CHAPTER 1

INTRODUCTION

Wind possesses energy by virtue of its motion. Any device capable of slowing down the mass of moving air, like a sail or a propeller can extract part of the energy and convert it into useful work. Such devices are usually called wind energy converters. The factors that determine the output of a wind energy converter include wind speed, cross section of the windswept by the rotor, the overall conversion efficiency of the rotor, transmission system and the generator.

The choice of the design parameters is critical for optimizing the wind turbine performance. For any fixed diameter, there are various parameters that influence energy production. They are wind velocity, rotor rotational velocity, number of blade, airfoil chord distribution and longitudinal blade twist. In this research work, the influence of rotor rotational velocity on wind turbine performance has been investigated and reported. In particular, it has been observed that the existing mathematical law on the variation of rotational velocity of rotor with wind speed, which allows the wind turbine to operate at its maximum power coefficient at all conditions. The power coefficient of a wind turbine yields a maximum value for a particular wind velocity and it decreases rapidly for all other wind velocities. Conversely, varying the rotor rotational velocity at different wind velocities, it is possible to have a power coefficient that is always at its maximum value. The power coefficient is the ratio of the extractable mechanical power to the power contained in the air stream (Hau 2006). Wind
turbine blades are twisted to obtain uniform angle of attack throughout the length of the blade. The distribution of chord and twist angles are the key parameters in the performance of wind turbine system.

In this thesis, an iterative approach is developed to identify the axial and tangential flow factors which play an important role in the performance of a wind turbine system. The axial and tangential flow factors are determined using Blade Element Momentum (BEM) method. The convergence of axial and tangential flow factors is identified and the power coefficient of the wind turbine is computed using proposed iterative approach. This approach assures the uniform angle of attack throughout the length of the blade.

The computational simulation techniques yield accurate results in shorter duration for evolving wind turbine performance and the related assessment, compared to various experimental methods. The above techniques are cost effective and alternative to the traditional practice of experimental prototype testing. Higher wind turbine efficiency can be achieved at various wind velocities, by optimizing the various parameters like angle of attack, tip speed ratio, blade geometry, chord and twist angles. The power coefficient can be maximized for various wind velocities by optimizing the above parameters. The optimum angle of attack and tip speed ratio for various airfoil sections to maximize the power coefficient have been identified in this work and presented. In order to understand the various parameters of wind turbine blades, the various types of the wind turbines and the related nomenclature with fundamental theories have been briefed in the following sections.
1.1 OVERVIEW OF WIND TURBINES

In this section different types of wind turbines with its components and the operating characteristics are discussed. Modern wind turbines fall into two basic groups viz. horizontal-axis and the vertical-axis. The orientation of the rotor axis with respect to the ground is normally either horizontal or vertical. Horizontal axis wind turbines (HAWT) have generally two or three-blades and generally designed to face upwind. But designed to face downwind are seldom found. Savonius and Darrieus type of wind turbine designs dominate the available vertical-axis wind turbine (VAWT). The different HAWT and VAWT designs make use of the aerodynamic principles of drag and lift. Drag and lift forces on a turbine blades are the forces acting in the directions parallel and perpendicular to the flow respectively. Rotors using aerodynamic lift achieve higher power coefficients than drag devices. Normally, HAWTs utilize aerodynamic lift and other drag-type designs are also exist. Savonius turbines are drag type devices, but Darrieus design uses lift (Manwell et al. 2002).

A horizontal axis machine has its blades rotating on an axis parallel to the ground and to the wind flow. These wind turbines can be operated either "upwind," with the blades facing the flow of the wind or “downwind” with the flow of the wind from backside of the blade. Upwind turbines face into the wind and the yaw drive or tail vane is used to keep the rotor facing into the wind as the wind direction changes. Downwind turbines do not require a yaw drive and the wind blows the rotor downwind. The yaw motor powers the yaw drive. In case of the upwind design, the direction of the rotor is controlled either by the tail vane or a motor driven mechanism. In the “downwind” type of design, the wind passes the tower before striking the blades and tail vane is not required. The turbine aligns to the wind direction automatically without the tail vane. The upwind and downwind turbines are shown in Figure 1.1.
The vertical axis wind turbines include Darrieus type, Giromill and Savonius. The Giromill has straight blades and Savonius type uses scoops to extract the energy from wind. By virtue of its design, a vertical axis machine need not be oriented with respect to wind direction. Further, the transmission system and generator have to be mounted at the ground level facilitating easier servicing. The VAWT turbines are usually of light weight in comparison with HAWT and reduce the cost of tower. Although vertical axis wind turbines have these advantages, their designs are not as efficient at collecting energy from the wind. Vertical axis wind turbines are not common because of their inherent demerits, as they cannot be installed at higher elevation to gain the advantages of higher wind velocity and their design is based on the utilization of drag forces of wind. As more emphasis is given in this thesis upon horizontal axis wind turbines, the next section is devoted to describe the basic elements, characteristics and various systems of horizontal axis wind turbines.

1.2 ELEMENTS OF HORIZONTAL AXIS WIND TURBINE

In horizontal axis wind turbines power is generated by the rotational motion of the aerodynamically shaped rotor blades. Blade pitch and yaw mechanism together help to extract maximum energy by controlling the
angle of attack and the direction of the rotor respectively. The other major components includes, main rotor, nacelle to house rotor brake mechanism, gearbox, power transmission shafts, control system, generator for electrical power generation. The wind vane and anemometers, to measure the wind direction and velocity are mounted over the nacelle. The main tower supports the entire structure against wind load and is constructed over a strong foundation. As large scale wind turbines are usually connected to national electrical networks, the transformer unit will be housed outside the entire tower to transmit the generated power to the power grid. The basic components of a typical modern HAWT is shown in Figure1.2.

