell 2007).

Combined effects of chemical reaction along with MHD for free convective flow over the moving cylinder in porous medium has not been analysed by the authors in the literature. The aim of this chapter is to study the effects of chemical reaction and Magnetohydrodynamics on flow, heat and mass transfer on a semi-infinite vertical cylinder in the presence porous medium.

5.2 MATHEMATICAL ANALYSIS

The free convection flow of a viscous incompressible, laminar flow over an impulsively started moving semi infinite vertical cylinder of radius \( r_0 \) with chemical and MHD effects in the presence of porous medium is considered. Initially both cylinder and the fluid are stationary at the same temperature \( T'_\infty \) and at the same concentration level \( C'_\infty \). At a time \( t' \geq 0 \), the cylinder starts moving in the vertical direction with uniform velocity \( u'_0 \). The temperature and the concentration on the surface of the cylinder are raised to \( T'_w \) and \( C'_w \). The axial and radial co-ordinates are taken to be \( x \) and \( r \), with \( x \) -axis measured vertically upward and \( r \) -axis perpendicular to the axis, respectively, from the leading edge where the boundary layer thickness is zero.

A uniform magnetic field is applied in the direction perpendicular to the cylinder. The fluid is assumed to be slightly conducting, and the induced magnetic field is negligible in comparison with the applied magnetic field. It is further assumed that there is no applied voltage, so that electric field is absent. Here the system is considered to be axis symmetric. The induced current does
not distort the magnetic field. The magnetic field is constant in a direction perpendicular to the cylinder. The coefficient of electrical conductivity is constant and scalar throughout the fluid.

The viscous dissipation effects are assumed to be negligible in the energy equations since the flow velocity magnitude is expected to be small. It is negligible because the motion is due to free convection only. It is also assumed that there exists a homogeneous first-order chemical reaction between the fluid and species concentration. Also, it is assumed that the level of species concentration is very low, and, hence, heat generated during chemical reaction can be neglected. Here, we assume that a chemically reactive species is emitted from the surface of the cylinder and diffuses into the fluid. The reaction is assumed to take place entirely in the stream.

On making the Boussinesq's approximation, the equations governing boundary layer flow of continuity, momentum, energy and concentration are as follows:

\[
\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{r}}{\partial \mathbf{v}} = 0
\]

\[
\frac{\partial T}{\partial \mathbf{t}} + \frac{\partial u}{\partial \mathbf{x}} + \frac{\partial v}{\partial \mathbf{r}} = \frac{\partial \beta}{\partial \mathbf{r}} (T^* - T) + \frac{\partial \beta}{\partial \mathbf{r}} (C^* - C) + \frac{\partial \mathbf{u}}{\partial \mathbf{r}} \left( \frac{\partial \mathbf{u}}{\partial \mathbf{r}} \right) \left( \frac{\partial \mathbf{u}}{\partial \mathbf{r}} \right) - \frac{\sigma B^2}{\rho} \mathbf{u} - \frac{\nu}{\mathbf{\lambda}} \mathbf{u}
\]

\[
\frac{\partial C}{\partial \mathbf{t}} + \frac{\partial u}{\partial \mathbf{x}} + \frac{\partial v}{\partial \mathbf{r}} = \frac{\partial C}{\partial \mathbf{r}} \left( \frac{\partial C}{\partial \mathbf{r}} \right) - K \mathbf{C}
\]

The corresponding initial and boundary conditions are given by
In order to transform the governing equations into non-dimensional form, we introduce the following dimensionless quantities:

\begin{align}
X &= \frac{x}{r_0}, \quad R = \frac{r}{r_0}, \quad U = \frac{u}{u_0}, \quad V = \frac{v}{v_0}, \quad t = \frac{t}{r_0^2}, \quad T = \frac{T' - T_x}{T_w - T_x}, \quad C = \frac{C' - C_x}{C_w - C_x}, \\
Gr &= \frac{g\beta r_0^2 (T_w - T_x)}{\nu u_0}, \quad Gc = \frac{g\beta r_0^2 (C_w - C_x)}{\nu u_0}, \quad Sc = \frac{\nu}{\alpha}, \quad Pr = \frac{\nu}{\alpha}, \quad \lambda = \frac{1}{r_0^2}, \quad k = \frac{k}{r_0^2}, \quad M = \frac{\sigma B_0^2 r_0^2}{\rho u_0^2}
\end{align}

which reduces the governing equations (5.1) – (5.4) into:

