Chapter 4

THEORY AND MEASUREMENTS
4.1 Introduction

Various theoretical analysis of cavity perturbation techniques are suggested by many researchers. The measurements of $\varepsilon$, $\mu$ and $\sigma$ are performed by inserting a small, appropriately shaped sample into a cavity and determining the properties of the sample from the resultant change in the resonant frequency and loaded Q-factor.

Bethe and Schwinger [20] suggested the cavity perturbation theory for the first time. They considered the cases that the perturbation causes (1) by the insertion of a small dielectric sample into a cavity and (2) by a small deformation on the boundary surface of the cavity. It was further modified by Casimir [50] to include the determination of the magnetic property of a small sphere.

The fundamental idea of the cavity perturbation is that the change in the overall geometrical configuration of the electromagnetic fields upon the introduction of the sample must be small. Experimentally, this indicates that the percentage change in the real part of the resonant frequency must be small [55]. Based on this assumption, a detailed derivation of perturbation formula for the frequency shift upon the introduction of a sample into a cavity was given by Waldron [57] and Harrington [60].
The method of measurement of complex permittivity based on cavity perturbation technique was first developed by Birnbaum and Franeu [48]. The assumption that "the electric field in the perturbing sample is equal to the electric field of the empty cavity" was made in their calculation. In their measurements they employed a rectangular cavity operating in the TE$_{106}$ mode. A practical application of TM$_{010}$ mode cylindrical cavity was reported by Nakamura and Furuichi [58]. They measured the dielectric parameters of cylindrical shaped BaTiO$_3$ single crystal. This technique has also been used for the measurement of ionic conductivity of β-alumina [78].

Generally, in these techniques, the length of the sample is equal to the height of the cavity so that both ends of the sample are in contact with the cavity walls. In the theoretical analysis suggested by Parkash et al. [82], the sample did not contact the cavity wall. They introduced a set of formulae for calculating $\sigma$ and $\varepsilon$ based on the assumption that the sample acts like a dipole with an effective depolarizing factor.

For the measurements, small holes can be drilled in the cavity walls and the sample can then be inserted into the sample holder. In this arrangement, as there is no need to disassemble the cavity, errors due to the misalignment of the cavity can be minimised.

In the first part of this chapter, theoretical analysis for the measurement of complex permittivity, complex permeability and conductivity of materials with rectangular waveguide cavities are discussed in detail. Theoretical analysis for a coaxial resonator for complex permittivity measurement over wide range of frequencies is also discussed.
The second part of this chapter deals with the measurement of dielectric and magnetic parameters of materials. The measurement of conductivity of materials is also incorporated.

4.2 Theoretical analysis for the determination of complex permittivity of materials using rectangular waveguide cavity

When a small sample is inserted in a cavity which has the electric field $E_o$ and magnetic field $H_o$ in the unperturbed state, the fields in the interior of the object are $E$ and $H$. Beginning with Maxwell's equations, Bethe and Schwinger [20] obtained an expression for the resonant frequency shift. For a lossless sample, the variation of resonant frequency is given as by Harrington [60] as

$$\frac{\omega - \omega_o}{\omega} = -\frac{\int (\Delta \varepsilon \ E \cdot E_o^* + \Delta \mu \ H \cdot H_o^*) \ d\tau}{\int (\varepsilon \ E \cdot E_o^* + \mu \ H \cdot H_o^*) \ d\tau}$$  \hspace{1cm} (4.1)$$

where $\varepsilon$ and $\mu$ are the permittivity and permeability of the medium in the unperturbed cavity respectively. $d\tau$ is the elemental volume. $\Delta \varepsilon$ and $\Delta \mu$ are the changes in the above quantities due to the introduction of the sample in the cavity. Without affecting the generality of Maxwell's equations, the complex frequency shift due to a lossy sample in the cavity is given by Waldron [57]

$$-\frac{\delta \Omega}{\Omega} \approx -\frac{(\varepsilon_r - 1) \varepsilon_0}{V_s} \int_{V_s} E \cdot E_o^* \ dV + (\mu_r - 1) \mu_0 \int_{V_s} H \cdot H_o^* \ dV +$$

