Chapter 3

3. MIMO CHANNEL CAPACITY

The theoretical aspects of the MIMO channels has been presented and discussed in the previous chapter. This chapter deals with results and discussions of Deterministic and Stochastic capacity of MIMO channels. The MIMO channels are also compared with correlation properties and XPD (Cross Polar Discrimination on the channel) factor. Different number of transmit and receive antenna selection strategies are considered for independent Rayleigh fading channels environment with channel state Information, Random channel nature, Average power allocation in sub channels Rician channels conditions and Co-allocation systems (CAS) conditions.

3.1. Literature Survey:

Survey is done to find the current state of art for MIMO channel capacity with existing methods as well as building novel methods with different assumptions and approaches. Analytical Expression for MIMO Channel Capacity is analyzed [1]. The Capacity results and analysis of channel parameters for MIMO measured channels is discussed in [2]. Capacity of multi-antenna Gaussian channels reported in [3]. The fading environments and its limits with Multiple Antenna systems discussed in [4]. The Fundamentals of MIMO channel Capacity is Explained in [5]. An over view about capacity of the MIMO channel presented in [6]. Clear analysis of Ergodic and Outage capacity for Narrowband MIMO Gaussian Channels is given [7]. Capacity scaling in MIMO wireless systems under correlated fading proposed [8]. Hence the simulation of various channel models, which describes MIMO channel capacity is done through mat lab, so web link [9] was very much help full to get more details. Impact of Antenna Spacing on the Capacity of Correlated MIMO Fading Channels is presented from [10]. Fading correlation and its effect on the capacity of multi-element antenna systems is discussed [11].
3.2. The MIMO Channel Capacity approaches:

The present work mainly focuses on two channel approaches for analyzing MIMO channel capacity viz.,

1. **Deterministic channel Capacity models**: Based on information theory;
   The Channel capacity measures the maximum amount of information that could be transmitted through a channel and received with negligible error. Here we analyse Ergodic, Outage capacity, using Singular Value Decomposition (SVD) / Water filling technique approaches.

2. **Stochastic channel capacity models**:

   Stochastic channel models can be split into two categories:

   a. **Correlation based models** such as the independent and identically distributed (i.i.d) model. This supposes that multi path channels in presence of scatterers are independent and uniformly distributed in all directions.

   b. **The Kronecker MIMO channel model**: Kronecker model assumes that spatial transmit correlation and spatial receive correlation are separable. Therefore, the full channel correlation matrix can be modelled by the Kronecker product of transmit and receive correlation matrix.

The channel approaches mentioned above indicate that the capacity gain obtained from multiple antennas, heavily depends on the amount of channel knowledge at either the receiver or transmitter, the channel SNR, and the correlation between the channel gains on each antenna element. MIMO uses multiple antennas at the transmitter and receiver to enable a variety of signal paths to carry the data, selecting separate paths for each antenna. The signal can take many paths, and get affected by fading.
As a result of the use multiple antennas, MIMO wireless technology is able to considerably increase the capacity of a given channel, while still obeying Shannon’s law. From the mathematical point of view, the MIMO communication is performed through a matrix called \textit{channel matrix} (H).

Some analytical expressions derived for the MIMO channel are Ergodic and outage capacity. These expressions are very complex, which limit their practical use. For instance, the Ergodic capacity can be obtained for Rayleigh MIMO channels using the integral of Eigen values of the channel matrix. A Gaussian approximation (AWGN) to the MIMO channel capacity with transmit and receive antennas can also be analyzed. Since the MIMO system performance depends on both the channel correlation properties and average receive signal to Noise Ratio (SNR), it is important and convenient to properly normalize the channel matrix for correct interpretation of results.

3.3. MIMO channel capacity plots with AWGN environment:

The first and foremost attempt is computing MIMO channel capacity in AWGN environment by number of iterations. The proposed technique has been simulated on MATLAB, considering AWGN model with transmit and receive antenna selection as \(N_t = [1,2,3,4]\) and \(N_r = [1,2,3,4]\) and generate plot shown from Figure. 3.1. This figure clearly shows the linear increment of capacity with average values of Signal to Noise Ratio (SNR). Around 27 bits/s/Hz of channel capacity is resulted in \(4 \times 4\) antenna configuration. Figure 3.2 shows the Three Dimensional (3D) plot for MIMO based AWGN channel. It shows the capacity value of around 1000 - 1200 bits/s/Hz at an average value of Signal to Noise Ratio (SNR) of around 30 dB (Not in negative side of SNR). One interesting property of MIMO wireless system is clearly depicted in this plot. That is despite of other values of SNR and capacity the Bandwidth used for the system shows the same value (i.e. from 0-10,000 Hz). The Band width of the system is properly utilized.
Figure 3.1: Capacity plot with Average SNR for AWGN model.

Figure 3.2: Capacity plot with Average SNR and Bandwidth for AWGN model (3D View).

In reality, transmission is always corrupted by noise whatever may be the type of channel assumed. AWGN is a very good model for the physical reality as long as only thermal noise is the source of disturbance. One must keep in mind that AWGN model is a mathematical function, because it implies that total power (all frequencies) is infinite.

The next section deals with deterministic approach and stochastic approaches with different fading conditions. Even Different antenna placing techniques Co-allocation Antenna Systems (CAS) based techniques are also developed in this proposed work.

3.4. The MIMO System Model (Deterministic approach):

Approximately all transmission schemes utilize the channel capacity as much as possible. Representing the input and output of a memory less wireless channel with the random variables X and Y respectively, the deterministic channel capacity is defined as

\[ C = \max_{p(x)} I(X;Y) \]  \hspace{1cm} (3.1)

Where \( I(X;Y) \) represents the mutual information between \( X \) and \( Y \) and states that the mutual information is maximized with respect to all possible transmitter statistical distributions \( p(x) \). Mutual information is a measure of the amount of information that one random variable contains about another variable.

The mutual information between \( X \) and \( Y \) can also be written as

\[ I(X;Y) = H(Y) - H(Y\mid X) \]  \hspace{1cm} (3.2)

Where \( H(Y \mid X) \) represents the conditional entropy between the random variables \( X \) and \( Y \). The entropy of a random variable can be described as a measure of the amount of information required on average to describe the random variable. It can also describe as a measure of the uncertainty of the random variable. Due to Equation (3.2), mutual information can be
described as the reduction in the uncertainty of one random variable due to the knowledge of the other. Note that the mutual information between X and Y depends on the properties of the channel (through a channel matrix $H$) and the properties of X (through the probability distribution of X).

![Diagram](image)

**Figure. 3.3: The MIMO System (Channel) Model.**

According to Figure 3.3, the channel matrix $H$ used in the representation of the input/output relations of a MIMO channel is defined in the next section. It is common to represent the input/output relations of a narrowband, single-user MIMO link by the complex base band vector notations as

$$ y = Hx + n $$  \hspace{1cm} (3.3)

Where $x$ is the $(N_T \times 1)$ transmit vector, $y$ is the $(N_R \times 1)$ receive vector, $H$ is the $(N_T \times N_R)$ channel matrix, and $n$ is the $(N_R \times 1)$ AWGN vector at a given instant of time.

