CHAPTER 1

INTRODUCTION

1.1 BASIC CONCEPTS IN GRAPH THEORY

A graph $G$ is a pair $(V, E)$, where $V$ is a non-empty set whose elements are called vertices of $G$ and $E$ is a set of 2-element subsets of $V$, whose elements are called edges of $G$. The sets $V$ and $E$ are the vertex set and edge set of $G$, respectively. We write $V(G)$ and $E(G)$ rather than $V$ and $E$ to emphasize that these are the vertex and edge sets of a particular graph $G$.

If $e = \{u, v\}$ is an edge, it can be written as $e = uv$ and $e$ is said to join the vertices $u$ and $v$; $u$ and $v$ are called adjacent vertices; $u$ and $v$ are said to be incident with $e$. If two vertices are not joined by an edge, then they are said to be nonadjacent. If two distinct edges are incident with a common vertex, then they are said to be adjacent with each other.

Definition 1.1.2. The number of elements in the vertex set of a graph is called the order of $G$ and is denoted by $p$. The number of elements in the edge set of a graph is called the size of $G$ and is denoted by $q$. A graph with $p$ vertices and $q$ edges is called $G\ (p, q)$-graph.

Definition 1.1.3. Two graphs $G = (V\ (G), E\ (G))$ and $H = (V\ (H), E\ (H))$ are identical if $V\ (G) = V\ (H)$ and $E\ (G) = E\ (H)$. $G$ and $H$ are isomorphic if there exist a bijective function $\phi: V\ (G) \rightarrow V\ (H)$ such that $uv$ is an edge of $G$ if $\phi (u)\phi (v)$ is an edge of $H$. Then $G \cong H$. 
Definition 1.1.4. A graph $H$ is called a subgraph of a graph $G$ if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. A spanning subgraph of $G$ is a sub graph $H$ with $V(H) = V(G)$.

For any subset $S$ of vertices of $G$, the induced sub graph $<S>$ is the maximal sub graph of $G$ with vertex set $S$. Therefore two vertices of $S$ are adjacent in $<S>$ if and only if they are adjacent in $G$.

Definition 1.1.5. The degree of a vertex $v$ (denoted $d(v)$) is the number of edges incident with $v$. The minimum degree of a graph $G$ is denoted by $\delta(G)$ and maximum degree is denoted by $\Delta(G)$. A vertex of degree $0$ (i.e. a vertex which is not joined to any other vertex) is said to be isolated vertex. A vertex of degree $1$ is said to be pendant vertex.

Definition 1.1.6. A path is a sequence of distinct vertices $(v_1, v_2, \ldots, v_n)$ such that consecutive vertices are adjacent.

Definition 1.1.7. A graph is said to be connected if there is a path between every pair of distinct vertices of the graph.

Definition 1.1.8. A directed graph or digraph is a graph such that each edge is directed. A directed edge is an edge such that one vertex incident with it, is designated as the head vertex and the other incident vertex is designated as the tail vertex. A directed edge $uv$ is said to be directed from its tail $u$ to its head $v$.

Definition 1.1.9. The indegree of a vertex $v \in V(G)$ counts the number of edges such that $v$ is the head of those edges. The outdegree of a vertex $v \in V(G)$ is the number of edges such that $v$ is the tail of those edges.
Definition 1.1.10. A **bipartite** graph is one whose vertex set can be partitioned into two subsets $X$ and $Y$ so that each edge has one end in $X$ and the other end in $Y$; such a partition $(X, Y)$ is called a bipartition of the graph. It is also called a bigraph.

Definition 1.1.11. The **complement** $G^c$ of a simple graph $G$ is the simple graph with the same vertex set $V(G)$, two vertices being adjacent in $G^c$ if and only if they are not adjacent in $G$.

Definition 1.1.12. For $k \geq 2$, a graph $G$ is a **$k$-partite graph** or **Multipartite** if $V(G)$ can be partitioned into $k$ non-empty subsets $V_1, V_2, \ldots, V_k$ such that no edge of $G$ joins vertices in the same set. The sets $V_1, V_2, \ldots, V_k$ are called partite sets of $G$. If $G$ is a $k$-partite graph (multipartite) having partite sets $V_1, V_2, \ldots, V_k$ such that every vertex of $V_i$ is joined to every vertex of $V_j$ where $1 \leq i, j \leq k$ and $i \neq j$, then $G$ is called a complete $k$-partite graph or complete multipartite.