\begin{align}
\frac{\partial}{\partial X}(RU) + \frac{\partial}{\partial R}(RV) &= 0 \tag{5.7} \\
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial R} &= GrT + GcC + \frac{1}{R} \frac{\partial}{\partial R}\left( \frac{R \frac{\partial U}{\partial R}}{\partial R} \right) - \frac{1}{\lambda} U - MU \tag{5.8} \\
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial R} &= \frac{1}{PrR} \frac{\partial}{\partial R}\left( \frac{R \frac{\partial T}{\partial R}}{\partial R} \right) \tag{5.9} \\
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial R} &= \frac{1}{ScR} \frac{\partial}{\partial R}\left( \frac{R \frac{\partial C}{\partial R}}{\partial R} \right) - KC \tag{5.10}
\end{align}

with the corresponding initial and boundary conditions:

\begin{align}
t \leq 0: & \quad U = 0, \quad V = 0, \quad T = 0, \quad C = 0 \quad \forall X \geq 0, \quad R \geq 0 \\
t > 0: & \quad U = 1, \quad V = 0, \quad T = 1, \quad C = 1 \quad \text{at} \quad R = 1 \\
& \quad U = 0, \quad T = 0, \quad C = 0 \quad \text{at} \quad X = 0 \quad \text{and} \quad R > 1 \quad \text{when} \quad t > 0 \\
& \quad U \to 0, \quad T \to 0, \quad C \to 0 \quad \text{as} \quad R \to \infty \tag{5.11}
\end{align}

### 5.3 SOLUTION TECHNIQUE

Crank – Nicolson implicit finite difference scheme described by Ganesan and Loganathan (Ganesan & Loganathan 2001) is used to solve the
system of two dimensional unsteady, non-linear coupled partial differential equations (5.7) – (5.10) with corresponding initial and boundary conditions (equation 5.11). The mesh sizes have been fixed at the level $\Delta X = 0.02$, $\Delta R = 0.2$ with time step $\Delta t = 0.01$. The equations at every internal nodal point for a particular, $i$ -level constitute a tridiagonal system. The system is solved by Thomas algorithm (Carnahan et al 1969). The scheme is unconditionally stable using the Von Neumann technique. Hence the scheme is compatible. Stability and compatibility ensures the convergence of the finite difference scheme.

5.4 RESULTS AND DISCUSSION

Numerical computations for the velocity, temperature, concentration, skin friction, Nusselt number, and Sherwood number are made for various physical parameters and are presented in Figures 5.1–5.13. Prandtl number is chosen to be $Pr = 0.71$ and $Pr = 7.0$. Schmidt number is taken to be $Sc = 0.6$ which corresponds to water vapour. Grashof number for heat transfer and modified Grashof number for mass transfer is chosen to be $Gr = Gc = 5.0$, since these values correspond to a cooling problem.

Figure 5.1 Comparison of temperature and concentration profiles
To determine the accuracy of the finite difference method, the finite difference solution of temperature and concentration profiles for the stationary cylinder, $Gr = Gc = 2.0$, $\lambda = 0$, $K = 0$ and $M = 0$ (corresponding to $\xi = 0$) are compared with the results of Chen and Yuh (Chen & Yuh 1980) and it is shown in Figure 5.1.

Velocity profiles for different values of K are presented in Figure 5.2. Velocity of the fluid elevates for the increase in the values of the permeability parameter $\lambda$. The velocity reduces when there is an increase in chemical reaction parameter $K$. Maximum value of the velocity is acquired in close proximity of the cylinder and then it vanishes. The magnitude of velocity for $Pr = 0.71$ is much higher than that of $Pr = 7.0$. The larger vales of $Pr$ increases velocity diffusion with high viscosity and hence move slowly.

Figure 5.2 Transient velocity profiles at $X = 1.0$ for $K$
Velocity profiles for different values of $M$ and $K$ are shown in Figure 5.3. The velocity of the fluid decreases for the increasing values of the permeability parameter. Here the chemical reaction parameter $K$ decelerates the fluid motion. Also the flow rate retards for an increase in the magnetic parameter which thereby reduces the velocity of the fluid. The significant reason behind this phenomenon is that application of magnetic field to an electrically conducting fluid gives rise to a resistive type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer. Hence as $M$ increases, the Lorentz force which is opposing the flow leads to enhance deceleration of the flow.

![Graph showing transient velocity profiles at $X = 1.0$ for $M$ and $K$](image)

**Figure 5.3 Transient velocity profiles at $X = 1.0$ for $M$ and $K$**
Figure 5.4 Transient temperature profiles at $X = 1.0$ for $K$

Figure 5.5 Transient temperature profiles at $X = 1.0$ for $M$ and $K$
Figure 5.4 shows the temperature profiles for different values of $K$ with fixed values of the other parameters such as Schmidt number, Grashof number and magnetic parameter. There is a rise in the temperature of the fluid when the chemical reaction parameter increases. Larger $Pr$ gives rise to decrease in the thermal boundary layer thickness, and hence, thinner temperature profiles. Clearly, the thermal diffusion from the wall does not prevail while the velocity diffusion moves away from the wall. From a physical point of view, when Prandtl number increases, the thermal diffusivity decreases and this phenomenon lead to the decreasing of energy ability that reduces the thermal boundary layer.