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3.5. Theoretical Analysis of MIMO Channel Capacity (Deterministic approach): 4

The capacity of a MIMO channel with power constraint \( P_T \) can be expressed as

\[
C = EH \left\{ \max_{P(w) \in \mathbb{R}^{N_T \times N_T}} I(X;Y) \right\}
\]  
(3.4)

Where \( Q = E \{xx^\dagger\}^2 \) is the covariance matrix of the transmit signal vector \( x \). The total transmit power is limited to \( P_T \), irrespective of the number of transmit antennas.

When the transmitter has no knowledge about the channel, it is optimal to use a uniform power distribution. The transmit covariance matrix is then given by

\[
Q = \frac{2\sigma^2}{N_T} \mathbf{I}
\]  
(3.5)

The capacity of a channel depends completely on the channel realization, noise, and transmitted signal power. The achievable capacity of MIMO systems is

\[
C = \log_2 \left( \det \left[ I + \frac{\sigma^2}{N_T} HH^* \right] \right)
\]  
(3.6)

Where \( \sigma \) is the average SNR per receive antenna caused by thermal noise at the antenna, \( ^* \) denotes transpose conjugate and \( I \) denoted the identity matrix. To ensure a fair comparison of capacity, the total power of the complex transmitted signals is constrained to \( P \), regardless of the number of transmit antennas. In the case of \( N_T \) number of transmitter antennas and \( N_R \) number of receiver antennas, the matrix of channel is defined by \( N_T \times N_R \) rank. Capacity grows linearly with \( m=\min(N_T, N_R) \), rather than logarithmically.

Important Definitions for **Deterministic approach**:

a. **Ergodic Capacity** is a long-term (Ergodic) behaviour of MIMO channel achieved by coding over large number of blocks determines the average achievable throughput.

b. **Outage capacity** is a short-term behaviour of MIMO channel Achieved by coding within one fading interval, determines the probability of failure in a delay-limited context Channel statistics determine the system performance.

### 3.5.1. Ergodic Capacity with CSIT and CSIR. (Constant channel MIMO capacity)

When the channel is constant and known perfectly at the transmitter and the receiver, then the MIMO channel can be converted to parallel, non-interfering single input single output (SISO) channels through a SVD of the channel matrix. Water filling the transmit power over these parallel channels whose gains are given by the singular values of the channel matrix “landas” leads to the power allocation.

The Ergodic Capacity with CSIT and CSIR is given by

$$ C = E_{H} \left\{ \max_{Q \in \mathcal{Q}} \log \left[ \det \left( I_{N_{R}} + H Q H^{H} \right) \right] \right\} $$

The optimum value Q is obtained by water filling (power filling) for each channel realization of H. Although the constant channel model is relatively easy to analyze, wireless channels in practice typically change over time due to multipath fading.
3.5.2. Fading in MIMO Channel Capacity:  

With slow fading, the channel may remain approximately constant long enough to allow reliable estimation at the receiver (perfect CSIR) and timely feedback of the channel state to the transmitter (perfect CSIT). However, in systems with moderate to high user mobility, the system designer is inevitably faced with channels that change rapidly. Fading models with partial CSIT or CSIR are more applicable to such channels. Capacity results under various assumptions regarding CSI are summarized in this section.

3.5.3. Capacity with No CSIT and Perfect CSIR:

Assuming perfect CSIR and no CSIT the mutual information with \( Q = \frac{SNR}{n_T} I_{n_T} \) is given by

\[
l = \log \left[ \det \left( I_{n_R} + \frac{SNR}{n_T} H H^H \right) \right]
\]  

(3.8)

Here \( P_{\text{out}} \) (outage Probability) = 1.

3.5.4. Ergodic Capacity with Channel Distribution Information Transmitter CDIT and CSIR:

The Ergodic capacity of MIMO fading channel with CDIT and CSIR is given by

\[
C = \max_{Q \in \mathcal{Q}} \mathbb{E}_H \log \left[ \det \left( I_{n_R} + H Q H^H \right) \right]
\]  

(3.9)

---

3.5.5. Ergodic Capacity without CSIT and CDIT for Gaussian fading conditions:

The Ergodic capacity of MIMO Gaussian fading with CDIT and CSIR is given by

\[
C = \max_{\mathbf{Q}, \mathbf{r}, \mathbf{r}_2} \mathbb{E}_H \log \left( \det \left( \mathbf{I}_N + \mathbf{S}_{\text{R}} \frac{\mathbf{H}}{N_0} \mathbf{H}^H \right) \right)
\]

(3.10)

The capacity gain is highly dependent on the multipath richness in the radio channel, since a fully correlated MIMO channel only offers one sub channel, while a completely de-correlated channel offers multiple sub channels, depending on the antenna configuration, the physical models are assumed partially correlated/de-correlated channels, since that is the case in practice.

3.5.6. Adaptive Transmit Power Allocation (Using Water Filling technique):

For the case when the CSI is known at the transmitter, the capacity can be increased by “water-filling” method.

\[
P_i = \left( \mu - \sigma^2 / \lambda_i \right)^+, \text{and } \ell = 1, 2, \ldots, T_0
\]

(3.11)

Where \( \mu \) is chosen to meet the power constraint so that

\[
\sum_{i=1}^{T_0} P_i = P
\]

Thus the received signal power at the \( i^{th} \) sub channel, gives the relation

\[
P_{ri} = (\lambda_i \mu - \sigma^2)^+, \text{ where } '+' \text{ denotes max (a,0) and } a = (\mu - \sigma^2 / \lambda_i).
\]
Thus the channel capacity is given by

\[ C = \sum_{i=1}^{r} \left[ \ln \left( \frac{\lambda_i}{\sigma^2} \right) \right]^{+} \]  

(3.12)

Where \( \sigma \) is the average SNR per receive antenna caused by thermal noise at the antenna.

3.6. Simulation Results for MIMO capacity in Rayleigh fading environment

In this work a highly scattered environment (Rayleigh fading) is considered. The Capacity of a MIMO channel with \( N_t \) transmit antennas and \( N_r \) receive antennas are analyzed. Figure 3.4. Shows, the simulated plot of theoretical Shannon’s capacity relation for SISO system with other different Transmit and Receiving antenna selections (\( 2 \times 2, 3 \times 3, 4 \times 4 \)). Comparing to SISO antenna combination, MIMO antenna combinations with \( 2 \times 2 \) results with more channel capacity value. Table 3.1 shows all antenna selection results. There is a linear increase in capacity observed from the plot.

Function (Capacity Power_Allocation) = Water_Filling_alg (Total Power (A, Ch A, B, N))  

(3.13)

The main parameters of the function are total power, bandwidth, sub channels (channel A) and number of bits (\( N_0 \)).

The MIMO Rayleigh channel capacity results are shown in Figure. 3.5 with respect to different antenna array sizes have been presented in table 3.1. \( 2 \times 2, 3 \times 3 \) configurations results lower values of MIMO capacity results at 10-20 dB SNR. The capacity results for 10 dB SNR versus 20 dB SNR shows that generally the channel capacity is not too far away from the Rayleigh distribution case of 21.66 bits /s /Hz. some locations suffering from the keyhole effect is reported for \( 4 \times 4 \) antenna configuration.
Figure 3.4: The MIMO Capacity (Vs Shannon’s) plot for Rayleigh environment.