Definition 1.1.13. An acyclic graph is called a **forest**. The components of a forest are trees.

Definition 1.1.14. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be any two graphs. Then their union $G_1 \cup G_2$ is the graph whose vertex set is $V_1 \cup V_2$ and edge set is $E_1 \cup E_2$.

Definition 1.1.15. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be any two graphs. Then their **intersection** $G_1 \cap G_2$ is the graph whose vertex set is $V_1 \cap V_2$ and edge set is $E_1 \cap E_2$.

Definition 1.1.16. The **join** $G_1 \vee G_2$ of two disjoint graphs $G_1$ and $G_2$ is the graph obtained from $G_1 + G_2$ by joining each vertex of $G_1$ to each vertex of $G_2$. For example, the join of the cycle graph $C_{n-1}$ with a single vertex graph is the wheel graph $W_n$. 
Definition 1.1.17. If \( G = (V, E) \) is any graph with at least two vertices, then the vertex deletion subgraph is the subgraph obtained from \( G \) by deleting a vertex \( v \in V \) and also all the edges incident to that vertex. The vertex deletion subgraph of \( G \) is sometimes denoted \( G - \{v\} \).

Definition 1.1.18. If \( G = (V, E) \) is any graph with at least one edge, then the edge deletion subgraph is the subgraph obtained from \( G \) by deleting an edge \( e \in E \), but not the vertices incident to that edge. The edge deletion subgraph of \( G \) is sometimes denoted \( G - \{e\} \).

Definition 1.1.19. Let \( G_1(V_1, E_1) \) and \( G_2(V_2, E_2) \) be two graphs. The Cartesian product \( G_1 \times G_2 \) of two graphs \( G_1 \) and \( G_2 \) is defined as \( G(V, E) \), whose vertex set \( V = V_1 \times V_2 \) and the edge set \( E \) is defined as follows if \( w_1 = (u_1, v_1) \) and \( w_2 = (u_2, v_2) \) be two typical vertices of \( G \) with \( u_i \in V_1 \) and \( v_j \in V_2 \) then \( w_1w_2 \in E(G) \) iff either (a) \( u_1 = u_2 \) and \( v_1, v_2 \in E_2 \) or (b) \( v_1 = v_2 \) and \( u_1, u_2 \in E_1 \).

Definition 1.1.20. (Wallis, 2001) A \((n, t)\)-kite graph consists of a cycle of length \( n \) with a \( t \)-edge path (the tail) attached to one vertex. We write its labeling as the list of labels for the cycle (ending on the attachment point); separated by a semicolon from the list of labels for the path (starting at the vertex nearest the cycle).

Definition 1.1.21. Let \( K_n = (V, E) \) be the complete graph with \( n \) vertices \( V(K_n) = \{1, 2, \ldots, n\} \) and \( n(n - 1)/2 \) edges \( E(K_n) = \{(i, j): 1 \leq i, j \leq n \text{ and } i < j\} \).

Definition 1.1.22. (Baskar Babujee, 2003) The class of planar graphs with maximal edges over \( n \) vertices derived from complete graph \( K_n \) is defined as \( Pl_n = (V, E) \), where \( V = \{1, 2, \ldots, n\} \) and \( E = E(K_n) \backslash \{(k, l): k = 3 \text{ to } n-2, l = k+2 \text{ to } n\} \). The number of edges in \( Pl_n \): \( n \geq 5 \) is \( 3(n-2) \). The construction of \( Pl_n \) is shown in the following Figure 1.1.
There are exactly two vertices with degree 3, two vertices with degree \((n - 1)\) and all other vertices with degree 4.

**Definition 1.1.23.** A circulant graph \(C_{\mathcal{I}}(a_1, a_2, \ldots, a_k)\) is a regular graph whose set of vertices is \(V = \{v_0, v_1, \ldots, v_{n-1}\}\) and whose set of edges is \(E = \{(v_i, v_{i+a_j}) \mod n): i = 0, 1, 2, \ldots, n-1; j = 1, 2, \ldots, k\}\). If \(a_k < \left\lfloor \frac{n}{2} \right\rfloor\) then \(C_{\mathcal{I}}(a_1, a_2, \ldots, a_k)\) is a 2\(k\)-regular graph; if \(a_k = \left\lfloor \frac{n}{2} \right\rfloor\) then \(C_{\mathcal{I}}(a_1, a_2, \ldots, a_k)\) is a \((2k-1)\)-regular one.