![Figure 1.2 Basic elements of horizontal axis wind turbine](http://ecee.colorado.edu/~ecen2060/wind.html)

1.2.1 Rotor Blade System

The rotor blade design is either based on lift or drag forces. The lift force acts in the direction perpendicular to the flow where as the drag force acts parallel to the flow of the wind. The wind turbine blade design employs the same principle as that of aircraft airfoils. When air flows over the blade, a wind speed and pressure difference is created between the upper and lower
surfaces of the blade. The pressure at the lower surface is greater and hence it creates the lift force. In fact, the blade is essentially an airfoil or wing as shown in Figure1.3.

![Figure1.3 Blade airfoil section](image)

As the blades are attached radially to a hub of the turbine rotor the lift force is converted into rotational motion. In general, wind turbines with lift based designs have much higher rotational speeds than drag types and hence, it is well suited for electricity generation as high speeds are necessary for electrical generators. The tip speed ratio (TSR) is the ratio of the velocity of the blade to the velocity of wind. At higher TSR, the velocity of the rotor is higher for specified wind velocity. The turbines based on lift forces have maximum TSR of around 10, while drag type turbines will have TSR approximately equal to 1. The number of blades that rotor contained and the total area of the rotor have influence over the performance of wind turbines. For the smooth rotation of rotor, wind must flow smoothly without turbulence over the blades; hence the spacing between blades should be sufficient. To achieve this requirement most of the wind turbines have only two or three blades. The wind turbines designed based on drag forces generally have lower rotational speeds with high torque capabilities. Because of this characteristic, it is well suited for farm windmills to lift water from deep well.
1.2.2 Brake System

Mechanical and aerodynamic brake systems are commonly used in wind turbines. The disc brakes are used to stop the rotor in emergencies. In disc brakes, a steel disc is rigidly fixed to the shaft of the rotor and during actuation, the calipers with brake pads are engaged to the disc with the hydraulic pressure that absorbs energy and the shaft is brought to rest as in the automotive braking systems. In the aerodynamic brake system, the rotor blade can turn about its own axis (by changing the pitch) and reduce the lift force acting on it and thereby acts like a brake. Thus the rotational velocity of the rotor can be adjusted by pitch mechanism.

1.2.3 Controller and Gear box

The function of the controller is to start or stop the turbine by applying brake system depending on the wind velocity. The cut-in speed of the wind turbine is 4 m/s and the cut-out speed is 25 m/s (Wind Energy Project Analysis Chapter, 2001-2004). Apart from this function, the controller senses and monitors the wind velocity and direction and transmits signals to operate yaw and pitch mechanisms.

Another vital device in wind turbines is the gear box. It serves the purpose of increasing the rotational speed of the rotor to the rated speed of the generator. The generator requires the rated speed to generate the power of specified frequency, voltage and current. Gear box connects the low-speed shaft (connected to rotor) to the high-speed shaft (connected to the generator) and increases the rotational speed which ranges from 30 to 60 rpm to 1,000 to 1,800 rpm which is the rotational speed required by most of the generators. Some of the DC type wind turbines do not require gear box as they are not required to run at rated speed. The gear box elimination in the transmission system reduces the complexity and maintenance requirements in the wind
turbines. However, larger generators are also required to generate the same power output as that of wind turbines generating AC power.

1.2.4 Generator

The generator converts the mechanical energy generated by wind turbines into electrical energy of specified rating. The generator can be either of AC or DC type. The generators are generally selected based on the output rating of the wind turbines. The generators are usually one of the following types: Squirrel-cage induction generator, Wound-rotor induction generator, Doubly-fed asynchronous generator or Synchronous generator.

A Squirrel-cage induction generator has a gear box to match the rotational speed of blades with that of the generator. Mechanical power is regulated through an inherent aerodynamic stall characteristic of blades or with active control of blade pitch. A wound-rotor induction generator has a gearbox for coupling the electrical generator to the rotor hub. They also have pitch control of blades for maximizing energy capture and controlling turbine speed within the range of generator. A doubly fed induction generator is an induction generator with a wound rotor and a four quadrant AC to AC converter, which is connected to the rotor winding. It has a variable frequency excitation of the rotor circuit, incorporating rotor current control via the power converter. The rotor circuit power converter may be of four quadrants, allowing independent control of real and reactive flow in either direction (rotor to grid or grid to rotor) or confined to unidirectional (grid to rotor) real power flow. These machines have a gear box for coupling the generator shaft to turbine hub, active control of turbine blade pitch for maximizing production and controlling mechanical speed and variable speed operation depending on the rating of power converter relative to the turbine rating. A synchronous induction generator has the generator coupled to the grid through a fully rated AC to DC and DC to AC power converters. They have a
gear box to match generator speed to variable rotational speed of blades and variable speed operation over a wide range depending on electrical generator characteristics (Burton et al. 2001).

1.2.5 Tower

Towers are made from tubular steel concrete or steel lattice. The tower supports the entire rotor system and also facilitates to locate the rotor at higher elevation to trap the wind energy effectively. It also has provision to reach the nacelle for maintenance of the turbine. It transmits vibration and mechanical load of the rotor to the ground. The towers are designed to withstand bending and buckling loads. Larger wind turbines are usually mounted on towers ranging from 120 to 210 feet in height.

1.3 OPERATING CHARACTERISTICS

A few of the important operating characteristics of a wind turbine such as the cut-in speed, cut-out speed, rated speed, Betz limit, power output and capacity factor are discussed in this section. These factors influence the performance of the wind turbine and work as constraints during optimization of power coefficient. A brief note on the above operating characteristics is presented in the following sub sections.