Figure 3.5: MIMO capacity with SVD in Rayleigh conditions \( N_T = [1, 2, 3, 2, 4], N_R = [1, 2, 2, 3, 4] \)

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**Keyhole effect:** A system with transmit and receive antennas surrounded by scatterers. These antennas are uncorrelated. If the channel were a Rayleigh i.i.d. channel, this would have yielded a channel matrix of full rank and size as $N_R \times N_T$.

Some observations from these simulation results are:

- Lower capacity results as Transmitter antennas are brought closer.
- Simulated capacity is a large fraction of capacity of Rayleigh distribution channel.
- Percentage of scattering with respect to Rayleigh distribution will vary from 80-90% from different antenna array sizes.

### TABLE 3.1
COMPARISON OF RAYLEIGH CHANNEL CAPACITY RESULTS AT 10-20 dB SNR.

<table>
<thead>
<tr>
<th>Antenna Array Size</th>
<th>Simulated Channel Capacity (bps /Hz at 10 dB Rayleigh distribution SNR)</th>
<th>Simulated Channel Capacity (bps /Hz at Rayleigh distribution at 20 dB SNR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Tx × 1 Rx</td>
<td>3.46</td>
<td>7.46</td>
</tr>
<tr>
<td>2 Tx × 2 Rx</td>
<td>8.8</td>
<td>11.88</td>
</tr>
<tr>
<td>3 Tx × 2 Rx</td>
<td>6.2</td>
<td>13.89</td>
</tr>
<tr>
<td>2 Tx × 3 Rx</td>
<td>6.2</td>
<td>13.89</td>
</tr>
<tr>
<td>4 Tx × 4 Rx</td>
<td>11.49</td>
<td>21.66</td>
</tr>
</tbody>
</table>

TABLE 3.1 shows the comparative analysis of Rayleigh channel capacity results at 10-20 dB SNR. It shows enhancement of Rayleigh distribution is an advantage with respect to different MIMO antenna selections.
Figure 3.6: Number of Transmitting & Receiving antenna vs. PDF plot.

Figure 3.6 shows Probability Density Function (PDF) of elements in a channel matrix with respect to all Eigen values Vs SNR using Number of Transmitting & Receiving antennas in a Rayleigh-Lognormal fading channel. Here the curve distribution shows more Eigen values generated for channel elements for 4 × 4 antenna selection than 2 × 2 and SISO. This analysis clearly gives the idea of linear increment of capacity with higher number of channel elements, generally assumed with RANK if a matrix. The MIMO channel with respect to different types of RANKS is discussed in the last section of this chapter.

MIMO Capacity plot for various antenna selections with CDI / CSI is shown Figure 3.7. MIMO Capacity with both CDI and CSI approaches clearly depicts the linear increment in MIMO channel capacities. The 6 × 6 antenna sizes can results in a capacity of around 30-40 bits / s / Hz at 20 dB. The different antenna array sizes also results increase in capacity, but not as good as same numbers of antennas at both transmitter and Receiver.

(By comparing 1 × 12 and 3 × 4 cases)
Comparing to only normal scattering (Rayleigh) environments, The CDI/CSI conditions for independent channels gives best MIMO channel capacities given in table 3.2.

<table>
<thead>
<tr>
<th>Antenna Array Size</th>
<th>Simulated Channel Capacity (bps /Hz at Rayleigh distribution at 20 dB SNR)</th>
<th>Simulated Channel Capacity For independent Channels In CDI / CSI conditions (bps /Hz at 20 dB SNR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Tx × 1 Rx</td>
<td>7.46</td>
<td>5.46</td>
</tr>
<tr>
<td>2 Tx × 2 Rx</td>
<td>11.88</td>
<td>13.5</td>
</tr>
<tr>
<td>3 Tx × 3 Rx</td>
<td>13.89</td>
<td>20.0</td>
</tr>
<tr>
<td>4 Tx × 4 Rx</td>
<td>21.66</td>
<td>26.5</td>
</tr>
</tbody>
</table>

Figure 3.7. MIMO Capacity plot for various antenna selections with both CDI/CSI.
SVD can be used to decompose the MIMO channel matrix $H$ into a set of equivalent SISO channels. The advantage of this is that the values of the diagonal matrix $D$ determine the number of independent parallel channels available in the channel $H$. This is given by the number of non-zero Eigen values; each of these gives the rank of that particular sub channel. Also the values obtained from the orthogonal matrices of the SVD gives the gains of the independent channels. These can be used to find weightings for the transmitting and receiving antennas. This creates beam forming as seen earlier and greatly increases the system performance.

The MIMO Capacity plot for various antenna selections using with CSIR in Rayleigh fading conditions is shown in Figure 3.8. In both Transmitter and Receiver antenna cases perfect CSIR is assumed. The individual channels are assumed to be spatially correlated. Figure 3.8. shows MIMO Capacity plot for various antenna selections with CSIR. Figure 3.9 shows
simulation results of the power allocation in a highly scattered Rayleigh environment channels with SVD.

![Power Allocation](image)

Figure 3.9. Power allocation for different MIMO antenna selection in a temporary correlated Rayleigh conditions.

The Capacity of a MIMO channel with Number of $N_t$ transmit antennas and $N_r$ receive antennas is analyzed. The power in all parallel channels (after % decomposition) is distributed according to water-filling algorithm that uses SVD decomposition with highest degree of correlation factor. The simulation results shown in figure 3.8 & figure 3.9 reveals the capacity of the channel, which is dominated by the free-space separation of transmit and receive regions from the scattering environment. It has been designed to calculate the Ergodic and Outage capacity of a MIMO Rayleigh channel considering without any CSIT and only perfect CSIR. The channel is assumed to be spatially correlated according to a Kronecker model but temporally uncorrelated. The analysis is mentioned for water filling and Equal power allocation in various sub channels are presented in tables 3.3 and table 3.4.
### TABLE 3.3
ERGODIC AND OUTAGE CAPACITY COMPUTATION
FOR WATER FILLING LEVELS (DIFFERENT) WITHOUT CSIT AND PERFECT CSIR

<table>
<thead>
<tr>
<th>No of Transmitting Antennas</th>
<th>No of Receiving Antennas</th>
<th>Ergodic capacity (Bits / s / Hz)</th>
<th>Outage Capacity (Bits / s / Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1.</td>
<td>5.8891</td>
<td>3.6074</td>
</tr>
<tr>
<td>2.</td>
<td>2.</td>
<td>7.7838</td>
<td>5.3881</td>
</tr>
<tr>
<td>3.</td>
<td>3.</td>
<td>8.9503</td>
<td>6.4661</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
<td>9.7489</td>
<td>7.2732</td>
</tr>
<tr>
<td>5.</td>
<td>5.</td>
<td>10.4095</td>
<td>8.0027</td>
</tr>
<tr>
<td>6.</td>
<td>6.</td>
<td>10.9344</td>
<td>8.5270</td>
</tr>
<tr>
<td>7.</td>
<td>7.</td>
<td>11.3584</td>
<td>8.8816</td>
</tr>
<tr>
<td>8.</td>
<td>8.</td>
<td>11.7740</td>
<td>9.3318</td>
</tr>
</tbody>
</table>

