CHAPTER II

PRICE DETERMINATION FOR AN EOQ MODEL FOR DETERIORATING ITEMS UNDER PERFECT COMPETITION

The problem of price determination for an EOQ model under perfect competition is of central importance in the field of inventory control and management, especially such kind of models which study the dynamics of the market economy over the time. In this chapter, an attempt has been made to analyze and compute the price of a unit item of inventory for an EOQ model for deteriorating items under the perfect competition as an important market structure. Along with optimization technique, marginal revenue and marginal cost approach has been employed to determine the price of a unit item of inventory. The present work is presumably believed to provide theoretical as well as applicable insights to the marketing experts and inventory managers engaged in the field of econo-operations research.
PRICE DETERMINATION FOR AN EOQ MODEL FOR DETERIORATING ITEMS UNDER PERFECT COMPETITION

2.0 INTRODUCTION

Deteriorating inventory models have been widely studied in the recent years because deterioration of goods is a common phenomenon in our practical life; pharmaceuticals, foods, vegetables and fruit are a few examples of such items. Therefore, the loss due to deterioration in the field of inventory cannot be neglected.

It is well known that the stock-level has motivational effect on the customers in a market that is the demand rate may go up or down if the on hand inventory level increases or decreases. Such a situation generally arises for a consumer item type of inventory. Therefore, it may be desirable to order large quantities resulting in stock remaining at the end of the cycle for the potential profits obtained from the increased demand.

In the beginning, Ghare and Schrader [1] considered continuous decaying inventory for constant demand and finally obtained economic order quantity for the model. Later, Shah and Jaiswal [2] investigated an order level inventory for deteriorating items with a constant rate of deterioration in a deterministic environment. Aggarwal [3] developed an order level inventory model by correcting and modifying the error in Shah and Jaiswal [2] while calculating the average holding cost of the

It is a common belief that a large pile of goods attracts more customers in the supermarket. This phenomenon is termed as stock dependent demand rate. In the recent past, there has been significant researches in this field that how does the stock dependent demand affect the inventory control policies. Levin et.al. [7] observed “large piles of consumer goods displayed in a supermarket will attract the customers to buy more items.” Silver and Peterson [8] noted that sales at the retail level tend to be proportional to the amount of inventory displayed. In order to quantify it, Baker and Urban [9] established an economic order quantity model for a power form inventory level with dependent demand pattern. Mandal and Phaujdar [10] then developed economics production quantity model for deteriorating items with constant production rate and linearly stock-dependent demand.

Later on, Datta and Pal [11] presented an EOQ Model in which the demand rate is dependent on instantaneous stocks displayed until a given level of inventory is reached, after which the demand rate becomes constant. In addition, they assumed the replenishment cycle ends with
zero stock. Urban [12] then relaxed the assumption of zero ending inventories because it may be desirable to order larger quantities resulting in stock remaining at the end of the cycle, due to the potential profits accruing from the increased demand. Further, for more extensive review of researches, vide the relevant references such as Chen and Simchi [13], Datta and Paul [14], Giri and Chaudhuri [15], Giri et.al. [16], Gupta et.al. [17], Lynn and David [18], Muth et.al. [19], Mishra et.al. [20, 21] Quyang et.al. [22], Padmanabhan and Vrat [23], Pal et.al. [24], Perakis and Sood [25], Pindyck [26], Ray and Chaudhari [27], Ray et.al. [28], Riordan and Michael [29], Robert and Larry [30], Schroder and Azzeddine [31], Spence and Michael [32], Santosh and Pakkala [33], Teng and Chang [34], Urban and Baker [35], Weliwita and Azzeddine [36], Zhao and Zheng [37].

Datta and Pal [11] developed an EOQ model in which demand rate is a power function of the on hand inventory until down to a certain stock level, at which the demand rate becomes constant. Teng et.al. [38] extended the work by allowing not only deteriorating items but also non-zero ending inventory. They have described different cases and presented an algorithm so that total profit per unit item is maximized.

Various competitions in the market structure are considered to be inevitable phenomena which seriously attract the attention of researchers engaged in this field, vide for example, David et.al. [39]. Robinson [40] in the “What is Perfect Competition” says that “perfect competition prevails when demand of the output of each producer is perfectly elastic.
This entails first that the number of sellers is large so output of any seller is negligibly small proportion of the total output of commodity and second, that buyers are alike in respect to their choice between rival sellers so that the market is perfect”. Moreover, eminent economists are unanimous to lay down the conditions of perfect competitions such as large numbers of buyers and retailers in the market for that particular item; firms are free for entering and escaping in the market; full marketing knowledge of both buyers and sellers; productive system should be fully dynamic and firms or industry should be near from the market; the number of firms should be large and there should be no constraints in the markets etc. In fact, distribution of inventory items is channelized through a strong marketing system. In a perfectly competitive market, there should be an economic efficiency encompassing the idea that a system proceeds with the minimum amount of deterioration (waste). An economically efficient system of distribution follows that no one can be made better off without making someone else worse off, the most output is obtained from a given amount of input inventory and production of inventory proceeds at the lowest possible per unit cost. It further comprises of allocative and productive efficiencies of the inventory system in which a locative efficiency occurs when price is equal to marginal cost at which point the item is available to the consumer at the lowest possible price and similarly, productive efficiency occurs when the firm produces at the lowest point of the average cost, implying it cannot produce the inventory any more cheaply.
In order to have an equal opportunity for each selling point (judged by economic efficiency) and enabling each customer to choose the selling point in equally likely manner (where raising of demands by the customers is perfectly elastic), an important dimension of the market structure in the form of perfect competition needs to be investigated with deteriorating item of EOQ model. Under this concept, marginal revenue and marginal costs interplay with each other to provide the logistic conditions so as to analyze EOQ and its price of a unit item of the inventory. Here, it is also worthwhile to mention that Teng et.al. [38] and other previous researchers in this series of works, made no attempt to analyze the EOQ models with deterioration under any sort of market structure.