**Definition 1.1.24.** A connected graph with at least one cut vertex is called a separable graph. The *cycle-cactus* is a connected separable graph in which every block is a cycle. The cycle-cactus \(C_k^{(n)}\), consists of \(n\) copies of \(C_k\), concatenated exactly at one vertex. The vertex at which all the cycles are concurrent is called the apex vertex.
Definition 1.1.25. (Acharya, 2008) A Shell $S_n$ of width $n$ is the graph obtained by taking $(n - 3)$ concurrent chords in a cycle $C_n$ on $n$ vertices. The vertex at which all the chords are concurrent is called the apex. Often the shell $S_n$ is called a fan and is denoted by $F_{n-1}$.

The Shell-cactus is a connected separable graph in which every block is a Shell $S_n$. The Shell-cactus $S^{(n)}_k$, consists of $n$ copies of $S_k$, concatenated exactly at one vertex. The vertex at which all the shells are concurrent is called the apex vertex.

Definition 1.1.26. (Selvaraju and Nirmala, 2009) The Structured Web Graph $S_n(P_m \times C_n)$ defined by star graph with $n$ pendent vertices is inserted at the center of $P_m \times C_n$ and has $|V| = n(m - 1) + 1$ vertex and has $|E| = 2n(m - 1)$ edges.

All the graphs studied in our thesis are finite and simple graphs without loops and multiple edges.
1.2 LABELING RELATED CONCEPTS

A labeling or a valuation of a graph is a map that carries the graph elements to the set of numbers, usually to the positive integers subject to certain conditions. If the domain is the set of vertices it is called as vertex labeling and if it is the set of edges, then the labeling is said to be edge labeling. If the labels are assigned to both vertices and edges of a graph, such labeling is called total labeling. Graph labeling was first introduced by Rosa (1967) as a way of attacking the problem of cyclically decomposing the complete graph into trees. Since then, many different types of graph labeling problems have been defined. This is not only due to its mathematical importance but also because of its wide range of the applications in areas like x-rays, crystallography, coding theory, cryptography, radar, astronomy, circuit design and communication design etc. An enormous body of literature has grown around this subject in the last fifty years and they have given birth to graph families like graceful graph, harmonious graph, felicitous graph, elegant graph, cordial graph, magic graph, antimagic graph, bimagic graph, prime graph, prime cordial graph and many more. A detailed survey about different kinds of graph labeling is given by Gallian (2011).

The thesis deals with $L(2,1)$ labeling, prime cordial labeling, magic type labeling and an application of graph labeling. A brief survey of these type of labeling are given in the following section.

1.2.1 $L(2, 1)$ Labeling and $L(3, 2, 1)$ Labeling

In ordinary graph coloring, adjacent vertices must be given different color, and the actual values of the colors used are irrelevant. However, in many applications, it is also important to separate labels on vertices at farther distances, where the labels used have some numerical meaning. A natural problem of this type is the channel assignment problem,
where channels (non-negative integers) are assigned to each radio transmitter (vertex) so that interfering (adjacent) transmitters get channels that are far apart. Hale (1980) introduced the graph theory model of the channel assignment problem where it was represented as a vertex coloring problem. F.S. Roberts proposed a variation of the channel assignment problem, which Griggs and Yeh introduced in (1992) and called the $L(2, 1)$ labeling problem. Keeping the radio transmitter analogy in mind, vertices in a graph need to be labeled such that “close” vertices (at distance two) get different labels while “very close” vertices (at distance one) get labels that are farther apart. More precisely, for a given graph $G$, a mapping $f: V(G) \to N \cup \{0\}$ is called an $L(2, 1)$ labeling (Distance two) if $|f(u) - f(v)| \geq 2$ for each edge $uv$ of $G$ and $|f(u) - f(v)| \geq 1$ for each pair $u, v \in V(G)$ at distance two apart. The $L(2, 1)$-labeling number of $G$ or span of $G$, denoted by $\lambda(G)$ is the smallest number $k$ such that $G$ has an $L(2, 1)$ labeling that does not use any label greater than $k$.