### TABLE 3.4
ERGODIC AND OUTAGE CAPACITY COMPUTATION FOR EQUAL POWER LEVELS WITH CSIT AND CSIR

<table>
<thead>
<tr>
<th>No of Transmitting Antennas</th>
<th>No of Receiving Antennas</th>
<th>Ergodic capacity (Bits / s / Hz)</th>
<th>Outage Capacity (Bits / s / Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1.</td>
<td>5.9586</td>
<td>3.6345</td>
</tr>
<tr>
<td>2.</td>
<td>2.</td>
<td>7.8118</td>
<td>5.5018</td>
</tr>
<tr>
<td>3.</td>
<td>3.</td>
<td>8.9446</td>
<td>6.090</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
<td>9.7756</td>
<td>7.2891</td>
</tr>
<tr>
<td>5.</td>
<td>5.</td>
<td>10.4215</td>
<td>7.9792</td>
</tr>
<tr>
<td>6.</td>
<td>6.</td>
<td>10.9419</td>
<td>8.4538</td>
</tr>
<tr>
<td>7.</td>
<td>7.</td>
<td>11.3895</td>
<td>8.9472</td>
</tr>
<tr>
<td>8.</td>
<td>8.</td>
<td>11.7707</td>
<td>9.2821</td>
</tr>
</tbody>
</table>

---

From Table 3.3, the Ergodic capacity values are more than Outage capacity due to channel state assumed perfectly at Receiver only. Table 3.4 shows Ergodic and Outage capacity values with Both Transmitter channel status and Receiver channel status is perfectly known. Only outage capacity results lesser values because of its short term behaviour property.

3.6.1. Ergodic Capacity in random MIMO channel:

Let us first assume that $H$ is a Gaussian random matrix whose realization is known at the receiver, or equivalently, that the channel output consists of the pair $(y, H)$. The input power is distributed equally over all transmitting antennas. Assuming a block fading model, then it is known that the Ergodic capacity of a random MIMO channel is given by:

$$
C = E_H \left\{ \log_2 \left[ \text{det} \left( I_N + \frac{\sigma}{N_H H} \right) \right] \right\}
$$

(3.13)

Where $E_H$ denotes that the expectation is taken with respect to the ensemble statistics of $H$, which are Gaussian distributed in this case, i.e., $H \sim N \left( 0, I_N \times I_N \right)$, and hence the envelopes are Rayleigh distributed. The closed form is given by (for unequal number of transmit/receive antennas)

$$
C = N \left( \log_2 (\sigma + 1) \right)
$$

(3.14)

Where $\sigma$ is the average SNR per receive antenna caused by thermal noise at the antenna.

3.6.2. Outage Capacity in random MIMO Channel:

Another measure of channel capacity is the outage capacity associated with an outage probability

$$
P_{\text{out}} = \inf \left\{ Q : Q \ni 0, P \left( \text{det} \left( I_N + H Q H^T \right) \leq C_{\text{outage}} \right) \right\}
$$

$$
\text{Tr}(Q) \leq 0
$$

(3.15)
Where $Q$ is covariance matrix and Let positive semi-definite $Q$ with the power constraint is $\text{tr} \ (Q) \leq \ P$, (trace operation) and the probability is taken over all realizations of the random matrix $H$. $P_{\text{out}}$, is the cumulative distribution function (CDF), of the outage probability, is related to CCDF by $P_{\text{out}} = 1 - \text{CCDF}$.

The narrowband Rayleigh MIMO channel capacity can be accurately approximated by Gaussian approximation, for the case when the receiver has the perfect CSI but the transmitter does not (equal power allocation). The implication of this result is that only mean and variance of the capacity are needed to get an accurate approximation of the outage capacity.

From (3.15), the instantaneous channel capacity is given by

$$C_{\text{inst}} = \sum_{i=1}^{N} \ln \left( 1 + \frac{P}{\sigma^2} \lambda_i \right)$$

(3.16)

Where $\lambda_i$ is the Eigen value matrix and also $i^{\text{th}}$ Eigen mode of $HH^*$. 

Figure 3.10: MIMO Channel Model shows pipes with different Eigen values but equal power levels

Figure 3.10 shows $m = \min (N_R, N_T)$ parallel channels, with equal power allocated to each pipe. If the channel is known at the transmitter then it is possible to direct the power into the stronger Eigen modes. In this case the capacity equation becomes:
\[ C = \sum_{i=1}^{m} \log_2 \left[ 1 + \frac{p_i \lambda_i}{\sigma^2} \right] \]  

(3.17.a)

Where the power distribution over “Pipes” are given by a water filling solution according to Figure 3.11. Where pipes with different power levels and different Eigen values, then Total power equation becomes:

\[ P_i = \sum_{i=1}^{m} p_i \]  

(3.17.b)

- **Figure 3.11:** Different Power distribution among the different Pipes with different Eigen values.

For the case when the channel matrix is equal to the identity matrix (the case of orthogonal parallel channels) all the Eigen modes are equal and the capacity increases linearly with the number of antennas as per Figure 3.12.

\[ C = \sum_{i=1}^{m} \log_2 \left[ 1 + \frac{P_T}{\sigma^2 N_t} \lambda_i \right] = \min(N_t N_R) \cdot \log_2 (1 + P_T/\sigma^2 N_t) \]  

(3.17 (c))
Figure 3.12: MIMO Channel Model shows Equal pipes with same Eigen values ($\hat{A}_1$).

3.6.3. Ergodic / Outage MIMO Channel Capacity Vs CCDF for a random channel:

The Channel cumulative distribution function (CCDF) of the capacity for the random MIMO channel when CSI is not available at the transmitter side. Figure 3.13 shows the CDFs of the random $2 \times 2$ and $4 \times 4$ MIMO channel capacities when SNR is 10dB, in which 10% outage capacity is indicated. It is clear from Figure 3.14 that the MIMO channel capacity improves with increasing the number of transmit and receive antennas. The same process is repeated for $8 \times 8$ and $10 \times 10$ antenna selection.

For Ergodic channels, the Ergodic capacity is equal to the Shannon capacity and is viewed as an important performance measure. On the other hand, for non-Ergodic channels, the Ergodic capacity has no physical significance and instead the proper performance measure is the outage probability which is actually the cumulative distribution function (CDF) of the Mutual Information (MI). In either Ergodic or non-Ergodic channels, the statistical distribution of the MI is fairly useful in obtaining corresponding performance measures, and more importantly it provides more comprehensive view about MIMO fading channels than the Ergodic capacity.

The observe the two different 10% outage capacity (from the CDF point 0.1) points from the two plots from the figures 3.13 and 3.14; there will be 5-7 bps /Hz capacity increment. It clearly indicates that CCDF is a very important property in the random channel.
Figure 3.13: MIMO Model showing 10 % outage capacity points for $2 \times 2$ & $4 \times 4$ Antenna selection.