In this chapter, a concept of perfect competition as an important market structure has been introduced and under its effect the price of a unit item of EOQ model has been analyzed and computed by employing approach of marginal revenue and marginal cost along with profit optimization technique. In addition, an exhaustive sensitivity analysis has been carried out with the help of computing algorithm to exhibit the use of the model under the environment of the perfect competition. The paper has been organized in various important sections which include introduction, mathematical analysis for various costs, price determination, sensitivity analysis, conclusion and future research apart from notations, assumptions and preliminary ideas which have been thoroughly used in this chapter.
2.1 Notations

The following notations have been thoroughly used in this chapter:

i. \( D(I) \) = stock dependent demand rate of inventory in the market.

ii. \( I(t) \) = inventory level at an instant of time \( t \). For the sake of brevity, we shall use \( I(t) \) as \( I \) for convenient handling of mathematical calculations in the chapter.

iii. \( D = \) constant demand rate in time interval \( t_i \leq t \leq T \).

iv. \( D(t) = \) demand at an instant of time \( t \).

v. \( \theta = \) constant rate of deterioration of item.

vi. \( S = \) stock level at initial time of a particular selling point. Selling points are final destinations where inventory items are ready for sale.

vii. \( S_0 = \) stationary stock of a particular selling point.

viii. \( i = \) end level stock of a particular selling point.

ix. \( N = \) number of selling points in the market at the end of cycle time.

x. \( N_0 = \) number of selling points at initial stage (often, it is assumed to be zero).

xi. \( N(t) = \) number of selling points at a time \( t \) in the market.

xii. \( T = \) horizon time (cycle time) of EOQ model.

xiii. \( HC = \) the holding cost of inventory.

xiv. \( PC = \) the purchasing cost of inventory.

xv. \( SC = \) the setup cost of the inventory.
xvi. \( Q = \) quantity of inventory in whole market.

xvii. \( TR = \) total revenue of inventory.

xviii. \( MR = \) marginal revenue.

xix. \( MC = \) marginal cost.

xx. \( C_1 = \) holding cost per unit item.

xxi. \( C_2 = \) purchasing cost per unit item.

xxii. \( C_3 = \) set up cost.

xxiii. \( P = \) price per unit item.

xxiv. \( \alpha \) and \( \beta \) are shape and size parameters of inventory level, where \( \alpha > 0 \) and \( 0 < \beta < 1 \).

### 2.2 Assumptions and Preliminary Ideas

We consider the following assumptions and preliminary ideas which have been used in the model development and its analysis:

i. The inventory system involves only one item, which is a necessary condition for perfect competition in the market of that particular item.

ii. The replenishment occurs instantaneously at an infinite rate.

iii. The deterioration rate \( \theta \) is constant and there is no replenishment or repair of deteriorated units during the period under consideration.

iv. The demand rate is deterministic and its functional form is given by

\[
D(t) = \begin{cases} 
\alpha I^\beta, & 0 \leq t \leq t_1 \\
D = \alpha S_0^\beta, & t_1 \leq t \leq T 
\end{cases},
\]

where level of inventory change
with time in linear form.

v. \( N(t) \) is assumed to be a continuous function.

We use the following ideas of marginal revenue and cost and their interrelationship, vide for example David et.al. [7]. Total revenue is defined as a product of price per unit quantity and total quantity of inventory sold by the organization. Marginal revenue is regarded as the change in the total revenue, which results from the change sold by a unit. Let at time \( t \) total revenue be \( TR \) for the number of total units \( Q \) and at time \( t+\Delta t \) total revenue be \( TR+\delta TR \) for the total number of units \( Q+\delta Q \). Then marginal revenue is given by first derivative of \( TR \) with respect to \( Q \).

Total cost of production is the sum of all expenditure incurred in producing a given volume of output and marginal cost is a cost incurred on a single unit, which increases the total cost of any organization. Let at time \( t \) total cost in production of \( Q \) quantity be \( TC \) and at \( t+\delta t \) total cost in producing \( Q+\delta Q \) quantity be \( TC+\delta TC \). Then marginal cost is given by first derivative of \( TC \) with respect to \( Q \). Apart from it, under perfect competition, price of a unit item for an EOQ model is determined when marginal revenue equalizes marginal cost.

2.3 Mathematical Analysis of the Model

Here, we consider the relative growth rate of selling points to be constant because this is a most common growth of selling points in any market structure. Mathematically, it is expressed by

\[
\frac{1}{N(t)} \frac{dN(t)}{dt} = k,
\]

where \( k \) is a constant.
It can give us
\[ \log N(t) = kt + c. \] It implies that \[ N(t) = e^{kt+c} \]

Putting \( t = T \), we obtain that \[ N(T) = e^{kT}e^c \]

It gives the value of \( c \) as \[ c = \log N - kT \] and it finally leads to \[ N(t) = Ne^{k(t-T)} \]

In view of perfect competition, we define average demand rate for a selling point by,

\[
\frac{d}{dt} \left( \frac{D(t)}{N(t)} \right) = \begin{cases} \frac{\alpha I^\beta}{Ne^{kT\left(\frac{t-t_1}{T}\right)}} \left( 1 - \frac{I_k}{(\beta+1)} \frac{dl}{dt} \right), & 0 \leq t \leq t_1 \\ \frac{D(t)}{N(1-kt)e^{kT\left(\frac{1-t}{T}\right)}}, & t_1 \leq t \leq T \end{cases}
\]

(2.3.1)

Further, we can easily observe that

\[
\frac{dl}{dt} + \theta I = -\frac{\alpha I^\beta}{Ne^{kT\left(\frac{t-t_1}{T}\right)}} \left( 1 - \frac{I_k}{(\beta+1)} \frac{dl}{dt} \right)
\]

(2.3.2)

And

\[
\left( \frac{dl}{dt} \right)^2 + \left( \theta I + \frac{\alpha I^\beta}{N(t)} \right) \frac{dl}{dt} = \frac{\alpha \theta I^\beta + 1}{N(t)(\beta+1)}
\]

(2.3.3)

Equation (2.3.3) can be rewritten as
\[ \zeta^2 + (\theta I)\zeta + \frac{\alpha}{N} \beta e^{kT\left[1-\frac{t}{T}\right]} \zeta = -\frac{\alpha}{N} \beta + 1 e^{kT\left[1-\frac{t}{T}\right]}, \]

where \( \zeta = \frac{dl}{dt}, \)

\[ (\zeta + \theta I) \left( \zeta + \frac{\alpha}{N} e^{kT\left[1-\frac{t}{T}\right]} I \beta \right) = 0 \]

It ultimately, after solving, gives us

\[ \zeta = -\theta I, \quad \text{or} \quad \zeta = -\frac{\alpha}{N} e^{kT\left[1-\frac{t}{T}\right]} I \beta \]

It can further be expressed as

\[ \frac{dl}{dt} = -\theta I \Rightarrow I = \lambda e^{-\theta t}, \quad \text{where} \ \lambda \text{ is a constant of integration} \]

\[ I = \lambda e^{\left(\beta+1\right)} \]

(2.3.4)

Moreover,

\[ \zeta = -\frac{\alpha}{N} e^{kT\left[1-\frac{t}{T}\right]} I \beta, \quad \text{which implies that} \]

\[ \frac{dl}{dt} = -\frac{\alpha}{N} e^{kT\left[1-\frac{t}{T}\right]} I \beta \]

(2.3.5)

2.4 Analysis of Holding Cost

Here, we examine the model in the following ways:

Case I (a)

In view of conditions \( S \geq i, \ 0 \leq t < t_i \) and equation (2.3.4), we can
easily obtain

\[ I(0) = S = \lambda e^t \quad \text{and} \]

\[ I(t_1) = S e^{\left(\frac{t_1^k}{(\beta + 1)}\right) \frac{k(t-t_1)}{(\beta+1)}} \]

which gives us \( I = i e^{\frac{t}{(\beta+1)}} \).