For general graphs, the research was focused mainly on $L(2, 1)$ labeling because of their practical applications. Another reason is the conjecture of Griggs and Yeh (1992) which assumes that $\lambda_{2,1}(G) \leq \Delta^2$ for every graph $G$ with maximum degree $\Delta \geq 2$. This conjecture was verified for several special classes of graphs, including graphs of maximum degree two, outer planar graphs by Calamoneri and Petreschi (2001), Hamiltonian cubic graphs (Kang, 2004), chordal graphs and unit interval graphs (Sakai, 1994). Bodlaender et al (2000) given a wide range of survey including algorithms, complexity and applications to communication networks. However, the core of the conjecture remains wide open even for 3-regular graphs. For general graphs, the original bound $\lambda_{2,1}(G) \leq \Delta^2 + 2\Delta$ of Griggs and Yeh (1992) was improved as $\lambda_{2,1}(G) \leq \Delta^2 + \Delta$ by Chang and Kuo (1996). A more general result of Karl and Skrekovski (2003) yields $\lambda_{2,1}(G) \leq \Delta^2 + \Delta - 1$ and the best
known bound of $\lambda_{2,1}(G) \leq \Delta^2 + \Delta - 2$ was given by Goncalves (2005). The illustration for $L(2, 1)$ labeling of $P_4 \times P_4$ is shown in the Figure 1.3.

![Figure 1.3 L(2, 1) labeling for P_4xP_4 with \lambda = 6](image)

$L(3, 2, 1)$ labeling naturally extends from $L(2, 1)$ labeling, taking consideration on vertices which are within a distance of three apart. Jia-zhuang and Zhen-dong (2004) introduced an $L(3, 2, 1)$ labeling of a graph $G$ as a function $f$ from the vertex set $V(G)$ to the set of positive integers such that for any two vertices $x, y \in E$, $|f(x) - f(y)| \geq 3$ if $d(x, y) = 1$; $|f(x) - f(y)| \geq 2$ if $d(x, y) = 2$ and $|f(x) - f(y)| \geq 1$ if $d(x, y) = 3$. The $L(3, 2, 1)$ labeling number $\lambda(G)$ or span of $G$ is the smallest positive integer $k$ such that $G$ has an $L(3, 2, 1)$ labeling with $k$ as the maximum label. $L(3, 2, 1)$ labeling number for paths, cycles, caterpillars, $n$-ary trees, complete graphs and complete bipartite graphs was determined and its upper bound of labeling using maximum degree was found in Jean Clipperton et al (2006).

### 1.2.2 Prime Cordial Labeling

Many Computer vision problems can be formulated as labeling problems. Applications of binary labeling problems include two-region image segmentation, shape de noising and 3D reconstruction. One such edge binary labeling problem is prime cordial labeling problem. The notion of prime
labeling originated with Entringer and was introduced in a paper by Tout et al (1982). A graph with vertex set $V$ is said to have a prime labeling if there exist a bijection $f : V(G) \rightarrow \{1, 2, \ldots, |V|\}$ such that for each edge $xy \in E(G)$, $f(x)$ and $f(y)$ are relatively prime. The illustration for prime labeling of double star $K_{1,6,6}$ is given in Figure 1.4.

![Figure 1.4 Prime labeling for $K_{1,6,6}$](image)

Cahit (1987) has introduced a variation of both graceful and harmonious labelings. Let $f$ be a function from the vertices of $G$ to $\{0, 1\}$ and for each edge $xy$ assign the label $\left| f(x) - f(y) \right|$. Call $f$ a cordial labeling of $G$ if the number of vertices labeled 0 and the number of vertices labeled 1 differs by at most 1, and the number of edges labeled 0 and the number of edges labeled 1 differs at most by 1.

Sundaram et al (2005) has introduced the concept called prime cordial labeling. A prime cordial labeling of a graph $G$ with vertex set $V$ is a bijection $f$ from $V$ to $\{1, 2, \ldots, |V|\}$ such that for each edge $uv \in E$

$$f(uv) = \begin{cases} 1 & \text{if } \gcd(f(u), f(v)) = 1 \\ 0 & \text{if } \gcd(f(u), f(v)) > 1 \end{cases}$$

and $|e(0) - e(1)| \leq 1$ where $e(0)$ is the number of edges labeled with 0 and $e(1)$ is the number of edges labeled with 1. Also they have exhibited the existence
of prime cordial labeling for certain kinds of graphs like $C_n$ if and only if $n \geq 6$; $P_n$ if and only if $n \neq 3$ or 5; $K_{1,n}$ (n odd); the graph obtained by subdividing each edge of $K_{1,n}$ if and only if $n \geq 3$; bistars; dragons; crowns; triangular snakes if and only if the snake has at least three triangles; ladders.