1.3.1 Cut-in Speed

The wind turbine effectively produces power in the specified range of wind velocities. The minimum wind velocity required for starting the power generation is known as Cut-in speed. If the speed of the wind is more than cut-in speed the turbine will start and generate power. For modern wind turbines it is generally around 10 mph.
1.3.2 **Cut-out Speed**

It is not safe to allow the wind turbines to operate beyond particular wind velocity. The speed of the wind at this specified limit is called Cut-out speed. The control systems monitor the wind velocity and shut down the turbine when the wind velocity exceeds the cut-out speed. Cut-out speed varies from 45 to 60 mph for various turbines. The most common method of shutting down a wind turbine is changing the pitch of the blade, so that the wind just passes through the blades without producing lift. The other methods incorporate some type of drag device to prevent the blades from turning in the high wind.

1.3.3 **Rated Speed**

The rated speed is the wind speed at which the wind turbine will generate its estimated rated power. For example, 2.5 MW wind turbine will typically produce its rated power once the wind speed exceeds about 27 mph. Most wind turbines are rated between 25 to 35 mph. At wind speeds between cut-in to rated speed, the power output from a wind turbine increases as the wind speed increases. The power output of a wind turbine is relatively flat above its rated speed till the wind speed reaches the cut-out speed. The typical graph showing variation of wind speed with respect to power for a wind turbine with rated power of 2.5 MW and rated wind speed of 25MPH is given in Figure 1.4.
1.3.4 Betz Limit

It is impossible for the wind turbine blades to convert all the energy exerted by the wind into power generation. Some of the wind energy must pass through the blades to make the turbine to turn. The wind turbine extracts energy from the momentum gained by the wind. After wind strikes the blade, it loses its momentum and the blade gains momentum. The change in momentum is converted into useful work. The theoretical maximum amount of energy in the wind that can be extracted by wind turbines is approximately 59.3% (Manwell et al. 2002). This value is known as the Betz limit. Further, there will be energy losses in gear box, generator and other mechanical moving parts. Hence, the useful energy available is only 15-25%.
1.3.5 Power Output

In general, the power generated by a wind turbine can be calculated using the following equation (1.1).

\[ P = 0.5 \rho A C_p N_g N_b v^3 \]  

(1.1)

Where,

- \( P \) - Power output of the wind turbine, watts
- \( \rho \) - Air density, kg/m\(^3\)
- \( A \) - Swept area of the blades, m\(^2\)
- \( C_p \) - Power Coefficient of the blades
- \( N_g \) - Generator efficiency
- \( N_b \) - Gear box efficiency
- \( v \) - Wind velocity, m/s

The capacity factor (CF) of a wind turbine is the actual energy output of a wind turbine during a given time period, usually one year, compared to its theoretical maximum energy output. The capacity factor is defined as,

\[ CF = \frac{kWh \text{ produced in a year}}{(8760 \times \text{Power Rating})} \]  

(1.2)

1.4 POWER CALCULATION

The basic function of the wind turbine rotor is to utilize the kinetic energy available in a moving air stream and convert it into rotational motion of shaft. The rotation of a disk in a stream of moving air is governed by a set of basic rules. An idealized or perfect rotor, under steady-state conditions, is a
simplification of the real system. Practical rotor design requires an understanding of the aerodynamics of rotor blades. Blades normally take the form of airfoils, which are specially shaped to maximize aerodynamic lift forces while minimizing drag. Blade design is of critical importance in turbine starting and operating performance. It plays a major role not only in power extraction but also in the structural integrity and fatigue endurance of the rotor. One of the basic theories of rotor aerodynamics is the one dimensional Betz’s Elementary Momentum Theory. More complicated theories, such as the two dimensional Blade Element Theory or Vortex Theory may provide more accurate determination of rotor performance (Hau 2006). Betz’s simple momentum theory considers a stream of air moving through a circular disk. The analysis is based on the following assumptions:

- Homogeneous, incompressible, steady state fluid flow
- No frictional drag
- Infinite number of blades
- Uniform thrust over the entire rotor area
- Non-rotating wake (air stream after passing through rotor)
- Upstream and downstream static pressures are equal to the undisturbed ambient static pressure

One dimensional theory, which considers a moving stream of air passing through wind turbine is shown in Figure 1.5.
Figure 1.5 Moving stream of air passing through wind turbine

The energy in the moving stream of air is given by the kinetic energy equation (1.3).

\[ E_k = \frac{1}{2} m v^2 \]  

Where

\( E_k \) - Kinetic energy of the air stream, Joule

\( m \) - Mass of the air, kg

\( v \) - Wind velocity, m/s

The power in the air stream is given by energy per unit time as stated by the equation (1.4)

\[ P = \frac{E_k}{t} = \frac{1}{2} \frac{m v^2}{t} = \dot{m} v^2 \quad (W) \]  

Where ‘\( \dot{m} \)’ is the mass flow rate of the air stream and is given by the equation (1.5)

\[ \dot{m} = \rho A v \quad (kg/s) \]
Thus the power of a moving stream of air with density ‘\( \rho \)’ and velocity ‘\( v \)’ that flows through a rotor of area ‘\( A \)’ is:

\[
P = \frac{1}{2} \rho A v^3
\]  

(1.6)

The above expression gives the power available in a moving stream of air. The power coefficient is defined as the ratio of power extracted to the available wind power.

\[
C_p = \frac{\text{Rotor Power}}{\text{Wind Power}}
\]  

(1.7)

The maximum possible rotor power coefficient (\( C_p \)) is given by the Betz Limit, \( C_p = 16/27 = 0.593 \) (Manwell et al 2002).

In practice, further energy is being lost due to the reasons listed below.