$$\frac{(D_o \cdot E_o^* + B_o \cdot H_o^*) \ dV}{V_c}$$  \hspace{1cm} (4.2)
Two approximations are made in applying equation (4.2), based on the assumptions that the fields in the empty part of the cavity are negligibly changed by the insertion of the sample, and that the fields in the sample are uniform over its volume. Both these assumptions can be considered valid if the object is sufficiently small relative to the resonant wavelength. The negative sign in equation (4.2) indicates that by introducing the sample the resonant frequency is lowered. Because the permittivity of practical materials is complex, the resonant frequency should also be considered as complex. In equation (4.2) $\delta \Omega$ is the complex frequency shift. $B_o$, $H_o$, $D_o$ and $E_o$ are the fields in the unperturbed cavity. $E$ and $H$ are the fields in the interior of the sample. $\bar{\varepsilon}_r = \varepsilon'_r - \varepsilon''_r$ and $\bar{\mu}_r = \mu'_r - \mu''_r$ and $V_c$ and $V_s$ are the volumes of the cavity and the sample respectively. In terms of energy, the numerator of equation (4.2) represents the energy stored in the sample and the denominator represents the total energy stored in the cavity. The total energy $W = W_c + W_m = 2W_c = 2W_m$. When a dielectric sample is introduced at the position of maximum electric field (Figure 4.1) only the first term in the numerator is significant, since a small change in $\varepsilon$ at a point of zero electric field or a small change in $\mu$ at a point of zero magnetic field does not change the resonance frequency. Thus equation (4.2) can be reduced to

$$\frac{-\delta \Omega}{\Omega} \approx \frac{\int_{V_s} (\bar{\varepsilon}_r - 1) E \cdot E_o^* \max dV}{\int_{V_c} 2 |E_o|^2 dV}$$

(4.3)
Figure 4.1 Sample positioned at the maximum electric field in the rectangular waveguide cavity
Let $Q_o$ be the quality factor of the cavity in the unperturbed condition and $Q_s$ the Q-factor of the cavity loaded with the object. The complex frequency shift is related to measurable quantities by [106],

$$\frac{\delta\Omega}{\Omega} \approx \frac{\delta\omega}{\omega} + \frac{j}{2} \left[ \frac{1}{Q_s} - \frac{1}{Q_o} \right]$$  \hspace{1cm} (4.4)

Equating the real and imaginary terms of equations (4.3) and (4.4) we get

$$- \frac{(f_s - f_o)}{f_s} = \frac{(\varepsilon'_r - 1) \int E.E_0^{* \text{ max}} \, dV}{V_s}$$

$$= \frac{2 \int |E_0|^2 \, dV}{V_C}$$  \hspace{1cm} (4.5)

$$\frac{1}{2} \left[ \frac{1}{Q_s} - \frac{1}{Q_o} \right] = \frac{\varepsilon''_r \int E.E_0^{* \text{ max}} \, dV}{V_s}$$

$$= \frac{2 \int |E_0|^2 \, dV}{V_C}$$  \hspace{1cm} (4.6)

We may assume that $E = E_o$ and the value of $E_o$ in $\text{TE}_{10}$ mode as $E_o = E_o^{\text{ max}} \sin (m\pi x/a) \sin (p\pi z/d)$ where $a$ is the broader dimension of the waveguide and $d$ the length of the cavity. Integrating and rearranging the above equations, we get

$$\varepsilon'_r - 1 = \frac{f_o - f_s}{2f_s} \left[ \frac{V_C}{V_s} \right]$$  \hspace{1cm} (4.7)
If the frequency shift is measured from the resonance frequency $f_t$ of the cavity loaded with empty capillary tube rather than that with empty cavity alone the above equations become.

$$\varepsilon''_r = \frac{V_C}{4V_s} \left[ \frac{1}{Q_s} - \frac{1}{Q_t} \right]$$ (4.8)

$$\varepsilon'_r - 1 = \frac{f_t - f_s}{2f_s} \left[ \frac{V_C}{V_s} \right]$$ (4.9)

$$\varepsilon''_r = \frac{V_C}{4V_s} \left[ \frac{1}{Q_s} - \frac{1}{Q_t} \right]$$ (4.10)

$Q_t$ is the quality factor of the cavity loaded with empty tube. $f_s$ and $Q_s$ are the resonance frequency and quality factor of cavity loaded with capillary tube containing the sample material.