Figure 3.14: MIMO Model showing 10 % outage capacity points for $8 \times 8$ & $10 \times 10$ Antenna selection.
3.6.4. The outage capacity for Rayleigh channel using Monte-Carlo Simulation Set-up:

The numerical results will be further extended with Monte-Carlo simulation methods with the channel objection to the Rayleigh distribution; From Monte-Carlo analysis, the numerical results are obtained with more parameters.

The Procedure set up is given below as:

1. **Initialize input parameters:**

   \[ M_R = [1 \ 2 \ 3 \ 4] ; \quad M_T = [1 \ 2 \ 3 \ 4] ; \]
   
   Test number = \( 2^4 \); Probability number = \( 10^2 \);
   
   Output number = \( 10^2 \);

2. **Channel Matrix and Eigen values Computation:**

   Using \( M_R \) and \( M_T \), generating Channel matrix \( H \) by randn function.
   
   Diagonal Channel matrix (D) is generated using \( H \times \) Transpose of \( H \).
   
   All Eigen Vectors are Computed form Diagonal matrix (D) from “eig” function.
   
   Full Rank matrix (F) is computed from: \( \text{eye (size (D))} + \text{SNR} \times \text{D.} / M_r \).
   
   Gain Matrix (G) = \( \text{sum (log}_2 (\text{diag (F)))} \) -- Diagonal elements of \( F \).

3. **Outage Capacity values Computation:**

   \( F_\text{matrix} = \text{sum (G_matrix) } / \text{test}_\text{number}; \quad H_\text{matrix} = \text{sort (F_matrix)}. \)
   
   Diagonal matrix (elements) = \( H_\text{matrix} \) (output number).
Figure 3.15 shows the simulated results of Outage Capacity for Rayleigh channel. Comparing to Figure 3.14 (where only two antenna selections are considered), here one can observe the 10% outage capacity for four different antenna array sizes and outage distribution is very promising for 4 × 4 antenna selection than 2 × 2 at 20 dB SNR.

3.6.4. Computation of MIMO channel capacity and complementary CDF & outage probability using Monte-Carlo simulation of around 10,000 channels

This computation method has three different functions. They are as follows;

1. This function compares Mean channel capacity for different MIMO realizations (SISO, SIMO, MISO, MIMO) as a function of SNR.

2. It also compares complementary CDF for different MIMO realizations (SISO, SIMO, MISO, and MIMO) as a function of capacity in bps/Hz.
3. It also compares Outage probability for different MIMO realizations (SISO, SIMO, MISO, MIMO) as a function of SNR.

   The simulation procedure is given below:

   i. **Simulation parameters:**

      - Setting SNR in dB in the range of 10dB to 30 dB.
      - Number of channel realizations=10,000.

   ii. **Mean channel capacity calculations:**

      - For one specific value of SNR, complex normal random value with variable is equal to 1/2 per dimension is given in terms of complex channel realizations.
      - The instantaneous capacitance is expressed in terms of complex Channel realizations as,

      \[ C_{\text{inst}} = \log_2 (1 + \text{SNR}_i \ast \text{Square of channel realizations}) \].

      - Mean capacity is expressed as

      \[ C_{\text{Mean}} (i) = \text{mean} (C_{\text{inst}}). \]

   iii. **Complementary CDF at SNR=10dB:**

      - Using channel realizations at 10dB SNR, Count all Channel realizations using histogram function.
      - In normal conditions, express it with **cum_sum** function as
cum_sum (count all realizations) / max (cum_sum (count all realizations)).

- complementary_CDF = 1 - count all realizations under normal conditions.

iv. Outage probability:

For (SNR=2 to 20 dB) then

Count all realizations =histogram (count all, realization range).

Counting all realizations of normal conditions

\[ \bar{\alpha}_s = \frac{\text{cum}_\text{sum} (\text{count})}{\max (\text{cum}_\text{sum} (\text{count}))}. \]

v. Power budget for MIMO water filling:

Initialization Parameters:

For Total Power \( P_t=1; N_R=4; N_T=4; \)

Mean channel capacity calculations:

For \( i1=1: \text{length} (\text{SNR}) \) for one specific value of SNR then

\[ H_{\text{vec}}=1/\sqrt{2}.*\text{complex} (\text{randn}(N_R*N_T, \text{ch realizations}), \text{randn}(N_R*N_T, \text{ch realizations})). \]

Reshaping elements of the channel matrix (H)

\[ H = \text{reshape} (H_{\text{vec}}, N_R, N_T, \text{channel realizations}). \]

-- reshape function returns Matrix elements are taken column wise
vi. **Instantaneous MIMO water filling capacity:**

Water filling parameter = \( \sum (1 + \frac{P_{\text{wf}(1,2,3,4)}}{N_{\text{wf}(1,2,3,4)}}) \).

Capacity due to water filling parameter = \( \log_2 \) (water filling parameter).

vii. **Mean capacity:**

\( C = \text{mean (Water filling parameter)} \). -- Using Mean Function.

viii. **Single Input Multiple Output (SIMO) Mean Capacity calculations:**

**Initial Parameters settings:**

\( N_T = 1; N_R = 4; \) SNR = 10 to 30 dB.

**Mean channel capacity:**

For \( i = 1: \) length (SNR) ---count

\( \text{SNR} = \text{SNR} (i_i) \); for one specific value of SNR

\( h_{14} = 1/\sqrt{2}.*\text{complex (randn (N_R*N_T, ch_realizations), randn (N_R*N_T, ch_realizations))}; \)

Repeat same procedure for **MISO (4x1) Realizations**
The simulation results for the channel realizations in normal conditions and computation of Outage probability (Outage capacity), Mean capacity analysis using water filling is done for SISO, MIMO (N_T=N_R=4 antennas), SIMO and MISO antenna selections with CCDF, Gain using water filling techniques. Figure 3.16 shows Mean Capacity computation for SISO, SIMO, 4×4 MIMO and MISO configurations. Here MIMO and 4×4 MIMO with water filling techniques give almost similar results.

![Mean Capacity vs SNR](image)

**Figure 3.16:** Mean Capacity computation for using Monte-Carlo method.

Figure 3.17. shows the simulated result of Outage Capacity Vs CCDF. The 10% outage probability for 4×4 MIMO with water filling technique results with 10 bps / Hz slightly better than MIMO. The simulation results show some tight lower and upper bounds on the CCDF of the MI. The results can be readily reduced to the case of Rayleigh fading. Compared with existing results, our bounds are not only given in closed form, but also readily applicable to the evaluation of the outage probability with sufficiently high accuracy.
Figure 3.17: Outage Capacity Vs CCDF using Monte-Carlo method.

Figure 3.18: Outage probability Vs SNR for 4 bps/Hz capacity using Monte-Carlo method.
Figure 3.18. shows the simulated result of Outage probability Vs SNR for 4 bps/Hz capacity. Here also $4 \times 4$ MIMO with water filling technique results better than $4 \times 4$ MIMO at 5 dB SNR.