- Rotation of the wake after the wind has passed through the rotor
- The number of blades used, and the losses at the blade tips
- Non-zero aerodynamic drag, as assumed by the one dimensional theory
- Electrical losses, such as heat loss through the transmission cabling or inefficiency of the slip rings

Hence, the net power output of the wind turbine can be determined by the power coefficient, the overall turbine efficiency, air density, rotor area and wind speed at the location.

\[
P = C_p \eta \frac{1}{2} \rho A v^3
\]  

(1.8)
Where,  \( C_p \) - Power coefficient of the blades

\( \eta \) - Efficiency of the turbine

\( P \) - Power output of the wind turbine

1.5 PERFORMANCE OF WIND TURBINE

International Electro technical Commission (IEC) (2005) stated that: “The wind turbine power performance characteristics are determined by the measured power curve and the Annual Energy Production (AEP).” The content in the standard IEC 61400-12-1:2005(E) - Wind turbines – Part 12-1: “Power performance measurements of electricity producing wind turbines” outlined the methodology used to establish these two performance criteria as follows:

- The measured power curve is collated by measuring wind speed and power output at a test site for a pre-determined period of time.

- The Annual Energy Production (AEP) is then calculated by applying the measured power curve to the reference wind speed distributions.

Manwell et al. (2002) explained that the power performance curve incorporates three key points namely cut-in, cut-out and rated speed are shown in Figure 1.6.
The ideal power coefficient for common wind turbine designs for various configurations is shown in the Figure 1.7 (Hau 2006). It is apparent that HAWTs are the most efficient with a power coefficient of about 0.49. Darrieus turbines achieve a maximum power coefficient of about 0.4, while Savonius designs typically only reach 0.15. Despite the differences in efficiency, each design has its own benefits and drawbacks. All turbines need to be positioned as high as possible, but VAWTs allow the generator to be located at ground level. VAWTs are generally quieter than HAWTs and do not need to yaw into the wind (Webb 2007).

Burton et al. (2001) have related the wind turbine performance to a different set of criteria in their work and presented the variations of power coefficient, thrust and torque coefficient with respect to varying TSR. The variation of power coefficient, thrust and torque coefficient with respect to TSR is briefed in the following sub sections.
1.5.1 Power Coefficient Vs Tip Speed Ratio ($C_p$ Vs $\lambda$) Performance Curve

The variations of Power coefficient ($C_p$) with respect to Tip speed ratio ($\lambda$) for various numbers of blades are illustrated in Figure 1.8 (Burton et al. 2001).
Burton et al (2001) derived the power coefficient ($C_p$) in terms of axial flow factor ($a$) which is defined as the fractional decrease in wind velocity between the free stream and the rotor plane.

$$C_p = 4a(1-a)^2 \quad (1.9)$$

Where, $C_p$ - Power coefficient

$a$ - Axial flow factor

and the Tip speed ratio (TSR) is

$$\lambda = \frac{\Omega r}{U_\alpha} \quad (1.10)$$

Where,

$\lambda$ - Tip speed ratio

$\Omega r$ - Rotational speed of rotor (rad/sec)

$U_\alpha$ - Free stream velocity (m/sec)

### 1.5.2 Torque Coefficient Vs Tip Speed Ratio ($C_Q Vs \lambda$) Performance Curve

Burton et al. (2001) revealed that the torque coefficient $C_Q$ is derived from the power coefficient ($C_p$) by dividing with tip speed ratio ($\lambda$). The $C_Q Vs \lambda$ performance curve is shown in Figure 1.9. When the rotor is connected to the gear box and the generator it is necessary to predict the torque of the rotor. This curve is used to predict the torque coefficient at various TSR for varying number of blades and the torque can be derived from the torque coefficient.
1.5.3 Thrust Coefficient Vs Tip Speed Ratio ($C_T$ Vs $\lambda$) Performance Curve

The graph as shown in the Figure 1.10 (Burton et al. 2001) shows the variation of thrust coefficient and TSR for varying number of blades. This performance curve is useful in determining the requirement of structural design of tower. Generally the thrust force is heavily influenced by the solidity of the rotor.
Burton et al. (2001) showed that:

\[ C_T = 4a(1 - a) \]  

(1.11)

Where,

- \( C_T \) - Thrust Coefficient
- \( a \) - Axial flow factor

### 1.6 MOMENTUM THEORY

In momentum theory, the equation for momentum transfer from air to rotor is derived. In this theory, the rotor is assumed to be a disc of area ‘\( A_d \)’ that is placed in the free stream flow of air. The velocity at the upstream (\( U_\alpha \)) gradually reduces towards the disc (\( U_d \)), it reduces further from disc to downstream (\( U_w \)). The pressure at upstream (\( P_\alpha \)) gradually increases towards the disc and there is a steep pressure drop (\( P_d^+ - P_d^- \)) across the disc (\( P_d^+ \) is upstream pressure and \( P_d^- \) is downstream pressure at the disc) then the pressure regains its original value at downstream (\( P_w \)). The air that passes through the disc undergoes an overall change from the upstream velocity (\( U_\alpha \)) to downstream velocity (\( U_w \)). The velocity and pressure distribution across the disc is shown in the Figure 1.13(Tony Burton et al 2001).