### 4.3 Theory for the determination of conductivity of the materials

For a dielectric material having non-zero conductivity, we may write the Ampere’s law in phasor form as

$$\nabla \times \mathbf{H} = (\sigma + j \omega \varepsilon) \mathbf{E}$$

$$= (\sigma + \omega \varepsilon'') \mathbf{E} + j \omega \varepsilon' \mathbf{E}$$ (4.11)
where $\varepsilon = \varepsilon' - j\varepsilon''$ is the absolute permittivity of the medium. The loss tangent

$$
\tan \delta = \frac{\sigma + \omega \varepsilon''}{\omega \varepsilon'}
$$

(4.12)

$\sigma_e = \sigma + \omega \varepsilon''$, is the effective conductivity of the medium.

But

$$
\tan \delta = \frac{1}{Q_m} = \frac{1}{Q_s} - \frac{1}{Q_t}
$$

(4.13)

where $Q_m$ is the loaded Q-factor of the cavity with sample alone.

The effective conductivity

$$
\sigma_e = \frac{\omega \varepsilon'}{Q_m} = \frac{\omega \varepsilon' r}{Q_m} \frac{\varepsilon_0}{Q_m}
$$

(4.14a)

when $\sigma$ is very small, the effective conductivity is reduced to

$$
\sigma_e = \omega \varepsilon'' = 2\pi f \varepsilon_0 \varepsilon'' r
$$

(4.14b)

### 4.4 Theory for the determination of complex permeability of materials using rectangular waveguide cavity

When a small sample is introduced into the cavity resonator, it causes a frequency shift. If the sample is lossy, the complex frequency shift is given by Harrington [60].
\[
\frac{\omega_s - \omega_o}{\omega_s} = - \frac{\iiint (\Delta \varepsilon \mathbf{E}_o^* \mathbf{E}_o^* + \Delta \mu \mathbf{H}_o^* \mathbf{H}_o^*) \, dV}{\iiint (\mathbf{E}_o^* \mathbf{E}_o^* + \mu_o \mathbf{H}_o^* \mathbf{H}_o^*) \, dV}
\] (4.15)

\(E_o\), \(H_o\) and \(\omega_o\) represent the fields and resonant frequency of the original cavity. \(E, H, \omega\) represent the corresponding quantities of the perturbed cavity. \(\Delta \varepsilon\) and \(\Delta \mu\) are the changes in permittivity and permeability of the medium of the cavity due to the introduction of the sample. \(dV\) is the elemental volume. Equation (4.15) can be written in terms of energy stored as

\[
\frac{\omega_s - \omega_o}{\omega_s} = - \frac{1}{W} \iiint \left[ \frac{\Delta \varepsilon}{\varepsilon} \frac{W_e}{W} + \frac{\Delta \mu}{\mu} \frac{W_m}{W} \right] \, dV \quad (4.16)
\]

\(W\) is the sum of the electric and magnetic energies in the cavity. \(W_e\) and \(W_m\) are respectively the electrical and magnetic energies stored in the sample. A small change in \(\varepsilon\) at a point of zero \(E\) or a small change in \(\mu\) at a point of zero \(H\) does not change the resonant frequency. Thus when a magnetic material is introduced at the position of maximum \(H\) (Figure 4.2) equation (4.15) is reduced to

\[
\frac{\omega_s - \omega_o}{\omega_s} = - \frac{\iiint \Delta \mu \mathbf{H}_o^* \mathbf{H}_o^* \, dV}{V_s \iiint 2\mu_o |\mathbf{H}_o|^2 \, dV} \quad (4.17)
\]
Figure 4.2 Sample positioned at the maximum magnetic field in the rectangular waveguide cavity
\[ \Delta \mu = \mu - \mu_0 \] where \( \mu = \mu' - j\mu'' \) is the absolute complex permeability of the material. Thus equation (4.17) becomes

\[
\frac{\omega_s - \omega_0}{\omega_s} = - \frac{\int (\mu_r - 1) H_H^* \, dV}{2 \int |H_0|^2 \, dV}
\] (4.18)