Now, we find holding cost as

\[ HC = C_1 \int_0^{t_1} e^{\left(\frac{t-t_1}{(\beta + 1)}\right) \frac{k}{k} i \left(\frac{t-t_1}{(\beta + 1)}\right)^{t_1}} \]

\[ = C_1 i \left(\frac{(\beta + 1)}{k}\right) (1 - e^{\left(\frac{t_1}{(\beta+1)}\right)}) \]

It further yields that

\[ \frac{dHC}{di} = C_1 \left(\frac{(\beta + 1)}{k}\right) \left(1 - e^{\left(\frac{t_1}{(\beta+1)}\right)}\right) \]  \hspace{1cm} (2.3.6)

and

\[ \frac{di}{dQ} = \frac{t_1 \beta k}{k \beta e^{(\beta+1)}} \]

\[ = \frac{t_1 \beta k}{(\beta+1) \alpha \beta i \beta^{-1} (e^{(\beta+1)} - 1)} \]  \hspace{1cm} (2.3.7)

**Case 1 (b)**

In the light of \( S \geq i, \ 0 \leq t < t_1 \) and equation (2.3.5), we have
\[ I^{1-\beta} = (\beta - 1) \alpha e^{k(T-t)}/kN + c, \]

where \( c \) is a constant of integration

If \( I(t_i) = i \), then it implies that

\[ i^{1-\beta} = (\beta - 1) \alpha e^{k(T-t_i)}/kN + c. \]

It can easily give the value of \( c \) as,

\[ c = i^{1-\beta} - (\beta - 1) \alpha e^{k(T-t_i)}/kN. \]

Further,

\[ I = \left[ \frac{\alpha (\beta - 1)}{kN} \left\{ e^{k(T-t)} - e^{k(T-t_i)} \right\} + i^{1-\beta} \right]^{1-\beta}. \]

Let

\[ U = \frac{\alpha (\beta - 1)}{kN} \left\{ e^{k(T-t)} - e^{k(T-t_i)} \right\} + i^{1-\beta}, \]

\[ A = \frac{\alpha (1-\beta)}{kN} e^{k(T-t_i)} + i^{1-\beta}, \quad B = \frac{\alpha (\beta - 1)}{kN}; \]

We find holding cost as

\[ HC = C \int_0^t \left\{ A + Be^{k(T-t)} \right\}^{1-\beta} dt \quad (2.3.8) \]

Let \( U = A + Be^{k(T-t)} \)
It can further give us \( B e^{k(T-t)}(-k)dt = dU \) and also

\[
\frac{dt}{dU} = -\frac{1}{Bke^{k(T-t)}} = \frac{1}{(A-U)k};
\]

In view of above, equation (2.3.8) turns out to be

\[
HC = C_1 t_1 \left\{ \frac{1}{U^{1-\beta}} \right\} \frac{dU}{(A-U)k} = -C_1 t_1 \left\{ \frac{1}{U^{1-\beta}} \right\} \frac{dU}{U-A}
\]

\[
HC = -\frac{C_1 t_1}{k} \left\{ \frac{1}{U^{1-\beta}} (U - A) \right\} \frac{dU}{U} \approx -\frac{C_1 t_1}{k} \left\{ \frac{1}{U^{1-\beta}} \right\} \left(1 + \frac{A}{U} \right) dU
\]

(Taking approximation for a term only)

\[
\approx -\frac{C_1}{k} \left\{ \frac{t_1}{U^{1-\beta}} \right\} \frac{dU}{U} - \frac{t_1}{0} \left\{ \frac{\beta}{U^{1-\beta}} \right\} dU
\]

\[
\approx -\frac{C_1}{k} \left( \begin{array}{c}
\frac{1}{U^{1-\beta}} \\
\frac{0}{U^{1-\beta}}
\end{array} \right) - \frac{t_1}{\beta} \left( \begin{array}{c}
\frac{1}{1-\beta} \\
\frac{0}{1-\beta}
\end{array} \right)
\]
\[
\approx \frac{C_1}{k} (\beta - 1) \left[ \begin{array}{c} U^{1-\beta} \\ 0 \end{array} \right]^{t_1} + \frac{C_1}{k} A(\beta - 1) \left[ \begin{array}{c} \beta \\ 0 \end{array} \right]^{t_1} \]

\[
\approx \frac{C_1}{k} (\beta - 1) \left[ \begin{array}{c} \frac{(1-\beta) \alpha T}{N} \left( e^{k (T-t_1)} - e^{k (T-t_1)} \right) + i^{1-\beta} \frac{1}{1-\beta} \\ - \frac{(1-\beta) \alpha T}{N} \left( e^{k T} - e^{k (T-t_1)} \right) + i^{1-\beta} \frac{1}{1-\beta} \end{array} \right]^{t_1} + \frac{C_1}{k} A(\beta - 1) \left[ \begin{array}{c} \frac{(1-\beta) \alpha T}{N} \left( e^{k (T-t_1)} - e^{k (T-t_1)} \right) + i^{1-\beta} \frac{1}{1-\beta} \\ - \frac{(1-\beta) \alpha T}{N} \left( e^{k T} - e^{k (T-t_1)} \right) + i^{1-\beta} \frac{1}{1-\beta} \end{array} \right]^{t_1} \]

\[
\approx \frac{C_1}{k} (\beta - 1) \left[ \begin{array}{c} i \frac{(1-\beta) \alpha T}{N} \left( e^{k T} - e^{k (T-t_1)} \right) + i^{1-\beta} \frac{1}{1-\beta} \\ \beta \end{array} \right]^{t_1} + \frac{C_1}{k} A(\beta - 1) \left[ \begin{array}{c} i \frac{(1-\beta) \alpha T}{Nk} \left( e^{k T} - e^{k (T-t_1)} \right) + i^{1-\beta} \frac{1}{1-\beta} \end{array} \right]^{t_1} \]
\[ \approx \frac{C_1}{k} (\beta - 1) \left\{ \frac{1}{1 - i} \left[ 1 + \frac{(1 - \beta) \alpha}{N k i^{1-\beta}} \left( e^{k T} - e^{k (T-t_1)} \right) \right] \right\}^{\frac{1}{1-\beta}} \]

\[ + \frac{C_1 A (\beta - 1)}{k} \left\{ i \beta - i \beta \left[ 1 + \frac{(1 - \beta) \alpha}{N k i^{1-\beta}} \left( e^{k T} - e^{k (T-t_1)} \right) \right] \right\}^{\frac{1}{1-\beta}} \]