![Figure 1.11 Velocity and Pressure distribution across the disc (Burton et al 2001)](image-url)
The rate of change of momentum is given by the following equation (1.12).

\[
\text{Rate of change of momentum} = (U_\alpha - U_w) \rho A_d U_d
\]

(1.12)

Where,

\begin{align*}
\rho & \quad \text{Density of air, kg/m}^3 \\
A_d & \quad \text{Area of the disc, m}^2 \\
U_d & \quad \text{Velocity at disc, m/s}
\end{align*}

It is always considered that the actuator disc induces a variation of velocity that must be superimposed on the free-stream velocity. The streamwise component of this induced flow at the disc is given by \( -aU_\alpha \), where \( a \) is known as the axial flow factor. The net stream velocity at the disc is given in the equation (1.13).

\[
U_d = U_\alpha(1 - a)
\]

(1.13)

The force exerted on disc is equal to the change of momentum of the air and it is calculated from the difference in pressure across the actuator disc and area of the disc which is given by the equation (1.14).

\[
(P_d^+ - P_d^-)A_d = (U_\alpha - U_w) \rho A_d U_\alpha U_d(1 - a)
\]

(1.14)

Bernoulli’s equation states that, under steady conditions, the total energy in the flow, comprising kinetic energy, static pressure energy and gravitational potential energy, remains constant provided no work is done on or by the fluid. Thus, for a unit volume of air the total energy is given by the equation (1.15).
\[
\frac{1}{2} \rho U^2 + p + \rho gh = \text{constant} \quad (1.15)
\]

Bernoulli’s equation is applied separately to the upstream and downstream sections of the stream-tube to obtain the pressure difference \((P_d^+ - P_d^-)\). The Bernoulli’s equation is applied between the section of upstream and disc as shown in the equation (1.16)

\[
\frac{1}{2} \rho_u U_u^2 + p_u + \rho_u gh_u = \frac{1}{2} \rho_d U_d^2 + p_d^+ + \rho_d gh_d \quad (1.16)
\]

Assuming the flow to be incompressible, the density at the upstream \((\rho_u)\) and at the disc \((\rho_d)\) are equal and as the elevation of the sections \((h_u = h_d)\) are the same, the potential energy at upstream and the disc are also same. Hence the difference of potential energy is zero and the equation (1.16) can be rewritten as equation (1.17).

\[
\frac{1}{2} \rho_u U_u^2 + p_u = \frac{1}{2} \rho_d U_d^2 + P_d^+ \quad (1.17)
\]

Similarly, for downstream and disc,

\[
\frac{1}{2} \rho_w U_w^2 + p_w = \frac{1}{2} \rho_d U_d^2 + P_d^- \quad (1.18)
\]

Subtracting the equations (1.17) and (1.18) and the resulting equation is shown in equation (1.19).

\[
(P_d^+ - P_d^-) = \frac{1}{2} \rho (U_u^2 - U_w^2) \quad (1.19)
\]

Substituting equation (1.19) in equation (1.14) the resulting equation is

\[
\frac{1}{2} \rho (U_u^2 - U_w^2) A_d = (U_u - U_w) \rho A_d U_u(1 - a) \quad (1.20)
\]
On simplification the equation (1.20) is reduced to

\[ U_w = (1 - 2a)U_x \]  \hspace{1cm} (1.21)

In the velocity head the 50\% loss occurs in axial direction on the upstream side and the remaining at the downstream side. The net force exerted by the air on the rotor is obtained by substituting the equation (1.21) in equation (1.14). On simplification the following equation is arrived.

\[ F = (P_d^+ - P_d^-)A_d = 2\rho A_d U_x^2 a(1 - a) \]  \hspace{1cm} (1.22)

The wind power \( P_{wind} \) available at the disc is arrived by the following equation (1.23).

\[ P_{wind} = FU_d = 2\rho A_d U_x^2 a(1 - a)^2 \]  \hspace{1cm} (1.23)

The power coefficient is calculated as in the equation (1.24)

\[ C_p = \frac{P_{wind}}{\frac{1}{2} \rho U_d^3 A_d} \]  \hspace{1cm} (1.24)

Substituting equation (1.23) in equation (1.24) and on simplification, the power coefficient obtained as in equation (1.25).

\[ C_p = 4a(1 - a)^2 \]  \hspace{1cm} (1.25)

From the above equation the maximum value of \( C_p \) is calculated by the equations (1.26) and (1.27).

\[ \frac{dC_p}{da} = 4(1 - a)(1 - 3a) = 0 \]  \hspace{1cm} (1.26)
From the equation (1.26) the value of axial flow factor ‘a’ is deduced as $a = 1/3$ and substituting it in equation (1.26) the maximum power coefficient is arrived as equation (1.27).

$$C_p = \frac{16}{27} = 0.593 \quad (1.27)$$

The above maximum value of the power coefficient is known as Betz limit derived by Betz (1919) the German aerodynamicist.

The thrust force (T) is given by the equation (1.28).

$$C_T = \frac{1}{2} \rho U_\infty^2 A_d \quad (1.28)$$

Thrust coefficient ($C_T$) is defined as the ratio of wind power to the thrust force ($T_{th}$).

$$C_T = \frac{P_{wind}}{\frac{1}{2} \rho U_\infty^2 A_d} \quad (1.29)$$

From the equations (1.23) and (1.29) the Thrust coefficient is obtained and given in the equation (1.30).

$$C_T = 4a(1-a) \quad (1.30)$$

The variation of power coefficient and thrust coefficient with respect to axial flow factor (a) is shown in Figure 1.12.
In the previous section, the momentum theory was discussed in detail to predict the maximum power coefficient of the wind turbine. The momentum theory considers only the axial flow factor ‘a’ but does not include the losses in the tangential direction due to tangential forces. In order to consider the energy losses in the tangential direction, the angular momentum theory is essential. The tangential flow factor (a’) is derived using angular momentum theory to evaluate the tangential losses.

The tangential velocity (\( \Omega r \)) and the axial velocity (U) will not be uniform for all radial positions of the wind blade. In order to predict the maximum power coefficient, the variation of both axial and tangential velocities is calculated. From the rotor disc area, an annular ring of width ‘\( \delta r \)’ and area ‘\( \delta A_d \)’ are considered at the radius ‘r’ from the centre of the rotor as illustrated in Figure 1.13. The trajectory of an air particle passing through the rotor disc is shown in Figure 1.14 and the increase in tangential velocity across the thickness of the disc is shown in Figure 1.15.