Baskar Babujee and Shobana (2009) have exhibited the prime cordial labeling for sun graph, coconut tree, $(n, t)$ kite graph for $n \geq 6$ and $t = n - 3$. Also prime cordial labeling for $(S_n^{(1)}:S_n^{(2)})$, full binary trees from second level, $K_2 \Theta C_n(C_n)$ and $K_{1,n,n}$ for $n \geq 3$ have been found by Baskar Babujee and Shobana (2010). The illustration for prime cordial labeling of Wheel graph $W_8$ is shown in the Figure 1.5.

![Figure 1.5 Prime cordial labeling for $W_8$](image)

Vaidya and Vihol (2010), Vaidya and Shah (2011) proved that the square graph of path $P_n$ is a prime cordial graph for $n = 6$ and $n \geq 8$ while the square graph of cycle $C_n$ is a prime cordial graph for $n \geq 10$. Also they show that the shadow graph of $K_{1,n}$ for $n \geq 4$ and the shadow graph of $B_n$ are prime cordial graphs. Certain cycle related graphs and the graphs obtained by mutual duplication of a pair of edges as well as mutual duplication of a pair of vertices from each of two copies of cycle $C_n$ admit prime cordial labeling. The prime cordial labeling of generalized Petersen graph was proved by Haque (2010).
**Theorem 1.2.1.** (Wiselet and Nicholas, 2010) In any binary edge labeling the following are equivalent.

i) \( |e(0) - e(1)| \leq 1 \)

ii) \( e(1) = \begin{cases} \left\lfloor \frac{q}{2} \right\rfloor & \text{if } q \text{ is even} \\ \left\lfloor \frac{q}{2} \right\rfloor \text{ or } \left\lceil \frac{q}{2} \right\rceil - 1 & \text{if } q \text{ is odd} \end{cases} \)

where \( \left\lfloor x \right\rfloor \) denotes the smallest integer greater than or equal to \( x \).

iii) \( \sum_{j=1}^{q} e_j + (q \mod 2) e_d = \left\lfloor \frac{q}{2} \right\rfloor \) where \( e_d \) is the binary label of a dummy edge which is introduced only when \( q \) is odd.

**1.2.3 Magic Type Labelings**

Sedláček (1963) introduced the magic labeling for a graph \( G = G(V, E) \) which is defined as a bijection \( f \) from \( E \) to a set of positive integers such that

(i) \( f(e_i) \neq f(e_j) \) for all distinct \( e_i, e_j \in E \), and

(ii) \( \sum_{e \in N(x)} f(e) \) is the same for every \( x \in V \), where \( N(x) \) is the set of edges incident to \( x \).

The original concept of total edge magic is due to Kotzig and Rosa (1970). A \( G(p, q) \) graph with \( p \) vertices and \( q \) edges is called total edge magic if there is a bijection \( f: V \cup E \rightarrow \{1, 2, \ldots, p+q\} \) such that for every edge \( uv \) in \( E \), \( f(u) + f(uv) + f(v) \) is a constant \( k \). A total edge magic graph is called a...
super edge magic if \( f(V(G)) = \{1, 2, \ldots, p\} \). Wallis (2001) called super edge magic as strongly edge magic. Super edge magic labeling for \( K_{1,8} \) is shown in below Figure 1.6.

![Figure 1.6 Super edge magic labeling for \( K_{1,8} \) with \( k = 20 \)](image)

Hartsfield and Ringel (1990) introduced the concept of antimagic graph. An *edge antimagic* total labeling of a graph with \( p \) vertices and \( q \) edges is a bijection \( f \) from the set of edges \( E \) to \( \{1, 2, \ldots, p+q\} \) such that for any edge \( uv \in E \), \( f(u) + f(uv) + f(v) \) all are distinct. A total edge antimagic graph is called *super edge antimagic* if \( f(V(G)) = \{1, 2, \ldots, p\} \).