Since \(\frac{(1 - \beta) \alpha}{N k i^{1-\beta}} (e^{k T} - e^{k (T-t_1)}) < 1\)

\[ \approx \frac{C_1}{k} (\beta - 1) i \left[ 1 - \left( 1 + \frac{1}{(1 - \beta)} \frac{(1 - \beta) \alpha}{N i^{1-\beta}} k \left( e^{k T} - e^{k (T-t_1)} \right) \right) \right] \]

\[ + \frac{C_1 A (\beta - 1)}{k} i \beta \left[ 1 - \left( 1 + \frac{\beta}{(1 - \beta)} \frac{(1 - \beta) \alpha}{N i^{1-\beta}} k \left( e^{k T} - e^{k (T-t_1)} \right) \right) \right] \]

(Using Binomial expansion)

\[ \approx \frac{C_1}{k} (\beta - 1) i \frac{\alpha}{N k i^{1-\beta}} \left( e^{k (T-t)} - e^{k T} \right) + \frac{C_1 A (\beta - 1)}{k} i \beta \frac{\alpha}{N k i^{1-\beta}} e^{k (T-t_1)} - e^{k T} \]

\[ HC \approx \frac{C_1}{k} (\beta - 1) i \frac{\alpha}{N k} \left( e^{k (T-t_1)} - e^{k T} \right) \left( 1 + i \beta - 1 A \right) \]

Substituting the value of A in the above expression, we get
\[ HC \approx \frac{C_1}{k} (\beta - 1) i \beta \frac{\alpha}{Nk} \left\{ e^{k(T-t_1)} - e^{kT} \right\}_{1+i^{\beta-1}}^\frac{1-i^{\beta-1}}{1+i^{\beta-1}} \left( 1-i^{\beta-1} \right) \left( -\frac{(1-\beta) \alpha e^{k(T-t_1)}}{Nk} \right) \]

\[ HC \approx \frac{C_1}{k} (\beta - 1) i \beta \frac{\alpha}{Nk} \left\{ e^{k(T-t_1)} - e^{kT} \right\}_{1+1-i^{\beta-1}}^\frac{1+1-i^{\beta-1}}{1+1} \left( -\frac{(1-\beta) \alpha e^{k(T-t_1)}}{Nk} \right) \]

\[ HC = \frac{C_1}{k} (\beta - 1) i \beta \frac{\alpha}{Nk} \left\{ e^{k(T-t_1)} - e^{kT} \right\}_{2-1-i^{\beta-1}}^\frac{2-1-i^{\beta-1}}{2} \left( -\frac{(1-\beta) \alpha e^{k(T-t_1)}}{Nk} \right) \]

**Case II**

For \( t_1 \leq t \leq T, \quad i \geq S_0 \)

\[ \frac{dl}{dt} + \theta I = -\frac{d}{dt} \left( \frac{D(t)}{N(t)} \right) \]

It implies that \( \frac{dl}{dt} + \theta I = -\frac{D}{N} \left\{ (1-kt)e^{k(T-t)} \right\} \]

Further, we can express it as

\[ I e^{\theta t} = \int \frac{D}{N} (kt - 1) e^{k(T-t)} e^{\theta t} dt + C \]

C is a constant of integration.

\[ = \frac{Dk}{N} \int t e^{k(T-t)+\theta t} dt - \frac{Dk}{N} \int e^{k(T-t)+\theta t} dt + C \]

\[ = \frac{Dk}{N} \left\{ t \int e^{k(T-t)+\theta t} dt - \int e^{k(T-t)+\theta t} dt \right\} - \frac{Dk}{N} \int e^{k(T-t)+\theta t} dt + C \]

\[ = \frac{Dk}{N} \left\{ t \frac{e^{k(T-t)+\theta t}}{(\theta - k)} - \frac{e^{k(T-t)+\theta t}}{(\theta - k)} \right\} - \frac{Dk}{N} \left\{ \frac{e^{k(T-t)+\theta t}}{(\theta - k)} \right\} + C \]
\[
\frac{Dk}{N} e^{kT} \left\{ t e^{t(\theta-k)} - e^{-t(\theta-k)} \right\} \frac{De^{(\theta-k)t+kT}}{N(\theta-k)} + C
\]

At time \( t=t_1 \), \( i \geq S_0 \) and for a constant demand rate \( D=\alpha S_0 \beta \), we obtain

\[
i e^{\theta_1} = \frac{\alpha S_0 \beta}{N} e^{kT} \left\{ t e^{t_1(\theta-k)} - e^{t_1(\theta-k)} \right\} \frac{\alpha S_0 \beta}{N} e^{t_1(\theta-k)} e^{kT} \frac{e^{t(\theta-k)}}{N(\theta-k)} + C
\]

\[
I e^{\theta_t} = \frac{\alpha S_0 \beta}{N} e^{kT} \left\{ t e^{t(\theta-k)} - e^{t(\theta-k)} - t e^{t_1(\theta-k)} - e^{t_1(\theta-k)} \right\} \frac{e^{t_1(\theta-k)+kT}}{(\theta-k)} + i e^{\theta_1}
\]

\[
I = \frac{\alpha S_0 \beta}{N} e^{(kT-\theta t)} \left\{ t e^{t(\theta-k)} - e^{t(\theta-k)} - t e^{t_1(\theta-k)} - e^{t_1(\theta-k)} \right\} \frac{e^{t_1(\theta-k)+kT}}{(\theta-k)} e^{-\theta t} + i e^{\theta(t_1-t)}
\]

\[
I = \frac{\alpha S_0 \beta}{N} e^{(kT-\theta t)} - \frac{\alpha S_0 \beta}{N} e^{(T-k)} - \frac{\alpha S_0 \beta}{N} e^{t_1(\theta-k)} \frac{e^{(kT-\theta t + \theta t_1-1k)}}{N(\theta-k)} + \frac{\alpha S_0 \beta}{N} e^{(kT-\theta t + \theta t_1-1k)} \frac{e^{(T-k-\theta t)}}{N(\theta-k)} + \frac{\alpha S_0 \beta}{N} e^{(T-\theta t)} \frac{e^{t_1(\theta-k)+kT-\theta t}}{N(\theta-k)} + i e^{\theta(t_1-t)}
\]

Now further,
\[ HC = C_1 \int_{t_1}^{T} \theta (t_1-t) \, dt, \text{ then} \]
\[ HC = C_1 \int_{t_1}^{T} \left[ \frac{\alpha S^\beta}{N(\theta-k)} \left( t \left( k T - k t \right) - k e^{t(T-k)} - t_1 e^{k T - \theta t + \theta t_1 - t_1} k \right) + k e^{k T - \theta t + \theta t_1 - t_1} k - e^{t(\theta-k)+k T - \theta t + t_1(\theta-k)+k T - \theta t} \right] \, dt \]
\[ + i \int_{t_1}^{T} e^{t_1-t} \, dt \]