Figure 1.12 Variations of \( C_p \) and \( C_T \) with Axial flow factor (a) (Burton et al. 2001)
Figure 1.13 Rotor disc area with annular ring (Burton et al. 2001)

Figure 1.14 Trajectory of an air particle passing through the rotor disc (Burton et al. 2001)

Figure 1.15 Increase of tangential velocity across the disc thickness (Burton et al. 2001)
The increase of rotor torque acting on the annular ring is responsible for imparting the tangential velocity component to the air whereas the axial force acting on the ring is responsible for the reduction in axial velocity. The whole disc is assumed to be consisting of multiple annular rings and each ring is imparting momentum independently to the air.

The torque (Q) on the ring will be equal to the rate of change of angular momentum of the air passing through the ring and is given in the general equation (1.31) (Burton et al. 2001).

\[
\text{Torque (Q)} = \text{Rate of change of angular momentum} \\
= (\text{mass flow rate}) \times (\text{change of tangential velocity}) \times (\text{radius})
\]

\[Q = \rho \delta A_d U_\alpha (1 - a) \Omega a' r^2 \]  \hspace{1cm} (1.31)

The torque in the annular ring is given in equation (1.32)

\[\delta Q = \rho \delta A_d U_\alpha (1 - a) \Omega a' r^2 \]  \hspace{1cm} (1.32)

The power developed on the annular ring is the product of torque \(\delta Q\) and the angular velocity \(\Omega\) and is given in the equation (1.33).

\[\delta P = \delta Q \Omega = \rho \delta A_d U_\alpha (1 - a) \Omega^2 a' r^2 \]  \hspace{1cm} (1.33)

The net power available at the ring is calculated from equation (1.23) by replacing \(P\) as \(\delta P\) and \(A_d\) as \(\delta A_d\) and is given in the equation (1.34).

\[\delta P = 2 \rho \delta A_d U_\alpha^2 a (1 - a)^2 \]  \hspace{1cm} (1.34)

Equating equations (1.33) and (1.34)

\[2 \rho \delta A_d U_\alpha^2 a (1 - a)^2 = \rho \delta A_d U_\alpha (1 - a) \Omega^2 a' r^2 \]  \hspace{1cm} (1.35)

On further simplification, the equation can be rewritten as
\[ U_\alpha^2 a(1 - a) = \Omega^2 a'r^2 \] (1.36)

Local speed ratio is defined as the ratio of tangential velocity at annular radius ‘r’ to free stream velocity. Local tip speed ratio \( \lambda_t \) is written as

\[
\text{Local tip speed ratio, } \lambda_t = \frac{\Omega r}{U_\alpha} \tag{1.37}
\]

By applying equation (1.37) in equation (1.36), and on simplification

\[ a(1 - a) = \lambda_t^2 a' \tag{1.38} \]

The area of the ring can be written as,

\[ \delta A_d = 2\pi r \delta r \tag{1.39} \]

By substituting equations (1.38) and (1.39) in equation (1.33), the power developed on the annular ring is,

\[ \delta P = dQ\Omega = \left(\frac{1}{2} \rho U_\alpha^2 2\pi r \delta r\right) \left(4a'(1 - a)\lambda_t^2\right) = (4\pi \rho U_\alpha^3 r \delta r) \left(a'(1 - a)\lambda_t^2\right) \tag{1.40} \]

The term \( \left(\frac{1}{2} \rho U_\alpha^2 2\pi r \delta r\right) \) represents the power flux through the annulus and the term \( \left(4a'(1 - a)\lambda_t^2\right) \) is the efficiency of the blade element in capturing the power or blade element efficiency \( \eta_t \).

\[ \eta_t = 4a'(1 - a)\lambda_t^2 \tag{1.41} \]

The equation (1.40) is written in terms of power coefficient,
\[
\frac{d}{d\mu} C_p = \frac{4\pi \rho \mu^2 \hat{a}'(1-a)\lambda^3}{\frac{1}{2}\rho \mu^2 \pi R^2} = \frac{8a'(1-a)\lambda^3 \mu^3}{R^2}
\] (1.42)

Substituting \(\mu = \frac{r}{R}\) in equation (1.41),

\[
\frac{d}{d\mu} C_p = 8a'(1-a)\lambda^3 \mu^3
\] (1.43)

The equation (1.43) can be integrated to determine the overall power coefficient for the disc for a given tip speed ratio (\(\lambda\)) and varying axial flow factor (\(a\)) and tangential flow factor (\(a'\)) radially.

1.8 MAXIMUM POWER COEFFICIENT

The values of axial flow factor and tangential flow factor will provide the maximum possible efficiency that can be determined by differentiating equation (1.38) by either factor or putting the result equal to zero.

\[
\frac{d}{d\alpha'} \alpha = \frac{1-a}{\alpha'}
\] (1.44)

From equation (1.38)

\[
\frac{d}{d\alpha'} \alpha = \frac{\lambda^2}{1-2a}
\] (1.45)

From equation (1.45)

\[
\alpha'\lambda^2 = (1-a)(1-2a)
\] (1.46)

The combination of equations (1.38) and (1.44) gives the required values of (\(a\)) and (\(a'\)) which maximize the incremental power coefficient.

\[
a = \frac{1}{3} \quad \text{and} \quad a' = \frac{\alpha(1-a)}{\lambda^2 \mu^2}
\] (1.47)
The axial flow factor (a) for maximum power extraction remains unchanged for non rotating wake case and it is 1/3. On the other hand a’ varies with radial position. From equation (1.43) the maximum power is

\[ C_p = \int_0^1 8(1-a)a'^2\mu^3 \, d\mu \]  

(1.48)

Substituting equation (1.47) in equation (1.48), the maximum power coefficient is derived as

\[ C_p = \int_0^1 8(1-a)a'\left[\frac{\alpha(1-\alpha)}{\lambda^2\mu^2}\right]\lambda^2\mu^3 \, d\mu = 4\alpha(1-\alpha)^2 = \frac{16}{27} = 0.593 \]

### 1.9 BLADE ELEMENT THEORY

Blade element theory is based on the following assumptions:

1. There are no aerodynamic interactions between different blade elements.
2. The span-wise component of velocity is ignored.
3. The effects of three dimensions are also ignored.
4. The forces on the blade elements are solely determined by the lift and drag coefficients.