![WF gain in capacity](image)

**Figure 3.19:** MIMO water-filling capacity gain with SNR using Monte Carlo method.

Figure 3.19. shows MIMO water-filling capacity gain with SNR. The maximum gain point is exactly at 10% outage probability for $4 \times 4$ MIMO with water filling technique at a -5 - 0 - 5 dB SNR region than other configurations like $1 \times 4$ SIMO, $4 \times 1$ MISO. The minimum WF gain point is for SISO nearly around 30 dB SNR.

### 3.6.5. Open loop and Closed loop MIMO Channel Capacity:

If MIMO channels change its capacity randomly, then, resulted $H$ is a random matrix. This means that its channel capacity is also randomly time-varying. In other words, the MIMO channel capacity can be given by its time average. In practice, we assume that the random channel is an Ergodic process, and then Ergodic channel capacity for the open-loop system without using CSI at the transmitter side, from Equation (3.7), is given as
\[ C_{\text{Open Loop}} = E \left\{ \sum_{i=1}^{\nu} \log_2 \left( 1 + \frac{E_i}{N_i N_u} \lambda_i \right) \right\} \]  
(3.18)

Where \(E_i\) is the energy of the transmitted signals, and \(N_o\) is the power spectral density of the additive noise. Symbol vector \(x\), received signal \(y\), \(E \{ \} \) represents total available power at the transmitter. Similarly, the Ergodic channel capacity for the closed-loop (CL) system using CSI at the transmitter side, from Equation (3.18), is given as If the transmit power for the \(i^{th}\) transmit antenna is given by \(p_i^{\text{opt}}\) (optimal= some fixed Number of Antenna selection) then Closed loop Ergodic capacity is given as

\[ C_{\text{Closed Loop}} = E \left\{ \sum_{i=1}^{\nu} \log_2 \left( 1 + \frac{E_i}{N_i N_u} p_i^{\text{opt}} A_i \right) \right\} \]  
(3.19)

The Performance analysis plot for Open loop and Closed loop for Ergodic MIMO channel capacity is shown in Figure 3.20. It is clear that the closed loop Ergodic channel capacity is having a slight upper edge over the open loop Ergodic channel capacity curve.

![Figure 3.20: Performance analysis of Open loop and Closed loop MIMO channel capacity.](image-url)
3.6.6. The MIMO outage capacity distribution in CAS environment:

Distributed multiple-input multiple-output (DMIMO) system is a combination of distributed antenna systems (DAS) and co-located antenna systems (CAS). When each DA port is equipped with multiple centralized antennas, the structure of the DAS can be easily extended to distribute multiple-input multiple-output (D-MIMO) systems, which is a combination of DAS and co-located MIMO systems. In DMIMO systems, antenna correlation within each DA port is often observed due to the placement of the array. Due to the large distances among different DA ports, the antenna correlation for each DA port is assumed independent. Simulation results verify the capacity improvement based on the various parameters from table 3.5.

The steps for the outage capacity calculations in CAS environment are given below.

1. Start.

2. Number of Transmitting and Receiving Antennas (N_T, N_R) as 2 × 2, 4 × 4, and 8 × 8 antenna selection strategy.

3. Assume that Input Transmit power is equally allocated to all transmitting antennas and Limiting the Number of receiving antennas (N_R);

4. Calculate the Channel matrix and calculate Mean capacity based on the relation

\[ C_{out} = \log_2 \det(1 + \frac{SNR}{N_R * H H^H}) \]

5. Take Absolute value of C_{out} and sort C_{out} also.

6. Mean value of C_{out} is given by \( C_{out\ (mean)} = \frac{\sum (C_{out})}{(N_T + N_R)} \).
7. Generate Capacity distribution plots.  
This algorithm computes channel matrix (H) with Co-allocated Antenna systems (CAS) method for various channel selection strategies. Figure 3.21. shows outage capacity distribution region for CAS with average SNR changes for 2×2, 4×4 and 8×8 antenna selection in co-related fading conditions. (SNR changes are not shown to a scale to show the capacity distribution only) It clearly indicates almost doubling of capacity for each selection according to the Red tip portion. The CAS model compassing the Rayleigh fading, logarithmic-normal shadow fading and path loss with different power allocation.

![Figure 3.21: Outage Capacity plot for 2X2, 4X4, 8X8 CAS model.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Approximate Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shadow Variance</td>
<td>Around 10 dB</td>
</tr>
<tr>
<td>Path Loss</td>
<td>-3.5</td>
</tr>
</tbody>
</table>

TABLE 3.5  
CAS ANTENNA PARAMETERS

---

The CAS model parameters are listed in table I. The frequency range is around 2-2.5 MHz and Bandwidth range is around 5MHz. Even the extensive analysis with Co-allocated Antenna system (CAS) with equal and uniform antenna allocation has been done and capacity distribution has compared for the Ergodic and outage capacity analysis. Here the antenna selection is chosen as $N_T=N_R=4$ with number of taps $L=4$ shown in Figure.3. 22. Which also indicate 3-Dimensional distribution of Ergodic and outage capacity in CAS antenna employment. It is clearly seen that outage variations are spread at the tail end of the plot. These variations are only due to the placement of Transmitting and Receiving antennas in different spacing or locations.
Figure 3.22: Comparison of Ergodic and Outage capacity in CAS environment for $N_r=N_t=4$ with $L=4$.

3.7 Correlation factors considered in MIMO Channel Capacity (Stochastic based approach)

The correlation degrees among the channel matrix gains in the MIMO system are closely related to the MIMO capacity with a reduced number of non-zero Eigen values of channel matrix, the capacity of the MIMO channel will be reduced, because of a rank deficient channel matrix. This is situation arises when the signals arriving at the receivers are correlated, even though a high channel rank, it is necessary to obtain high spectral efficiency on a MIMO channel. Low correlation is not a guarantee of high capacity. One of the causes of
correlated fading is lack of rich scattering environment. For the channel model mentioned in three cases arise for the lack of rich scattering environment.

Case 1: The channels between multiple transmitted node and single receiving node are correlated, this is known as transmit correlation.

Case 2: The channels between single transmitting node and multiple receiving nodes are correlated, this is known as receive correlation.

Case 3: Both of them at the same time. The correlation between the fading of two distinct antenna pair is the product of the corresponding transmits correlation and receives correlation.

3.7.1. Correlation factors in basic MIMO channel capacity equation:

Consider a fixed linear matrix channel with additive white Gaussian noise and when the transmitted signal vector is composed of statistically independent equal power components, then each with a Gaussian distribution and the receiver knows the channel, its capacity is

\[ C = \log_2 \det \left( I + \frac{\sigma^2}{N,HH^t} \right) \text{ bits/s/Hz.} \] (3.20)

Where “N” is the number of transmit / receive antennas (for the sake of simplicity we consider here the case when the number of transmit and receive antennas are equal, but a general case can be considered in a similar way); \( \sigma \) is the average signal-to-noise ratio (SNR), \( I \) is \( N \times N \) identity matrix; \( H \) is the normalized channel matrix, which is considered to be frequency independent over the signal bandwidth; and “†” means transpose conjugate.