It implies that
\[ HC = C_1 \int_{t_1}^{T} \left[ \frac{\alpha S^\beta}{N(\theta-k)} \left( t k e^{k T - k t} - k e^{t(T-k)} - t_1 e^{k T - \theta t + \theta t_1 - t_1} k \right) + k e^{k T - \theta t + \theta t_1 - t_1} k - e^{k T - t} + t_1(\theta-k)+k T - \theta t \right] \, dt + i \int_{t_1}^{T} e^{t_1-t} \, dt \]
\[ HC = C_1 \int_{t_1}^{T} \left[ \frac{\alpha S^\beta}{N(\theta-k)} \left( t k e^{k T - k t} - k e^{t(T-k)} - t_1 e^{k T - \theta t + \theta t_1 - t_1} k \right) + k e^{k T - \theta t + \theta t_1 - t_1} k - e^{k T - t} + t_1(\theta-k)+k T - \theta t \right] \, dt \]
\[ + i \int_{t_1}^{T} e^{t_1-t} \, dt \]
\[ HC = C_1 \frac{\alpha S_0^B}{N(\theta - k)} \left[ \begin{array}{c} T_k e^{k(T-T)} - t_{1k} e^{k(T-t_1)} \\ -k e^{T(T-k)} - t_{1k} e^{T-t_1} \end{array} \right] - \left[ \frac{e^{k(T-T)}}{(-k)} - \frac{e^{k(T-t_1)}}{(-k)} \right] \\
\left[ \begin{array}{c} -k e^{T(T-k)} - t_{1k} e^{T-k} \\ (T-k) e^{T-k} - (T-k) \end{array} \right] + i \left[ \theta (t_1 - T) - 1 \right] \\
+ \left[ \begin{array}{c} (k(T-\theta T+t_1-t_1k)) \frac{e^{k(T-k)}}{(-\theta)} \\ (k(T-\theta t_1+\theta t_1-t_1k)) \frac{e^{k(T-t_1)}}{(-\theta)} \end{array} \right] \\
+ \left[ \begin{array}{c} k e^{k(T-k)} (T-k) \\ (-\theta) \end{array} \right] - \left[ \begin{array}{c} k e^{k(T-k)} (T-k) \\ (-\theta) \end{array} \right] \]
\[
HC \approx C_1 \frac{\alpha S^B_0}{N(\theta - k)} \left[ \begin{array}{c}
T_k e^{k(T-T)} - t_1 e^{k(T-t_1)} \\
\frac{(-k)}{(T-k)} - \frac{t_1 e^{k(T-t_1)}}{(-k)}
\end{array} \right] - \left( \begin{array}{c}
e^{k(T-T)} - e^{k(T-t_1)} \\
\frac{(-k)}{(T-k)} - \frac{t_1 e^{k(T-t_1)}}{(-k)}
\end{array} \right)
\]
\[
+ \left( \begin{array}{c}
\left( \frac{k(T-\theta T + \theta t_1 - t_1)}{(-\theta)} \right) + \frac{(k T - t_1)}{(-\theta)} \\
\left( \frac{k(T-\theta T + \theta t_1 - t_1)}{(-\theta)} \right) + \frac{(k T - t_1)}{(-\theta)}
\end{array} \right) + \left( \begin{array}{c}
e^{\theta(t_1-T)} - 1 \\
e^{\theta(t_1-T)} - 1
\end{array} \right)
\]
\[
HC \approx C_1 \frac{\alpha S^B_0}{N(\theta - k)} \left[ \begin{array}{c}
-T_1 e^{k(T-t_1)} \\
\frac{(-k)}{(T-k)} - \frac{t_1 e^{k(T-t_1)}}{(-k)}
\end{array} \right] - \left( \begin{array}{c}
\frac{1}{(-k)} - \frac{e^{k(T-t_1)}}{(-k)} \\
\frac{1}{(T-k)} - \frac{t_1 e^{k(T-t_1)}}{(T-k)}
\end{array} \right)
\]
\[
- \left( \begin{array}{c}
\left( \frac{k(T-\theta T + \theta t_1 - t_1)}{(-\theta)} \right) + \frac{(k T - t_1)}{(-\theta)} \\
\left( \frac{k(T-\theta T + \theta t_1 - t_1)}{(-\theta)} \right) + \frac{(k T - t_1)}{(-\theta)}
\end{array} \right) + \left( \begin{array}{c}
k^{(k T - t_1)} \left( \frac{k(T-\theta T + \theta t_1 - t_1)}{(-\theta)} \right) + \frac{(k T - t_1)}{(-\theta)} \\
k^{(k T - t_1)} \left( \frac{k(T-\theta T + \theta t_1 - t_1)}{(-\theta)} \right) + \frac{(k T - t_1)}{(-\theta)}
\end{array} \right)
\]
\[
+ \left( \begin{array}{c}
e^{\theta(t_1-T)} - 1 \\
e^{\theta(t_1-T)} - 1
\end{array} \right)
\]
\[
HC \approx C_1 \frac{\alpha S^B_0}{N(\theta - k)} \left[ \begin{array}{c}
-T_1 e^{k(T-t_1)} \\
\frac{(-k)}{(T-k)} - \frac{t_1 e^{k(T-t_1)}}{(-k)}
\end{array} \right] + \left( \begin{array}{c}
\frac{k(T-t_1)}{(-k)} \\
\frac{k(T-\theta T)}{(-k)}
\end{array} \right) - \left( \begin{array}{c}
\frac{e^{T(T-k)}}{(T-k)} - e^{t_1(T-k)} \\
\frac{e^{T(T-k)}}{(T-k)} - e^{t_1(T-k)}
\end{array} \right)
\]
\[\begin{align*}
&+ \frac{t_1}{\theta} \left[ e^{\left( kT - \theta t + \theta t_1 - t_1 \right)} - e^{\left( kT - t_1 \right)} \right] - \frac{k}{\theta} \left[ e^{\left( kT - \theta t + \theta t_1 - t_1 \right)} - e^{\left( kT - t_1 \right)} \right] \\
&+ i \left[ e^{\theta (t_1 - T)} - 1 \right]
\end{align*}\]

### 2.5 Total Revenue and its Quantities

We analyze, here, the total revenue and obtain corresponding quantities of inventory for the following cases:

**Case I (a)**

We know that

\[TR = P \int_0^{t_1} \alpha \left( \frac{t}{e^{\beta}} \right) dt = P \int_0^{t_1} \left( \frac{k \left( t - t_1 \right)}{\beta + 1} \right) dt\]

\[= P \alpha i^\beta \int_0^{t_1} e^{\frac{k \left( t - t_1 \right)}{\beta + 1}} dt = P \alpha i^\beta \left( \frac{1 + \beta}{\beta k} \right) e^{\frac{k \left( t - t_1 \right)}{\beta + 1}}\]

\[= P \alpha i^\beta \left( \frac{1 + \beta}{\beta k} \right) \left( e^{\frac{t_1}{\beta + 1}} - e^{\frac{-t_1}{\beta + 1}} \right) = P \alpha i^\beta (1 + \beta) \left( e^{\frac{-k t_1}{\beta + 1}} - e^{\frac{k t_1}{\beta + 1}} \right)\]

\[= P \alpha i^\beta \left( \frac{1 + \beta}{k \beta t_1} \right) \left( e^{\frac{k t_1}{\beta + 1}} - 1 \right)\]