The angle of attack is determined by wind speed, the flow factors, rotational speed of the rotor. The aerofoil characteristic coefficients such as Coefficient of lift (C_l) and Coefficient of drag (C_d) vary with the angle of attack and wind velocity. For given values of axial flow factor (a) and tangential flow factor (a’), the forces on the blades can be determined.
A turbine with ‘N’ blades of tip radius ‘R’ each with chord (c) and set pitch angle β measured between the aerofoil zero lift line and the plane of the disc is considered. Both the chord length and the pitch angle may vary along the blade span. The blades rotate at angular velocity \( \Omega \) and the wind speed is \( U_w \). The tangential velocity \( \Omega r \) of the blade element shown in Figure 1.13 combined with the tangential velocity of the wake \( a' \Omega r \) means that the net tangential flow velocity experienced by the blade element is \( (1+a')\Omega r \). Figure 1.18 shows all the velocities and forces relative to the blade chord line at radius \( r \).

**Figure 1.16 Rotating annular stream tube: notation (Burton et al. 2001)**

**Figure 1.17 Blade element model (Burton et al. 2001)**
A blade divided up into $N$ elements is considered as shown in Figure 1.13. Each of the blade elements will experience a slightly different flow as they have a different rotational speed ($\Omega r$), a different chord length ($c$) and a different twist angle. Blade element theory involves dividing up the blade into a sufficient number (usually between ten and twenty) of elements and calculating the flow at each one. Overall performance characteristics are determined by numerical integration along the blade span. The resultant relative velocity ($W$) at the blade acting at an angle $\phi$ to the plane of rotation is given in equation (1.49). The inflow angle ($\phi$) and the angle of attack ($\alpha$) is derived from the Figure 1.18 and is given in the equations (1.50) and (1.51).

\[
W = \sqrt{U_0^2(1 - \alpha)^2 + \Omega^2 r^2(1 + \alpha')^2}
\]  

(1.49)

**Figure 1.18 Blade element velocities and forces (Burton et al. 2001)**

\[
sin\phi = \frac{U_0(1-\alpha)}{W} \quad \text{and} \quad \cos\phi = \frac{\Omega r(1+\alpha')}{W}
\]

(1.50)

The angle of attack ($\alpha$) is given by

\[
\alpha = \phi - \beta
\]

(1.51)

The lift force on a span-wise length $\delta r$ of each blade, normal to the direction of $W$, is

\[
\delta L = \frac{1}{2} \rho W^2 c r C_l \delta r
\]

(1.52)
and the drag force parallel to W is

\[ \delta D = \frac{1}{2} \rho W^2 c \theta C_d \delta r \] (1.53)

1.10 THE BLADE ELEMENT MOMENTUM (BEM) THEORY

According to the BEM theory, the force of a blade element is responsible for the change of momentum of the air passing through the annulus swept of the element. Further, it is assumed that there is no radial interaction between the flows through contiguous annuli, if the axial flow factor does not vary radially. The axial flow factor is seldom uniform and experimental examination shows that the assumption of radial independence is acceptable (Lock 1924).

The component of aerodynamic force on ‘N’ blade elements resolved in the axial direction is given as

\[ \delta L \cos \phi + \delta D \sin \phi = \frac{1}{2} \rho W^2 N (C_L \cos \phi + C_D \sin \phi) \delta r \] (1.54)

The rate of change of axial momentum of the air passing through the swept annulus is

\[ \rho U_a (1 - a) 2\pi r \delta r 2\pi U_a = 4\pi \rho U_a^2 a (1 - a) r \delta r \] (1.55)

The drop in wake pressure caused by wake rotation is equal to the increase in dynamic head, which is

\[ \frac{1}{2} \rho (2a \dot{\omega} r)^2 \] (1.56)

Therefore the additional axial force on the annulus is

\[ \frac{1}{2} \rho (2a \dot{\omega} r)^2 2\pi r \delta r \] (1.57)
Combining the equations (1.54), (1.55) and (1.57),

\[ \frac{1}{2} \rho W^2 N c (C_l \cos \phi + C_d \sin \phi) \delta r = 4 \pi \rho [U_d^2 a (1 - a) + (a' \Omega r)^2] r \delta r \]  

(1.58)

Simplifying the equation (1.58)

\[ \frac{W^2}{U_d^2} N c \frac{c}{R} (C_L \cos \phi + C_D \sin \phi) = 8 \pi (a(1 - a) + (a' \lambda \mu)^2 \mu) \]  

(1.59)

The element of axial rotor torque caused by aerodynamic forces on the blade elements is

\[ (\delta L \sin \phi - \delta D \cos \phi) r = \frac{1}{2} \rho W^2 N c (C_l \sin \phi - C_d \cos \phi) \delta r \]  

(1.60)

The rate of change of angular momentum of the air passing through the annulus is

\[ \rho U_d (1 - a) \Omega r^2 a' r^2 \pi r \delta r = 4 \pi U_d (\Omega r) a' r^2 (1 - a) \delta r \]  

(1.61)

Equating the equations (1.60) and (1.61), the equation (1.62) is arrived as

\[ \frac{1}{2} \rho W^2 N c (C_l \sin \phi - C_d \cos \phi) r \delta r = 4 \pi U_d (\Omega r) a' (1 - a) r^2 \delta \]  