The components of the magnetic field for TE_{10p} mode are

\[
H_x = - \frac{\beta a}{\pi} C \sin \frac{\pi x}{a} \sin \beta z
\] (4.19a)

\[
H_z = C \cos \frac{\pi x}{a} \cos \beta z
\] (4.19b)

The denominator of equation (4.18) becomes

\[
\int |H_0|^2 \, dV = \int H_x^2 \, dV + \int H_z^2 \, dV
\]

\[
= \int \int \int \frac{\beta^2 a^2}{\pi^2} C^2 \sin^2 \left[ \frac{\pi x}{a} \right] \sin^2(\beta z) \, dx \, dy \, dz
\]

\[
+ \int \int \int C^2 \cos^2 \left[ \frac{\pi x}{a} \right] \cos^2(\beta z) \, dx \, dy \, dz
\]

\[
= \frac{C^2 \, V_c}{4} \left[ \frac{\lambda^2 g + 4a^2}{\lambda^2 g} \right]
\] (4.20)

where \( V_c = abd \) is the volume of the cavity.
The numerator of equation (4.18) is reduced to
\[
(\mu_r - 1) \int \frac{\mathbf{H} \cdot \mathbf{H}^*}{\lambda} dV = \frac{4 \frac{c^2}{g} \frac{a^2}{v_s} (\mu_r - 1)}{\lambda^2 g} \quad (4.21)
\]

Substituting (4.20) and (4.21) in (4.18) we get
\[
\frac{\omega_s - \omega_o}{\omega_s} = - (\mu_r - 1) \frac{8a^2}{(\lambda^2 g + 4a^2)} \frac{v_s}{v_C} \quad (4.22)
\]

The real and imaginary parts of the complex frequency shifts are given by
\[
\frac{\omega_s - \omega_o}{\omega_s} = \frac{\delta \omega}{\omega_s} + \frac{j}{2} \left[ \frac{1}{Q_s} - \frac{1}{Q_o} \right] \quad (4.23)
\]

where $Q_s$ and $Q_o$ are the loaded quality factors of the cavity with and without the sample in the sample holder. From equations (4.22) and (4.23) we get
\[
(\mu_r' - 1) = \left[ \frac{\lambda^2 g + 4a^2}{8a^2} \right] \frac{f_o - f_s}{f_s} \frac{v_C}{v_s} \quad (4.24)
\]
\[
\mu_r'' = \left[ \frac{\lambda^2 g + 4a^2}{16a^2} \right] \frac{v_C}{v_s} \left[ \frac{1}{Q_s} - \frac{1}{Q_o} \right] \quad (4.25)
\]
f_s and f_0 are the resonant frequencies of the cavity with and without the sample in the sample holder. \( \lambda_g \) is the guided wavelength.

For TE_{10p} mode,

\[
\lambda_g = \frac{2d}{n}
\]  

(4.26)

\( n = 1, 2, 3, \ldots \)

4.5 Theory for the determination of complex permittivity of materials using coaxial cavity resonator

The coaxial transmission line resonator consists of a circular waveguide which operates below cut-off for the TM_{01} mode. Along the axis of the waveguide there is a removable centre conductor (Figure 4.3). Thus the TEM mode can propagate up to the end of the centre conductor.

