

APPENDIX 1

Texture Spectrum

In a square raster digital image each pixel is surrounded by eight neighboring pixels. The local texture information for a pixel can be extracted from a neighborhood of 3 X 3 pixels, which represents the smallest complete unit (in the sense of having eight directions surrounding the pixel). Given a neighborhood of 3 x 3 pixels (which will be denoted by a set containing nine elements: \( V = \{ V_o, V_1, \ldots, V_8 \} \), where \( V_o \) represents the intensity value of the central pixel and \( V_i \) (where \( i = 1, 2, \ldots, 8 \)) is the intensity value of the neighboring pixel \( i \)), we define the corresponding texture unit by a set containing eight elements, \( TU = \{ E_1, E_2, \ldots, E_8 \} \), where \( E_i \) (where \( i = 1, 2, \ldots, 8 \)) is determined by the formula:

\[
E_i = \begin{cases} 
0 & \text{if } V_i < V_o \\
1 & \text{if } V_i = V_o \\
2 & \text{if } V_i > V_o 
\end{cases}
\]

for \( i = 1, 2, \ldots, 8 \) and the element \( E \), occupies the same position as the pixel \( I \). As each element of \( TU \) has one of three possible values, the combination of all eight elements results in \( 3^8 = 6561 \) possible texture units in total.
**B1. Labeling Texture Units**

There is no unique way to label and order the 6561 texture units. In our study, the 6561 texture units are labeled by using the following formula:

\[
N_{TU} = \sum_{i=1}^{8} E_i \cdot 3^{i-1}
\]  

where \(N_{TU}\) represents the texture unit number and \(E_i\) is the \(i^{th}\) element of texture unit set \(TU=\{E_1, E_2, \ldots, E_8\}\). In addition, the eight elements may be ordered differently. If the eight elements are ordered clockwise as shown in Fig. 2, the first element may take eight possible positions from the top left \((a)\) to the middle left \((h)\), and then the 6561 texture units can be labeled by the above formula under eight different ordering ways (from \(a\) to \(h\)). Fig. 3 gives an example of transforming a neighborhood to a texture unit with the texture unit number under the ordering way \(a\).

**B2. Texture Spectrum**

The previously defined set of 6561 texture units describes the local-texture aspect of a given pixel; that is, the relative grey-level relationships between the central pixel and its neighbors. Thus the statistics of the frequency of occurrence of all the texture units over a large region of an image should reveal texture information. We termed the texture spectrum the frequency distribution of all the texture units, with the abscissa indicating the texture unit number \(N_{TU}\) and the ordinate representing its occurrence frequency.

In practice, a real texture image is usually composed of two parts: Texture elements and random noise or background. The greater the proportion of texture components compared to the background, the better that texture can
be perceived by human vision. In the texture spectrum the increase in percentage of texture components in an image will result in a tendency to form a particular distribution of peaks. In addition, different textures are composed of particular texture units with different distributions in their texture spectra. In this way the texture of an image can be characterized by its texture spectrum.

It should be noted that the labeling method chosen might affect the relative positions of the texture units in the texture spectrum, but will not change their frequency values in the latter. It should be also noted that the local texture for a given pixel and its neighborhood is characterized by the corresponding texture unit, while the texture aspect for a uniform texture image is revealed by its texture spectrum calculated within an appropriate window. The size of the window depends on the nature of the texture image.

![Figure A1.1](image.png)

**Figure A1.1** Eight clockwise, successive ordering ways of the eight elements of the texture unit

Let $V = [40, 63, 28, 45, 35, 21, 40, 67, 88]$ Then, $TU = [2, 0, 2, 0, 0, 1, 2, 2]$ Texture Unit Number $= 6005$
Figure A1.2 Sample Textures

Figure A1.3 Texture Spectrum
Figure A1.4 Test Images

Figure A1.5 Texture Spectrum -1
Figure A1.6 Texture Spectrum -2

Figure A1.7 Texture Spectrum -3
Figure A1.8 Texture Spectrum -4

Figure A1.9 Texture Spectrum -5
Figure A1.10 Texture Spectrum -6

Figure A1.11 Texture Spectrum -7
Figure A1.12 Texture Spectrum -8

Figure A1.13 Texture Spectrum -9
Figure A1.14 Texture Spectrum -10

Figure A1.15 Texture Spectrum -11
Figure A1.16 Texture Spectrum -12

Figure A1.17 Texture Spectrum -13
Figure A1.18 Texture Spectrum -14

Figure A1.19 Texture Spectrum -15
Figure A1.20 Texture Spectrum -16

Figure A1.21 Texture Spectrum -17
Figure A1.24 Texture Spectrum -20
APPENDIX 2

MASKS

VS, RBS, LG and JR

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Figure A2.1 Output of texture identification and texture defect detection Masks
APPENDIX 3

A3.1 FRACTAL DIMENSION

Fractal geometry can be used to discriminate between textures. The word “fractal” is really more of an adjective than a noun, and it refers to entities (especially sets of pixels) that display a degree of self-similarity at different scales. A mathematical straight line displays a high degree of self-similarity any portion of the line is the same as any other at any magnification.

The fractal dimension $D$ of a set of pixels $I$ is specified by the relationship.

$$I = N r^D \quad (A3.1)$$

where the image $I$ has been broken up into $N$ nonoverlapping copies of a basic shape, each one scaled by a factor of $r$ from the original. Equation A 3.1 can be rewritten as

$$D = \frac{\log N}{\log \left( \frac{1}{r} \right)} \quad (A3.2)$$

From this it can be seen that there is a log-log relationship between $N$ and $r$. If log $(N)$ were plotted against log $(r)$ the result should be a straight line whose slope is approximately $D$. 
A3.2 STEPS TO COMPUTE HURST COEFFICIENT

1. The first step in computing the Hurst Coefficient is to determine the maximum grey level difference for each distance class of pixels.

2. A line to be fit to this data using a log-log relationship, so the next step is to take the log of both the distance and the grey-level difference.

3. A straight line is fit to the points, using a least-squares approach.

4. The slope of this line is the Hurst coefficient.