(1.62)

Simplifying the equation (1.62)

\[ \frac{W^2}{U_d^2} N c \frac{c}{R} (C_l \sin \phi - C_d \cos \phi) = 8 \pi \lambda \mu^2 a' (1 - a) \]  

(1.63)

Where,

\[ \mu = r/R \]

It is convenient to write
\[ C_T \cos \phi + C_d \sin \phi = C_X \quad (1.64) \]
\[ C_T \sin \phi - C_d \cos \phi = C_Y \quad (1.65) \]

Solving equations (1.59) and (1.63) to obtain values for the flow induction factors \( a \) and \( a' \) using two-dimensional aerofoil characteristics requires an iterative process. The following equations derived from (1.59) and (1.63), are convenient in which the right-hand sides are evaluated using existing values of the flow induction factors yielding simple equations for the next iteration of the flow induction factors.

\[ \frac{a}{1-a} = \frac{\sigma_Y}{4 \sin^2 \phi} \left( C_X - \frac{\sigma_Y}{4 \sin^2 \phi} C_Y^2 \right) \quad (1.66) \]
\[ \frac{a}{1+a'} = \frac{\sigma_Y C_Y}{4 \sin \phi \cos \phi} \quad (1.67) \]

Chord solidity \( \sigma_r \) is defined as the total blade chord length at a given radius divided by the circumferential length at that radius and is given in equation (1.68).

\[ \sigma_r = \frac{N_c}{2 \pi r} = \frac{N_c}{2 \pi \mu R} \quad (1.68) \]

It is argued by Wilson and Lissaman (1974) that the drag coefficient should not be included in equations (1.66) and (1.67) because the velocity deficit caused by drag is confined to the narrow wake which flows from the trailing edge of the aerofoil. Furthermore, Wilson and Lissaman (1974) stated that the drag-based velocity deficit is only a feature of the wake and does not contribute to the velocity deficit upstream of the rotor disc. The basis of the argument for excluding drag in the determination of the flow induction factors is that, for attached flow, drag is caused only by skin friction and does not affect the pressure drop across the rotor. Clearly, in stalled flow the drag is overwhelmingly caused by pressure. In attached flow it has been shown by Young and Squire (1938) that the modification to the inviscid
pressure distribution around an aerofoil caused by the boundary layer has an affect both on lift and drag. The ratio of pressure drag to total drag at zero angle of attack is approximately the same as the thickness to chord ratio of the aerofoil and increases as the angle of attack increases.

1.11 TIP LOSS CORRECTION FACTOR

The tip loss correction factor \( F_t \) is given by (Grant Ingram 2005) in his research work as the following equation

\[
F_t = 2 \pi \exp \left\{ - \left[ \frac{N/2}{1 - r/R} \right]^2 \left( \frac{r}{R \cos \beta} \right)^3 \right\}
\]

(1.69)

Where

\[
\begin{align*}
N & \quad \text{Number of blades} \\
r & \quad \text{Radius} \\
R & \quad \text{Tip radius} \\
\beta & \quad \text{Pitch angle}
\end{align*}
\]

The value of \( F_t \) varies from 0 to 1 and characterizes the reduction in forces along the blade.

1.12 OVERVIEW OF THE THESIS

This thesis is divided into two main parts. The first part deals with the introduction to wind turbine, literature review, development of correlations for coefficient of lift and coefficient of drag and the effect of Reynolds number on lift and drag coefficient of airfoil. The second part deals with the optimization of wind turbine blade using the Iterative method and Genetic Algorithm with computer simulation and performance evaluation of the wind turbine system. Chapters 1 and 2 serve as an introduction to the
study. These chapters give the overall view of the recent trend of research works being carried out on wind turbine performance and the application of computer simulation codes especially on optimization of wind turbine. Chapter 3 presents the development of correlations for coefficient of lift and drag and the validation of coefficients with experimental work. The analysis of various NACA airfoils at various Reynolds number is also presented in this chapter. Chapters 4 and 5 present the second part of this work i.e. methods for optimization of power coefficient. In Chapter 4 an Iterative approach to optimize the power coefficient is discussed and in Chapter 5 Optimization of power coefficient using Genetic Algorithm using computer simulation for various airfoils is presented. Chapter 6 presents the conclusions of the research work and the suggestions for further study.

Chapter 1: This chapter provides comprehensive information on wind turbine system and its characteristics. In this chapter the basics of wind turbine and its characteristics, aerodynamics theories, objectives of the research work and overview of chapters are presented.

Chapter 2: This chapter deals with a detailed review of literature related to the wind turbine system.

The literature review is classified into

a) Airfoil aerodynamics

b) Design and performance of wind turbine

c) Optimization of wind turbine and simulation techniques

Chapter 3: The correlations for coefficient of lift and drag are developed in this chapter. The results of the correlations are validated with the
experimental work. This chapter also deals with the analysis of various NACA airfoils for wind turbine system at various Reynolds number.

**Chapter 4:** In this chapter an Iterative method is proposed to identify the convergence of axial and tangential flow factors and optimization of power coefficient. The Computational Fluid Dynamics is used to optimize the coefficient of lift and drag. This chapter also deals with the effect of coefficient of drag and tip loss correction factor on the power coefficient. The power coefficient is computed for the two cases and compared in this chapter.

**Chapter 5:** An optimization of wind turbine power coefficient using Genetic Algorithm is performed and the corresponding optimum angle of attack and tip speed ratio are also computed for wind turbine with various NACA airfoils. The results of the optimization for various categories are presented.

**Chapter 6:** The conclusions of the work are presented in this chapter and the suggestions for further study on this work are also highlighted in this chapter.