The standing wave field components of the resonant TEM mode are obtained by combining the forward and backward propagating waves [107] as

\[
E_{\rho S}^\circ = \frac{A}{\rho} e^{j\beta z} + \frac{B}{\rho} e^{-j\beta z}
\]  

(4.27)

\[
H_{\phi S}^\circ = \frac{-A}{\omega \rho} e^{j\beta z} + \frac{B}{\omega \rho} e^{-j\beta z}
\]  

(4.28)

where \( \omega \) is the free space impedance and A and B are constants. The boundary conditions require that \( E_{\rho S}^\circ \) must vanish at \( z = 0 \) and \( z = L \), where L is the length of the centre conductor. The first condition gives \( A = -B \), while the second gives the resonance condition.
Figure 4.3 Cross-sectional view of the coaxial transmission line resonator
Now the equations (4.27) and (4.28) become

\[
E_{jS}^* = \frac{A}{\mu \rho} \sin \beta z = E_{o \max} \sin \beta z \quad (4.29)
\]

\[
H_{jS}^* = \frac{A}{\epsilon \rho} \cos \beta z = H_{o \max} \cos \beta z \quad (4.30)
\]

The electrical energy stored in the cavity

\[
w_e = \frac{\epsilon_o}{2} \int_{V_c} E_{jS}^* \quad (4.31)
\]

On substituting the value of \( E_{jS}^* \) from (4.29) in (4.31) we get

\[
w_e = \frac{\epsilon_o}{2} \int_{0}^{2\pi} \int_{0}^{L} \int_{a}^{b} E_{o \max}^2 \sin^2 \beta z \quad (4.32)
\]

where \( b \) and \( a \) are the outer and inner radii respectively of the coaxial resonator.

When the sample is introduced into the cavity, the relative complex frequency shift is given by Waldron [57] as

\[
-\frac{\delta \Omega}{\Omega} \approx \frac{1}{V_s} \int_{0}^{V_s} \left( \frac{\epsilon_{\max}}{E_{o \max}} + (\epsilon_{\max} - 1) \frac{\epsilon_{\max}}{E_{o \max}} + \frac{\epsilon_{\max}}{E_{o \max}} \right) dV
\]

\[
\int_{V_c} \left( (D_{o \max} + B_{o \max}) \right) dV
\]

\[
(4.33)
\]
The numerator of equation (4.33) represents the energy stored in the sample and the denominator represents the total energy stored in the cavity.

Here

\[
\left( D_{0} \cdot E_{0}^{*}_{\text{max}} + B_{0} \cdot H_{0}^{*}_{\text{max}} \right) dV = 4 w_{e} \quad (4.34)
\]

When the dielectric sample is introduced at the position of maximum electric field, the relative frequency shift is given by

\[
\frac{\delta \Omega}{\Omega} \approx \frac{(\varepsilon_{r} - 1) \varepsilon_{0} \int_{V_{s}} E_{pS}^{0} \cdot E_{0}^{*}_{\text{max}} dV}{4 w_{e}} \quad (4.35)
\]

Substituting for \( w_{e} \) equation (4.35) becomes

\[
\frac{\delta \Omega}{\Omega} \approx \frac{(\varepsilon_{r} - 1) \int_{V_{s}} E_{pS}^{0} \cdot E_{0}^{*}_{\text{max}} dV}{2 \pi b L} \frac{2}{2} \int_{0}^{2 \pi} \int_{0}^{L} E_{0}^{2}_{\text{max}} \sin^{2} \beta z \rho d\rho d\phi dz \]

\[
\approx \frac{(\varepsilon_{r} - 1) V_{s}}{2 \pi \frac{(b^{2} - a^{2})}{2} L} \]

\[
\approx \frac{(\varepsilon_{r} - 1)V_{s}}{\pi L (b^{2} - a^{2})} \quad (4.36)
\]
Here \( \mathbf{E}_{ps}^0 = \mathbf{E}_0^0 \) because the field inside the sample is assumed to be equal to the field in the cavity. The volume of the sample \( V_s = \pi r^2 (b-a) \) where \( r \) is the radius of the sample. Also \( \varepsilon_r = \varepsilon_r' - j\varepsilon_r'' \). Then equation (4.36) becomes

\[
- \frac{\delta \Omega}{\Omega} \approx \frac{(\varepsilon_r' - 1)r^2}{L(b+a)} - \frac{j \varepsilon_r'' r^2}{L(b+a)}
\]

i.e.,

\[
- \frac{\delta \Omega}{\Omega} \approx \frac{-(\varepsilon_r' - 1)r^2}{L(b+a)} + \frac{j \varepsilon_r'' r^2}{L(b+a)} \quad (4.37)
\]