It becomes an interesting problem when one arrives at exactly two distinct constants \( k_1 \) or \( k_2 \) for \( f(u) + f(uv) + f(v) \). Edge bimagic total labeling was introduced by Baskar Babujee (2004) and studied the same for \( B_{n,n} \) and \( K_{1,n} \) graph(Baskar Babujee, 2004a). A graph \( G(p, q) \) with \( p \) vertices and \( q \) edges is called *total edge bimagic* if there exists a bijection \( f : V \cup E \rightarrow \{1, 2, \ldots, p+q\} \) such that for any edge \( uv \in E \), we have two constants \( k_1 \) and \( k_2 \) with \( f(u) + f(v) + f(uv) = k_1 \) or \( k_2 \). A total edge bimagic graph is called *super edge bimagic* if \( f(V(G)) = \{1, 2, \ldots, p\} \). Super edge bimagic total labeling of \( P_7 \) is given in Figure 1.7.
Figure 1.7 Super edge bimagic total labeling of $P_7$ with $k_1 = 16$, $k_2 = 21$

Super edge bimagic labeling was introduced by Baskar Babujee and Jagadesh (2008) and proved the same for the Path $P_n$, Star $K_{1,n}$, the double star $K_{1,n,n}$, Hoffman tree $P_n \odot K_1$, $\{B_{m,n} : 2\}(m,n \geq 1)$, $\{K_{1,n} : 3\}(n \geq 3)$. Further Super edge bimagic labeling have been proved for some class of graphs derived from fundamental graphs (Baskar Babujee and Jagadesh, 2008a), certain classes of disconnected graphs (Baskar Babujee and Jagadesh, 2008b) and cycle related graphs (Baskar Babujee and Jagadesh, 2008c).

Mirka Miller et al (2003) defined the 1-vertex magic vertex labeling of a graph with $p$ vertices as a bijection $f$ taking the vertices to the integers $\{1, 2, ..., p\}$ with the property that there is a constant $k$ such that at any vertex $x$, $\sum_{y \in N(x)} f(y) = k$, where $N(x)$ is the set of vertices adjacent to $x$.

Lemma 1.2.2 (Mirka Millar et.al, 2003) If $f$ is 1-vertex magic vertex labeling then $\sum_{x \in V} d(x) f(x) = kp$ where $d(x)$ is the degree of vertex $x$.

A 1-vertex magic vertex labeling is same as sigma labeling or $\Sigma$-labeling which was introduced by Vilfred (1994). Swamininathan and Jeyanthi (2007) introduced and exhibited $(a,d)$-1-vertex antimagic vertex labeling for certain graphs. It becomes an interesting problem when one arrives at exactly two distinct constants $k_1$ or $k_2$ for $\sum_{y \in N(x)} f(y)$. This motivated us to introduce and work in 1-vertex bimagic vertex labeling.
1.3 CHAPTER ORGANIZATION

The thesis is organized into six chapters as follows:

**Chapter 1** is the introductory chapter in which the basic concepts of graph theory are outlined. Also in light of our work, the definitions, the literature survey of graph labeling and some results regarding them have been listed.

In **Chapter 2**, distance two labeling and span value has been obtained for planar graph with maximum edges $Pl_n$, circulant graph $Ci_n(1, 2)$ and $G \times P_m$ graphs. Also $L(3, 2, 1)$ labeling is studied for $G \times P_m$ graphs and upper bound span is calculated.

In **Chapter 3** some characterization results and new construction related to prime cordial graphs are studied. Further prime cordial labeling for $(n, t)$ kite graph, $C_n(P_m)$ ($n \geq 4$), cycle-cactus $C_k^{(a)}$ ($n > 3$), shell-cactus $S_k^{(a)}$ ($n > 3$ and $k > 4$), $P_n(P_m)$ ($n \geq 4$), $\langle S_n (P_2 \times C_n) : K_2 \rangle$, $m$-star $K_{1,n,n,...,n}$ for $(n, m > 2)$ are obtained. Also, the duality of the prime cordial graphs is studied.

In **Chapter 4**, 1-vertex bimagic vertex labeling has been introduced and the same has been exhibited for certain classes of graphs. Also new construction on edge bimagic and edge anti magic are given.

In **Chapter 5**, we apply labeling techniques and encrypt as well as decrypt numbers using the concept of residue class $Z_3$ using the complete graph $K_n$.

In **Chapter 6**, concluding remarks of the over all work along with few open problems and future research work to be done are given.