Chapter 6

Conclusion and Scope

In this chapter we briefly describe some of the major applications of our work and the open problems identified in the topic of interest for further research, and which, we hope, will be investigated in the future.

Motivation for studying set-assignment problems on graphs mainly stem from behavioral sciences. In real life situations, our attitudes are continuously influenced and undergo modifications by the nature of attitude-behavior, content of interactions ongoing in the social system we live in, especially due to attitudes and behaviors of persons in our direct contact. In general, interpersonal relationships depend on personal attitudes of the individuals in any social group. When attitudes are expressed by the individuals to others in the group, the types of interpersonal interactions get affirmed and/or modified. On the other hand,
such affirmations and/or modifications in various types of interpersonal interaction in the group induce change in attitudes of the persons in the group whose instantaneous interaction structure is represented by a graph $G = (V, E)$. This dynamically revisory socio-psychological phenomenon motivates a study of set-assignments $h : V \cup E \rightarrow 2^Z$ where $Z$ is a set of attitudes which are likely to be expressed by an individual member of the social group $V$ to any of his/her neighbors with a probability. In this perspective, the ordered triple $(G, Z, h)$, where the graph $G = (V, E)$ represents the acquaintance pattern existing amongst the individuals in the social group $V$ and $h$ represents attitude distribution amongst the elements in $G$, may be regarded as a social network, has an important role to play in the day to day life of an individual.

Hamming distance named after Richard Hamming is used in telecommunication to count the number of flipped bits in a fixed-length binary word as an estimate of error, and therefore is called the signal distance. Hamming weight analysis of bits is used in several disciplines including information theory, coding theory and cryptography. However, comparing strings of different lengths, or strings where not just substitutions but also insertions or deletions have to be expected, a more sophisticated metric like Levenshtein is more appropriate. For
$q$-ary strings over an alphabet of size $q \geq 2$ the Hamming distance is applied in case of orthogonal modulation while the Lee distance is used for phase modulation. The concept of Hamming distance is also used in systematics as a measure of genetic distance.

The theory of isometric set-labelings are rich in theory, with many applications. The main motivation to study isometric set-labeling is due to the problem in communication theory posed by Pierce in 1972. In a telephone network one wishes to be able to establish a connection between two terminals $A$ and $B$ without $B$ knowing that a message is on its way. The idea is to let the message be proceeded by some ‘address’ of $B$, permitting to decide at each node of the network in which direction the message should proceed. The message will proceed to the next node if its hamming distance to the destination node $B$ is shorter or, at a constant proportionality distance or, at various fixed constants of proportionality. The most natural way of devising such a scheme is by labeling the nodes by strings of subsets of a set $X$, which amounts to try to embed the graph in a dcsl-graph.
Interesting problems and conjectures are identified in both the dcsl-graphs, bitopological graphs and hypergraph representation of dcsl-graphs. They are already pointed out in the respective chapters. However, we list below some of the most important problems which are open for further research and investigation.

**Problem 1.** Characterize a dispersible dcsl-graph.

**Problem 2.** Consider any structure-activity relationship $\mathcal{R}$ of a molecular graph that has been identified to be well correlated with the Weiner index. Is it possible to achieve such a correlation using $M$-Weiner index for a low cardinality dcsl-sets $X$ as possible?

**Problem 3.** For any dcsl-graph $G$, the *dispersivity* of $\nu(G)$ of $G$ is the least cardinality of a ground set $X$, such that $G$ admits a dispersive dcsl. Also, find $\nu(K_n)$.

**Problem 4.** Find the necessary condition for a graph to be $l_1$-embeddable, $k$-uniform dcsl-graph.
**Problem 5.** What are the characteristic polynomials of the associated distance matrices of various dcsl-graphs, namely dispersible dcsl-graphs, $k$-uniform dcsl-graphs, $(k, r)$-arithmetic dcsl-graphs and in particular 1-uniform dcsl-graphs.

**Problem 6.** Calculate the dcsl-index of $k$-uniform dcsl-graph and the dcsl index of 1-uniform graph (or equivalently, calculate the isometric dimension of partial cubes).

**Problem 7.** Characterize bitopological set-indexed graphs, in particular, characterize bitopological set-indexed trees.

**Problem 8.** Characterize the transitive digraphs that are bitopologized.

**Problem 9.** Characterize $k$-uniform dcsl hypergraph.

**Problem 10.** Characterize $(k, r)$-arithmetic dcsl hypergraphs.
**Problem 11.** Characterize \((0,1)\)-matrix of 1-uniform dcsl hypergraphs.

**Problem 12.** Characterize hypergraph representation of dispersible-dcsl graphs, \((k,r)\)-arithmetic dcsl-graphs, \(k\)-uniform dcsl-graphs and study their various properties.