Also

\[
- \frac{\delta \Omega}{\Omega} \approx \frac{(f_s - f_0)}{f_s} + \frac{j}{2} \left[ \frac{1}{Q_s} - \frac{1}{Q_0} \right] \quad (4.38)
\]

From (4.37) and (4.38) we can write

\[
(\varepsilon_r' - 1) = \frac{L(b+a)}{r^2} \frac{(f_0 - f_s)}{f_s} \quad (4.39)
\]

\[
\varepsilon_r'' = \frac{L(b+a)}{2r^2} \left[ \frac{1}{Q_s} - \frac{1}{Q_0} \right] \quad (4.40)
\]

If the frequency shift is measured from the resonance frequency \( f_1 \) of the cavity loaded with empty tube rather than that with empty cavity alone, the above equations become
\[ (\varepsilon'_{r} - 1) = \frac{L (b + a)}{r^2} \frac{(f_{t} - f_{s})}{f_{s}} \quad (4.41) \]

\[ \varepsilon''_{r} = \frac{L (b + a)}{2r^2} \left[ \frac{1}{Q_{t}} - \frac{1}{Q_{s}} \right] \quad (4.42) \]

\( Q_{t} \) be the quality factor of the cavity loaded with empty tube. \( f_{s} \) and \( Q_{s} \) be the resonance frequency and quality factor of the cavity loaded with capillary tube containing the sample respectively.

4.6 Measurement of complex permittivity, complex permeability and conductivity of materials using rectangular waveguide cavities

The rectangular waveguide cavity resonator is connected to the two parts of the HP 8514B S-parameter test-set of the measuring system. It is operated in the TE_{10p} mode. The network analyzer generates 801 discrete frequencies within a range of 200 MHz scanning the resonant frequency of the cavity. This allowed measurement of the transmission through the cavity in increments of 20 kHz by reading the co-ordinates of a marker on the CRT display of network analyzer. With the help of HP 9000/300 series instrumentation computer, the measurements at different resonant frequencies can be carried out automatically for a single resonator. Separate computer programmes (HP Basic) are developed for measurement and calculation procedures. The block diagram of the experimental set-up is shown in Figure 4.4.
Figure 4.4 Block diagram of the experimental set-up
The material whose permittivity is to be measured is taken in a capillary tube. The capillary tube is made up of low-loss fused silica. The capillary tube containing the sample material (liquid/powder) is introduced into the cavity through the non-radiating slot on the broad wall of cavity (Figure 4.5). This non-radiating slot has little effect on the Q-factor of the cavity. Since the cavity is operated in the \( \text{TE}_{10p} \) mode it has a number of resonant frequencies. The positions of maximum electric field will be different for different resonant frequencies. With the help of movable sample chamber, the tube can be positioned at the maximum electric field corresponding to each resonance peak of the cavity. Due to the introduction of the sample, the resonance frequency lowers from \( f_t \) to \( f_s \) and Q-factor lowers from \( Q_t \) to \( Q_s \). Figure 4.6 shows the amplitude response of the cavity in both cases.

The procedure for the determination of complex permittivity is described as follows.

1. The resonance frequency \( f_t \) and unloaded Q-factor \( Q_t \) of the cavity resonator are measured with the empty capillary tube inserted in the cavity at the position of maximum electric field.

2. The sample material is filled in the capillary tube. It is positioned at maximum electric field. The resonance frequency \( f_s \) and loaded quality factor \( Q_s \) are measured.

3. Knowing the diameter of the sample tube, and feeding the series instrumentation computer with proper HP Basic Programme, conductivity \( \sigma_r \) and \( \varepsilon''_r \) of the material can be computed.

It is noted that the characteristics of the cavity loaded with empty tube is same as that of empty cavity alone because of negligibly small wall thickness of tube and low-loss nature of material of the tube.
Figure 4.5 Schematic diagram of the rectangular waveguide cavity resonator (side view)
Figure 4.6 Amplitude response of the rectangular waveguide cavity.  
(a) Unloaded cavity,  (b) Cavity loaded with sample.
The procedure for the determination of complex permeability is as follows.