Hence, quantity of inventory is obtained as
\[ Q = \frac{\alpha \beta (1 + \beta)}{\frac{k \beta t_1}{e^{(\beta+1)}}} \left( \frac{k \beta t_1}{e^{(\beta+1)}} - 1 \right) \]

(2.5.1)

**Case I (b)**

Total revenue is calculated

\[
TR = P \alpha \int_0^{t_1} \left[ \left( 1 - \beta \right) e^{\left( \frac{k (T-t)}{Nk} \right)} + \left( \beta - 1 \right) e^{\left( \frac{k T}{Nk} \right)} \right]^{\beta} dt
\]

\[
= P \alpha \int_0^{t_1} \left[ 1 - \beta + \left( \beta - 1 \right) e^{\left( \frac{k T}{Nk} \right)} \right]^{\beta} dt
\]

\[
= P \alpha \int_0^{t_1} \left[ A + Be^{\left( \frac{k T}{Nk} \right)} \right]^{\beta} dt
\]

\[
\approx P \alpha \int_0^{t_1} \left( A - U \right)^{-1} dU
\]

(Due to only first term of the binomial expansion)

\[
\approx P \alpha \int_0^{t_1} \frac{1}{k A - U} dU = P \alpha \int_0^{t_1} \frac{1}{k} \left( \frac{A}{A - U} \right)^{-1} dU
\]

\[
\approx P \alpha \int_0^{t_1} U^{\beta - 1} \left( 1 + \frac{A}{U} \right) dU
\]

\[
\approx P \alpha \int_0^{t_1} U^{\beta - 1} \left( 1 + \frac{A}{U} \right) dU
\]
\[
\approx -PA \frac{\alpha}{k} \int_0^1 U^{1-\beta} dU - PA \frac{\alpha}{k} \int_0^{t_1} U^{1-\beta} dU
\]

\[
\approx \frac{P \alpha (\beta - 1)}{\beta k} \left( \frac{\beta}{1-\beta} \right)_{t=0}^{t=t_1} + \frac{P \alpha A (\beta - 1)}{(2 \beta - 1)k} \left( \frac{2^{-1}}{\beta-1} \right)_{t=0}^{t=t_1}
\]

\[
\approx \frac{P \alpha (\beta - 1)}{k \beta} \left( i \beta - i \beta \left( 1 + (\beta - 1) \alpha (e^{kT} - e^{-k(T-t_1)}) \right)^{\beta - 1} \right)
\]

\[
+ \frac{(\beta - 1)P \alpha A}{k (2 \beta - 1)} \left( i (2 - \beta - 1) + (\beta - 1) \alpha (e^{kT} - e^{-k(T-t_1)}) \right)^{\frac{2 \beta - 1}{1 - \beta}}
\]

\[
\approx \frac{P \alpha (\beta - 1)}{k \beta} \left( 1 - \frac{\beta}{(1 - \beta)} (\beta - 1) \alpha (e^{kT} - e^{-k(T-t_1)}) \right)^{\frac{1}{1 - \beta}}
\]

\[
+ \frac{(\beta - 1)P \alpha A}{k (2 \beta - 1)} \left( 1 - \frac{(2 \beta - 1)(\beta - 1)(e^{kT} - e^{-k(T-t_1)})}{N k l^{1-\beta}} \right)^{\frac{1}{1 - \beta}}
\]
\[ \approx \frac{P \alpha^2 i^\beta (\beta-1)(e^{kT} - e^{k(T-t_1)})}{k^2 N i^{1-\beta}} + \frac{(\beta-1) P \alpha A i^{2\beta-1}}{k(2\beta-1)} \\
- \frac{(\beta-1) (\beta-1) \alpha (e^{kT} - e^{k(T-t_1)})}{N k i^{1-\beta}} \]

\[ \approx \frac{P \alpha^2 i^\beta (\beta-1)(e^{kT} - e^{k(T-t_1)})}{k^2 N i^{1-\beta}} + \frac{(\beta-1) \alpha^2 P A (e^{kT} - e^{k(T-t_1)}) i^{2\beta-1}}{N k^2 i^{1-\beta}} \]

\[ \approx \frac{P \alpha^2 (\beta-1)(e^{kT} - e^{k(T-t_1)})}{N k^2} \left\{ i^{2\beta-1} + i^{3\beta-2} \left[ i^{1-\beta} - (\beta-1) \alpha e^{k(T-t_1)} \right] \right\} \]

Hence, quantity is given as

\[ Q = \frac{\alpha^2 (\beta-1)(e^{kT} - e^{k(T-t_1)})}{N k^2} \left\{ i^{2\beta-1} - (\beta-1) \alpha e^{k(T-t_1)} i^{3\beta-2} \right\} \]

**Case II**

We calculate the revenue for the linear demand as

\[ \text{Revenue} = P \int_{t_1}^{T} \alpha S_0^\beta \, dt = P \alpha S_0^\beta (T - t_1) \]

And, hence, corresponding quantity is given as

\[ Q = \alpha S_0^\beta (T - t_1) \quad (2.5.2) \]

**2.6 Purchasing Cost**

We obtain purchasing cost for the model as follows:
Case 1(a)

PC = C₂ S. It finally gives as

\[ PC = C₂ i e^{(\beta+1)} \]  
(2.6.3)

Case I (b)

PC = C₂ S, which implies that

\[ PC = C₂ \left[ t^{1-\beta} + \frac{(\beta-1)\alpha}{Nk} \left( e^{kT} - e^{k(T-t)} \right) \right]^{1-\beta} \]  
(2.6.4)

Case II

In order to find the initial stock, we substitute here \( t = 0 \)

which gives us

\[ I(0) = \frac{\alpha S₀ β k₀ e^{(kT-k₀)}}{N(θ-k)} - \frac{\alpha S₀ β k₁ e^{0(T-k)}}{N(θ-k)} - \frac{\alpha S₀ β t₁ e^{(kT-θ+θt₁-1t₁k)}}{N(θ-k)} \]

\[ + \frac{αS₀ β}{N(θ-k)} e^{(kT-θ+θt₁-1t₁k)} + \frac{αS₀ β}{N(θ-k)} e^{t₁(θ-k)+kT-θ0} + ie^{θ(t₁-0)} \]