Temperature profiles for different values of $M$ and $K$ are shown in Figure 5.5. Time taken to reach steady-state depends upon both the Prandtl and Schmidt number. The time required to reach the steady state increases as $Pr$ increases. It is evident that the temperature of the fluid increases for the increasing values of the permeability, magnetic and chemical reaction parameter.

Figure 5.6 depicts the concentration profiles for different values of $K$. The effect of the chemical reaction parameter is very important in the concentration field. The fluid concentration drastically decreases as the chemical reaction parameter or the buoyancy ratio parameter increases.

Figure 5.7 depicts the concentration profiles for different values of $M$ and $K$ with fixed values of other parameters. The concentration of the fluid decreases with the increase in magnetic and chemical reaction parameter.

In order to find the local as well as average skin friction, the rate of heat and mass transfer, the numerical values of velocity, temperature and concentration profiles are taken into account.
The dimensionless form of these values are given by

\[ \tau_\epsilon = \left( \frac{\partial U}{\partial r} \right) \]  \hspace{1cm} (5.12)

\[ \overline{\tau} = -\int_0^1 \left( \frac{\partial U}{\partial r} \right) dX \]  \hspace{1cm} (5.13)

\[ Nu_s = -X \left( \frac{\partial T}{\partial r} \right) \]  \hspace{1cm} (5.14)

\[ \overline{N}u = -\int_0^1 \left( \frac{\partial T}{\partial r} \right) dX \]  \hspace{1cm} (5.15)

\[ Sh_s = -X \left( \frac{\partial C}{\partial r} \right) \]  \hspace{1cm} (5.16)

\[ \overline{Sh} = -\int_0^1 \left( \frac{\partial C}{\partial r} \right) dX \]  \hspace{1cm} (5.17)

The derivatives (equations 5.12 – 5.17) are evaluated using five-point approximation formula and integrals are evaluated using Newton cotes formula. Here \( \tau_\epsilon \) represents local skin friction, \( Nu_s \) is local Nusselt number, \( Sh_s \) is local Sherwood number, \( \overline{\tau} \) is average skin friction, \( \overline{N}u \) is average Nusselt number and \( \overline{Sh} \) is average Sherwood number.

Figures 5.8 – 5.10 depicts the local skin friction, local Nusselt number and local Sherwood number are plotted for different values of permeability, magnetic and chemical reaction parameter respectively. From Figures 5.8 and 5.10, the local shear stress and the local Sherwood number increases for the increasing values of permeability, magnetic and chemical reaction parameter; whereas from Figure 5.9 the local Nusselt number decreases for the increasing values of permeability, magnetic and chemical reaction parameter.
Figure 5.6 Transient concentration profiles at $X = 1.0$ for $K$

Figure 5.7 Transient concentration profiles at $X = 1.0$ for $M$ and $K$
Figure 5.8 Local skin friction

Figures 5.11 – 5.13 shows the average skin friction, average Nusselt number and average Sherwood number are plotted for different values of permeability, magnetic and chemical reaction parameter respectively.

From Figures 5.11 and 5.13, it is clear that the average skin friction and average Sherwood number increases with the increase in magnetic and chemical reaction parameter. It is seen that the average skin friction increases with time and become steady state after some time. But there is a decrease in average Nusselt number when there is an increase in magnetic and chemical reaction parameter as seen in Figure 5.12.
Figure 5.9 Local Nusselt number

Figure 5.10 Local Sherwood number
Figure 5.11 Average skin friction

Figure 5.12 Average Nusselt number
5.5 CONCLUSIONS

Natural convective flow over a moving vertical cylinder is investigated with chemical reaction and MHD effects in the presence of porous medium. The governing equations are solved by Crank Nicolson finite difference scheme. The numerical calculations are carried out for various values of physical parameters, namely, permeability parameter, magnetic parameter, and chemical reaction parameter. The conclusions of the study are:

- The chemical reaction parameter decelerates the fluid motion.
- The flow rate retards for an increase in the magnetic parameter.
- Increase in magnetic and chemical reaction parameter enhances the fluid temperature but reduces the concentration of the fluid.
- The average skin friction and average Sherwood number increases with the increase in magnetic and chemical reaction parameter.