1. The resonant frequency $f_0$ and unloaded quality factor $Q_0$ of the rectangular cavity are measured with empty capillary tube inserted at the position of maximum magnetic field.

2. The sample material is filled in the capillary tube. It is positioned at the maximum magnetic field (Figure 4.7).

3. Knowing the diameter and length of the sample tube, $\mu'_r$ and $\mu''_r$ can be computed with the help of series instrumentation computer.

The measurement of the resonant frequency and Q-factor are performed by using the slow scan technique. The sweep rate of frequency is kept slow enough (about 0.01 MHz/Sec.) to avoid errors in the measurements of the properties of materials due to uncertainty principle in swept frequency [68].

Any misalignment in the positioning of the tube in the cavity can be avoided by using copper plugs fitted with sample chamber (Figure 3.4).

If the sample tube does not terminate at the end walls of the cavity but extends beyond, an additional correction may be made in the equations because of the electric field perturbations at the end walls. Since the correction is almost negligible to begin with, no appreciable change in accuracy in the measurements is expected because of this effect.
Figure 4.7 Experimental set-up for the measurement of complex permeability
The diameter of the sample tube is small enough that the cavity characteristics are rigorously applicable and large enough that the observed effects on the resonant frequency shift as well as the changes in Q-factor of the cavity are easily measurable. In addition we require that the Q-factor of the cavity loaded with sample in the capillary tube is sufficiently large that the resonance is still well defined. With these criteria in mind, sample tubes of different diameters are selected for different materials (polar, non-polar etc.) to enhance the sensitivity in measurement. The sizes are chosen to give an approximate frequency shift of 0.0015 GHz and a decrease in the Q-factor of the cavity from \( Q_0 \) by about a factor of 10 to 15%. The following chart gives an idea of the sizes of the capillary tube for the measurements for each cavity.

<table>
<thead>
<tr>
<th>Type of cavity</th>
<th>Dimensions of the tube (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outer Diameter</td>
</tr>
<tr>
<td>S-Band</td>
<td>1.0, 0.9, 0.8</td>
</tr>
<tr>
<td>C-Band</td>
<td>0.7, 0.6, 0.5</td>
</tr>
<tr>
<td>X-Band</td>
<td>0.3, 0.2, 0.1</td>
</tr>
</tbody>
</table>

The significant error that usually occurs in the complex permittivity/permeability measurements is due to the non-uniformity of cross-section of the tube. It causes error in the value of \( V_s \). So the diameter of the tube should be accurately measured at different cross-sections of the tube and average value is taken.

For the measurement of the complex permittivity and conductivity of vapours, the method described in the previous section is adopted. The experimental arrangement for this case is shown in Figure 4.8. Care has been taken that the meniscus of the liquid in the tube should be well below the cavity so that it will not enter the cavity field region due to expansion during heating.
Figure 4.8 Experimental set-up for the measurement of complex permittivity of vapours
4.7 **Measurement of complex permittivity of materials using coaxial cavity resonators**

Coaxial transmission line resonator is a reflection type cavity i.e., the microwave power is coupled into or out of the cavity through the same coupling loop. The resonator is connected to one port of HP 8514B S-parameter test. With the help of HP 8510B Network analyzer and HP 9000/300 series instrumentation computer, the resonance frequency and Q-factor can be measured automatically.

The procedure for the measurement of complex permittivity is described in the following steps.

1. The empty capillary tube introduced into the cavity at the position of maximum electric field. The resonant frequency $f_1$ and Q-factor $Q_1$ are measured and recorded in computer.

2. The capillary tube is filled with the sample material and its both ends are sealed. It is introduced into the cavity. The resonant frequency $f_s$ and Q-factor $Q_s$ are measured and recorded in computer.

3. The feeding of the values of inner radius of capillary tube ($r$), inner radius of cavity ($b$), radius of the centre conductor ($a$) and length of the centre conductor ($L$) in the computer furnishes the values of $\varepsilon'_r$ and $\varepsilon''_r$ of the material.