It further shows that

\[ S = -\frac{αS₀ β k₀}{N(θ-k)} - \frac{αS₀ β t₁ e^{(kT+θt₁-1t₁k)}}{N(θ-k)} + \frac{αS₀ β k₁ e^{(kT+θt₁-1t₁k)}}{N(θ-k)} - \frac{αS₀ β e^{kT}}{N(θ-k)} \]

\[ + \frac{αS₀ β}{N(θ-k)} e^{t₁(θ-k)+kT} + ie^{θt₁} \]
Hence,

\[ PC = C_2 S \]

\[
PC = C_2 \left\{ -\frac{\alpha S_0^\beta k}{N(\theta-k)} - \frac{\alpha S_0^\beta t_1 e^{(kT+\theta t_1-k)\frac{\theta}{t_1}}}{N(\theta-k)} + \frac{\alpha S_0^\beta k e^{(kT+\theta t_1-k)\frac{\theta}{t_1}}}{N(\theta-k)} \right\}
\]

\[
\alpha S_0^\beta e^{kT} + \frac{\alpha S_0^\beta t_1(\theta-k)+kT}{N(\theta-k)} + ie^{\theta t_1}
\]

2.7 Price Determination for the Model

Under this section, we determine the price under the following cases:

**Case 1(a)**

For \(0 \leq t \leq t_1\) and \(S > i\)

Profit = TR - HC - PC - Set-up Cost

For optimum profit

\[
\frac{d \text{profit}}{dQ} = \frac{\partial TR}{\partial Q} - \frac{\partial HC}{\partial Q} - \frac{\partial PC}{\partial Q} - \frac{\partial SC}{\partial Q} = 0
\]

\[
\frac{\partial}{\partial Q} TR = \frac{\partial HC}{\partial Q} + \frac{\partial PC}{\partial Q} + \frac{\partial SC}{\partial Q}
\]

Hence,

\[
P = \frac{\partial HC}{\partial i} \times \frac{\partial i}{\partial Q} + \frac{\partial PC}{\partial i} \times \frac{\partial i}{\partial Q} + \frac{\partial SC}{\partial i} \times \frac{\partial i}{\partial Q}
\]

Now, in view of equations (2.3.6), (2.3.7) and (2.6.3) we obtain
\[
d\frac{HC}{dQ} = C_1 \frac{-t_1 k \beta k}{(1-e^{(\beta+1)})e^{(\beta+1)}}
\]
and
\[
\frac{\partial PC}{\partial i} = C_2 e^{(\beta+1)}
\]

It finally gives us

\[
p = C_1 \frac{-k t_1 \beta k}{(1-e^{(\beta+1)})e^{(\beta+1)}} + \frac{-t_1 k \beta k}{C_2 e^{(\beta+1)} k e^{(\beta+1)}}
\]

\[
\alpha i \beta^{-1}(e^{(\beta+1)} - 1) + \alpha (\beta + 1)(e^{(\beta+1)} - 1) i \beta^{-1}
\]

Case I (b)

For \(0 \leq t \leq t_1\)

We use the above formula of profit as in case I (a) and obtain that

\[
\frac{\partial HC}{\partial Q} = \frac{\partial HC}{\partial i} \times \frac{\partial i}{\partial Q}
\]

\[
\frac{\partial PC}{\partial Q} = \frac{\partial PC}{\partial i} \times \frac{\partial i}{\partial Q}
\]

\[
\frac{\partial PC}{\partial i} = C_2 \left[1 - \beta + \frac{(\beta-1)\alpha}{Nk} \left(e^{kT} - e^{-k(T-t_1)}\right)\right] \frac{\beta}{(1-\beta)} i^{-\beta}
\]
\[
\frac{\partial HC}{\partial i} = \frac{C_1 \alpha}{k^2 N} \beta (\beta - 1) \left( e^{k(T-t_1)} - e^{kT} \right) \left( 2 \beta i \beta - 1 - \frac{(1-\beta) e^{k(T-t_1)(2\beta - 1)}i^{2\beta - 2}}{N k} \right)
\]

\[
\frac{\partial Q}{\partial i} = \frac{\alpha^2 (\beta - 1) e^{kT} - e^{k(T-t_1)}}{N k^2} \left( 2(2\beta - 1)i^{2\beta - 2} - (\beta - 1)(3\beta - 2) \alpha e^{k(T-t_1)i^{3\beta - 3}} \right)
\]

\[
C_1 \beta (\beta - 1) \alpha \left( e^{k(T-t_1)} - e^{kT} \right)
\]

\[
\times \left\{ 2\beta N k i \beta - 1 + \alpha (\beta - 1)(2\beta - 1) e^{k(T-t_1)i^{2\beta - 2}} \right\}
\]

\[
\frac{\partial HC}{\partial Q} = \frac{C_1 \beta i^{\beta - 1} \left( \alpha (1-\beta)(2\beta - 1) e^{k(T-t_1)i^{\beta - 1}} - 2\beta N k \right)}{\alpha \left\{ 2(2\beta - 1)N k i^{2\beta - 2} - (\beta - 1)(3\beta - 2) \alpha e^{k(T-t_1)i^{3\beta - 3}} \right\}}
\]

\[
\frac{\partial PC}{\partial Q} = \frac{\partial PC}{\partial i} \times \frac{\partial i}{\partial Q}
\]

\[
\frac{\partial PC}{\partial i} = C_2 \left[ i^{1-\beta} + \frac{(\beta - 1)\alpha}{N k} \left( e^{kT} - e^{k(T-t_1)} \right) \right] \frac{\beta}{(1-\beta) i^{\beta - 1}}
\]
\[
\frac{\partial PC}{\partial Q} = \frac{C_2 \left[ N k \left( 1 - \beta \right) + (\beta - 1) \alpha \left\{ e^{kT} - e^{k\left( T-t_1 \right)} \right\} \right]^{\beta}}{(1-\beta)^{1-\beta} N k^2} \\
\times \frac{2(2\beta-1)N k i^{2\beta-2} - (\beta - 1)(3\beta - 2) \alpha e^{k\left( T-t_1 \right) i^{3\beta-3}}}{\alpha^2 (\beta - 1)(e^{kT} - e^{k\left( T-t_1 \right)})} \\
\]

Hence, finally price can be given as

\[
P = \frac{\partial HQ}{\partial Q} + \frac{\partial PC}{\partial Q} \\
\]

\[
P = \frac{C_1 \beta i^{\beta-1} \left\{ \alpha (1 - \beta)(2\beta - 1)e^{k\left( T-t_1 \right) i^{\beta-1}} - 2\beta N k \right\}}{\alpha \left\{ 2(2\beta-1)N k i^{2\beta-2} - (\beta - 1)(3\beta - 2) \alpha e^{k\left( T-t_1 \right) i^{3\beta-3}} \right\}} \\
\]

\[
+ \frac{C_2 \left[ N k \left( 1 - \beta \right) + (\beta - 1) \alpha \left\{ e^{kT} - e^{k\left( T-t_1 \right)} \right\} \right]^{\beta}}{(1-\beta)^{1-\beta} N k^2} \\
\times \frac{2(2\beta-1)N k i^{2\beta-2} - (\beta - 1)(3\beta - 2) \alpha e^{k\left( T-t_1 \right) i^{3\beta-3}}}{\alpha^2 (\beta - 1)(e^{kT} - e^{k\left( T-t_1 \right)})} \\
\]