Let us adopt here the following normalization condition:
\[ \sum_{i,j=1}^{N} |h_{ij}^2| = N \]  \hspace{1cm} (3.21)

Where \( h_{ij} \) component denotes the components of \( H \). Hence, when \( H = I \) (completely uncorrelated parallel sub-channels), \( \sigma / N \) is the signal-to-noise ratio per receive branch. To study the effect of correlation in an explicit way and to separate it from the effect of unequal received powers, we assume that all the received powers are equal.

In this case, \( \sigma_i = \text{def} \sum |h_{ij}^2| = 1 \), and equation (3.21) reduces to:

\[ C = \log_2 \det (I + \frac{\sigma^2}{N}) \text{bits/s/Hz} \]  \hspace{1cm} (3.22)

Where is the normalized channel correlation matrix, \( |r_{ij}| \leq 1 \), whose components are:

\[ r_{ij} = \frac{1}{\sigma_i \sigma_j} \sum_k h_{ik} h_{jk}^* = \sum_k h_{ik} h_{jk}^* \]  \hspace{1cm} (3.23)

Where \(^*\) denotes complex conjugate. The last equality in (3.23) holds due to the assumption of equal received powers. It immediately follows from (3.23)

\[ r_{ij} = r_{ji}^* \]

### 3.8. Correlated MIMO System Capacity using its replica:

In the case of closed form expression for Ergodic capacity for correlated MIMO case, we assume that the correlation is on one side of the transmission system. Let \( \Sigma \) is the correlation matrix and now Carrying out transformation \( H \Sigma^{-\frac{1}{2}} = H' \). The capacity expression (at relatively high SNR) is given by:
\[ C = E[H] \left\{ \log \det \left( I_N + \frac{\mathbf{G}}{N \sum_{HH}} \right) \right\} \text{bits/s/Hz} \]  \hspace{1cm} (3.24)

The equation (3.24) shows that the capacity degrades as the correlation increases between antennas. We can also extend the above result to correlations at both ends. Here we give the result directly for correlations on both ends as

\[ C = \log_2 \left( \ln \left( \det \mathbf{\Sigma}_1 \right) + \ln \left( \det \mathbf{\Sigma}_2 \right) + N(\log(\sigma) - 1) \right) \text{bits/s/Hz} \]  \hspace{1cm} (3.25)

Where \( \mathbf{\Sigma}_1, \mathbf{\Sigma}_2 \) are correlation matrices.

3.9. Modeling and capacity analysis of MIMO channels with the joint effect of spatial and temporal correlation:

Recent studies on the capacity of MIMO channels are focused on the effect of spatial correlation. In reality, the channel is not ideally Rayleigh i.i.d. There are various factors that cause it to deviate from this and, as a result, the performance of MIMO systems deteriorates. One of these is correlation. Correlation problems arise because of the separation distance between antenna elements. Usually, this separation distance is in the order of a few centimeters, whereas the separation between the Transmitter and the Receiver is in the order of a few kilometers. Hence, the theoretical capacity of the spatial correlated Rayleigh MIMO channel is an important issue in MIMO technology.

In this section, the joint effect of spatial and temporal correlation has been investigated. When \( N_T \) transmit and \( N_R \) receive antennas are employed, it is shown that the outage capacity still increases linearly with respect to \( N_T N_R \), despite the presence of spatial and temporal correlation. The analysis is very general, as it is based on the transmit and receive antenna correlations matrices. Some of the main parameters like steering channel matrix \( \mathbf{H} \) and its rank, spatial correlation coefficients, power location in sub channels with transmitter knows Channel State Information (CSI) are taken into account.
Figures 3.23 & 3.24 shows the analysis of Transmitting and Receiving antennas based on SNR selection at four values. In both cases (Transmit & Receive antenna selections) the improvement of capacity is observed at higher SNR (i.e. 15 dB) than at lower SNR (i.e. 0 dB).

Figure 3.23: Capacity Performance of various transmit antenna selections based on Spatial Correlation and depiction of capacity with $\text{SNR} = [0, 5, 10, 15] \text{ dB}$. 
Figure 3.24: Capacity Performance Analysis of various receive antenna selections based on Spatial Correlation and depiction of capacity with $SNR = \{0, 5, 10, 15\}$ dB.

Table 3.6 shows the capacity computation for both Transmit selection and Receive selection at 10 dB SNR. The Receive selection shows better results than Transmit selection by using best Receiver detection schemes. (Discussed in next chapter)

<table>
<thead>
<tr>
<th>No of Transmitting Antennas (Selection)</th>
<th>SNR dB</th>
<th>capacity (Bits / s / Hz)</th>
<th>No of Receiving Antennas (Selection)</th>
<th>SNR dB</th>
<th>capacity (Bits / s / Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
<td>11</td>
<td>5</td>
<td>10</td>
<td>12.5</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>12.5</td>
<td>10</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
<td>13</td>
<td>15</td>
<td>10</td>
<td>19</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>13.4</td>
<td>20</td>
<td>10</td>
<td>21</td>
</tr>
<tr>
<td>25</td>
<td>10</td>
<td>13.6</td>
<td>25</td>
<td>10</td>
<td>22</td>
</tr>
<tr>
<td>30</td>
<td>10</td>
<td>13.8</td>
<td>30</td>
<td>10</td>
<td>23</td>
</tr>
</tbody>
</table>

Figure 3.25 shows the Comparison of various transmit and receive antenna selection with average and water filling allocation of power. The curves plotted are almost similar with range of values of system capacity with respect to various values of SNR. Water filling technique is already explained in chapter 2.

![MIMO capacity Analysis for NT=NR=[10,8,6,4,2]](image)

**Figure 3.25: Comparison of various transmit and receive antenna selection with average and water filling allocation of power.**

### 3.11. Working Algorithm for MIMO Capacity analysis with Correlation factor:

The working procedure for $2 \times 2$ MIMO systems with a correlation value with 0.5 and an XPD of 0.5 for Ergodic capacity or outage capacity is as follows:

1. Choose number of transmit and receive antennas as $N_T = N_R = 2$.
2. Assume Correlation coefficient as 1 for Correlation present and 0 for Un-correlated antennas.
3. Assume antenna XPD (cross polarization discrimination in horizontal and vertical directions) value (1 if antenna XPD is to be investigated, 0 if not).
   Equate alpha ($\alpha_{mn}$ any phase shift resulted) to XPD value.
4. Vary SNR through 0 to 20 dB.
v. By assuming capacity either for water-filling or determining the capacity in the presence of a Rician Component (0.8) to obtain the output.

Rician fading model is similar to that for Rayleigh fading, except that in Rician fading, a strong dominant component is present. It may be LOS wave.

Figures 3.26 & 3.27 (Next page), shows the comparison of Ergodic and outage capacities in 2 \( \times \) 2 MIMO system. Here three different plots are seen, one theoretical increment in capacity, middle one with Rician factor (component) added in a simulation model and the other one with water level procedure. Here the correlation factor and XPD factor are added as certain fixed value. The results clearly show the capacity distribution in fixed and variable environments.

The depolarization effect is characterized by the Cross Polarization Discrimination (XPD) which is defined as the power ratio of the co-polarization and cross-polarization components of the mean incident wave.

3.12: Impact of Cross Polarization Discrimination (XPD) on system capacity

For the sake of simplicity, consider the 2 \( \times \) 2 MIMO channel generated according to the Kronecker channel modeling. Assuming that the CSI is known at the receiver side, the MIMO system capacity can be derived by exploiting the SVD technique.

Figure 3.25 shows plotted curves of the Ergodic capacity for XPD =0.5 and the curve of the MIMO channel capacity as a function of the SNR. Simulation results show that XPD affects the performances of the MIMO system capacity. The MIMO system capacity is shown to be seriously reduced for high level of the polarization discrimination. Thus, mismatch in polarization results in losses in the MIMO channel capacity. Figure 3.26 shows plotted curves of the Outage capacity for XPD =0.5 and the curve of the MIMO channel capacity as a function of the SNR. Simulation results show that XPD affects the performances of the MIMO system capacity and CCDF.
Figure 3.26: Comparison of Ergodic capacity analysis for $N_T=N_R=2$ with Correlation factor=0.5 & XPD =0.5.

Figure 3.27: Comparison of Outage capacity analysis for $N_T=N_R=2$ with Correlation factor=0.5 & XPD =0.5

---

Not much difference is reported from Figure 3.26 and Figure 3.27. But a slight variation in the outage capacity and Ergodic capacity can be easily observed.

3.12. Rician component on system capacity

The Rayleigh assumption typically holds in fixed wireless links. However, there are situations where there may exist a strong coherent component. This Rician component does not experience, any fading over time contributions. Similarly, in fixed wireless access scenarios, most reflected and diffracted contributions add coherently, as the transmitter and the receiver are fixed. All these situations lead to a Rician distribution of the received signals. If we assume a sufficiently large separation between Transmitter and Receiver, The channel matrix H clearly denotes a rank decrease. It leads to Rician K-factor. We have considered K=0.8 for our simulation results shown in Figure 3.25 and Figure 3.26.

3.13. Effect of channel Correlation on Ergodic capacity for 4×4 MIMO.

Consider the computation of Ergodic MIMO (4×4) channel capacity when there exists a correlation between the transmit and receive antennas.

Correlated Rayleigh MIMO Channel Coefficients generated through Mat lab code is described as follows:

**Important contents of the mat lab code:**

```
N_T : number of transmitters
N_R : number of receivers
N  : length of channel matrix
R_m : correlation vector/matrix of Transmitter, For example: [1 0.5], [1 0.5;0.5 1]
R_n : correlation vector/matrix of Receiver.
hh  : N_R x N_T x N correlated channel

h=sqrt(1/2) *( randn ( N_T*N_R, N)+j* randn (N_T*N_R,N) ); Channel matrix Generation.
C = sqrtm ( sqrt ( kron (R_m, R_n) )); Capacity Computation with Power (field) correlation
Hh = zeros (N_R, N_T, N) ; Apply correlation to channel matrix
```
With the following channel correlation matrices states that even though no correlation exists between the receive antennas. But From the simulated result from Figure. 3.28, it can be shown that a capacity of 3.3 bps / Hz is lost due to the channel correlation when SNR is 18dB.

**Result of Co-related channel Matrix (Through Simulation)**

0.4724 - 0.4302i  0.5911 + 0.7620i  0.3162 + 0.4831i  1.2412 - 0.5614i  
0.4832 + 0.1355i -0.4850 + 0.5367i  0.3808 - 0.0020i  0.0888 + 1.4366i  
0.3033 + 0.7064i -0.1493 + 0.4232i  0.2533 - 1.1226i  0.0619 - 0.1745i  
0.3213 - 0.3105i -1.1870 - 0.6228i  1.4263 - 0.1375i  0.0821 - 1.4521i

![Graph](image)

**Figure 3.28:** Capacity reduction due to the channel correlation.

In practice, correlation exists between the signals transmitted by or received at different antenna elements. Correlation can arise if the elements are not spaced sufficiently far apart. The capacity of MIMO systems depends on the statistical properties of the channel and the amount of knowledge about those properties. While for no transmitter channel knowledge
correlated fading results are worst. Having the transmitter acquire the channel properties on average can actually lead to capacity improvement over uncorrelated fading channels. However, in a real world scenario the fades are usually not independent, but will exhibit certain fading correlations. It has been observed [12] that channel capacity degrades significantly in the presence of fading correlations. However, these observations were built on the assumption of having zero transmitter channel knowledge and no other source of diversity, like time or frequency available. Extending the investigations to compute the capacity of the channel in different environments for the best possible concussions. In the first attempt, Figure 3.29 shows channel capacity of $4 \times 4$ MIMO channel with correlation conditions (One-to-One antenna) and uniform Angle spread conditions. Here the plot computes correlation parameters, scattering conditions and calculation of Mean capacity. Similarly Figures 3.30 depicts Un-Correlated High Rank (UHR), Un-Correlated Low Rank (ULR) and Correlated Low Rank (CLR) channel matrix generation for comparing mean channel capacity with $8 \times 4$ antenna combinations. Even Correlation factors equality is also shown (Equal to 1 indicates perfect Linearity).

![Figure 3.29: Computation of $4 \times 4$ MIMO Channel capacity under different conditions.](image_url)
Figure 3.30: Computation of $8 \times 4$ MIMO Channel capacity with different RANKS of matrix.

Table 3.7, shows the capacity computation for UHR, ULR and CLR for $8 \times 4$ MIMO channel Models. Here UHR model results large value of capacity.

**TABLE 3.7**
CAPACITY COMPUTATION FOR UHR, ULR AND CLR FOR $8 \times 4$ MIMO CHANNEL

<table>
<thead>
<tr>
<th>RANK Type</th>
<th>SNR (dB)</th>
<th>Capacity (b/s/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLR</td>
<td>15</td>
<td>3.1423</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>10.1342</td>
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<td>15</td>
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<td>21.0052</td>
</tr>
<tr>
<td>UHR</td>
<td>15</td>
<td>23.878</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>48.2224</td>
</tr>
</tbody>
</table>

---

3.14. Conclusions:

From all these investigations, the main outcomes are

1. The capacity of MIMO systems depends on the statistical properties of the channel and the amount of knowledge about those properties.

2. Deterministic channel Capacity models measures the maximum amount of information that could be transmitted through a channel and received with negligible error using SVD, Water Filling techniques.

3. Stochastic channel models with Correlation based models such as i.i.d model and Kronecker model assumes that spatial transmit correlation and spatial receive correlation are separable. Therefore, the full channel correlation matrix can be modelled by the Kronecker product of transmit and receive correlation matrix. Correlation decreases channel capacity.

4. Other channel models like, closed / open loop, Random, Rayleigh, Rician and AWGN are also investigated.

5. Finally simulations are carried out on the basis of RANK of a channel matrix like ULR, UHR and CLR capacity results are discussed.

The next Chapter introduces the Theoretical analysis and numerical results of Spatial Diversity scheme in which Transmit diversity and Receive Diversity techniques are computed and numerical results are discussed.
REFERENCES