Page 82 of 224
\[ P = C_1 \beta i^{-1} \left\{ \alpha (1 - \beta) (2\beta - 1) e^{k(T-t_1)} \right\} (\beta - 1) \left( e^k T^{-e^k(T-t_1)} \right) \]

\[ C_2 \left[ N k i^{-1} - \beta + (\beta - 1) \alpha \left( e^k T^{-e^k(T-t_1)} \right) \right] \frac{\beta}{(1 - \beta)} i^{-1} \beta N k^2 \]

\[ \alpha^2 (\beta - 1) \left( e^k T^{-e^k(T-t_1)} \right) \times \left\{ 2(2\beta - 1) N k i^{2\beta - 2} (\beta - 1) (3\beta - 2) \alpha e^{k(T-t_1)} i^{3\beta - 3} \right\} \]

**Case II**

For \( t_1 \leq t \leq T \)

As we know that

\[ P = \frac{\partial HC}{\partial S_0} \times \frac{\partial S_0}{\partial Q} + \frac{\partial PC}{\partial S_0} \times \frac{\partial S_0}{\partial Q} \]

Since \( P = \frac{\partial HC}{\partial Q} + \frac{\partial PC}{\partial Q} + \frac{\partial SC}{\partial Q} \)

We have

\[ \frac{\partial HC}{\partial S_0} \approx C_1 \alpha \beta S_0^{-1} \frac{T + t}{N(\theta - k)} \left[ T + t e^{k(T-t_1)} + \frac{1 - e^{k(T-t_1)}}{k} - k \frac{e^{T(T-k)} - e^{(T-t_1)(T-k)}}{(T-k)} \right] \]

\[ + \frac{t}{\theta} \left( e^{(kT-\theta T+\theta t_1-t_1k)} - e^{(kT-t_1k)} \right) - \frac{1}{\theta} \left( e^{(kT-\theta T+\theta t_1-t_1k)} - e^{(kT-t_1k)} \right) \]
\[
\frac{\partial P}{\partial S_0} = C_2 \left\{ \frac{\alpha \beta S_0^{\beta-1}}{N(\theta - k)} \cdot \left[ -k - t_1 e^{k(T+\theta t_1-t_1 k)} + k e^{k(T+\theta t_1-t_1 k) - e^{k T + e^{t_1 (\theta-k)+k T}}} \right] \right\}
\]

\[
\frac{\partial Q}{\partial S_0} = \alpha \beta S_0^{\beta-1} (T-t_0)
\]

\[
\frac{\partial Q}{\partial S_0} = \frac{C_1}{k(T-t_1)N(\theta-k)(T-k)\theta} \cdot \left[ -k(T-k)\theta^{T+t_1 e^{k(T-t_1)}} + \theta(T-k)\left( e^{k(T-t_1)} - k^2 \theta \left( e^{T(k-k) - e^{t_1 (T-k)}} \right) \right) + t_1 k(T-k) e^{k(T-\theta T+\theta t_1-t_1 k) - e^{k(T-t_1 k) - e^{k(T-t_1 k) - e^{k(T-t_1 k)}}}} \right]
\]

\[
\frac{\partial P}{\partial Q} = \frac{C_2}{N(\theta - k)(T - t_1)} \left\{ -k - t_1 e^{k(T+\theta t_1-t_1 k)} + k e^{k(T+\theta t_1-t_1 k) - e^{k T + e^{t_1 (\theta-k)+k T}}} \right\}
\]

Therefore, finally we get

\[
P = \frac{C_1}{k(T-t_1)N(\theta-k)(T-k)\theta} \cdot \left[ -k(T-k)\theta^{T+t_1 e^{k(T-t_1)}} + \theta(T-k)\left( e^{k(T-t_1)} - k^2 \theta \left( e^{T(k-k) - e^{t_1 (T-k)}} \right) \right) + t_1 k(T-k) e^{k(T-\theta T+\theta t_1-t_1 k) - e^{k(T-t_1 k) - e^{k(T-t_1 k) - e^{k(T-t_1 k)}}}} \right]
\]
\[ + \left\{ -k - t_1 e^{(kT + \theta t_1 - t_1 k)} + k e^{(kT + \theta t_1 - t_1 k)} - e^{kT} \right\} \frac{t_1(\theta - k) + kT}{N(\theta - k)(T - t_1)} \]

Finally, we present in brief the price of an item of the inventory given for the various cases as under:

**Case 1 (a)**

\[ k \left( \frac{t - t_1}{\beta + 1} \right) \]

For \( 0 < t < t_1 \) and \( l = i e^{-\beta} \)

We find out the price as

\[ P = \frac{C_1(1 - e^{(\beta + 1)})e^{(\beta + 1)}}{t_1 \beta k} + \frac{C_2 e^{(\beta + 1)}k e^{(\beta + 1)}}{t_1 \beta k} \]

\[ \alpha i \beta^{-1} (e^{(\beta + 1)} - 1) \quad \alpha (\beta + 1)(e^{(\beta + 1)} - 1)i \beta^{-1} \]

**Case 1 (b)**

For \( 0 < t < t_1 \)

and \( l = \left[ 1 - \beta + (\beta - 1)\alpha \left( e^{k(T - t)} - e^{k(T - t_1)} \right) \right]^{1 - \beta} \)

We obtain the price as
\[ P = C_1 \beta i^{\beta-1} \left[ (1-\beta)(2\beta-1)e^{k(T-t_1)} \beta - 2\beta Nk \right] \alpha (\beta-1)(e^{\frac{kT}{1-\beta}} - e^{\frac{k(T-t_1)}{1-\beta}}) + C_2 \left[ Nk i^{1-\beta} + (\beta-1)\alpha \left( e^{\frac{kT}{1-\beta}} e^{\frac{k(T-t_1)}{1-\beta}} \right) \right] \beta \frac{2}{(1-\beta)^2} - \beta Nk^2 \]
\[
\frac{\alpha^2 (\beta-1)(e^{\frac{kT}{1-\beta}} - e^{\frac{k(T-t_1)}{1-\beta}})}{2 (2\beta-1) Nk i^{2\beta-2} - (\beta-1)(3\beta-2)\alpha e^{\frac{k(T-t_1)}{1-\beta}}^{3\beta-3}}
\]

**Case II**

For \( t_1 \leq t < T \) and

\[
I = \frac{\alpha S^\beta}{N(\theta-k)} \left( te^{(kT-kt)} - ke\left(\frac{kT-\theta t + \theta t_1 - t_1 k}{1-\beta}\right) + ke\left(\frac{kT-\theta t + \theta t_1 - t_1 k}{1-\beta}\right) - e^{t(\theta-k)+kT-\theta t} + e^{t_1(\theta-k)+kT-\theta t} + i e^{\left(\theta(t_1-t)\right)} \right)
\]

We determine the price as

\[
P = C_1 \left[ -\frac{k}{T-t_1} \theta \left( T + t_1 e^{\frac{kT-t_1}{1-\beta}} \right) + \theta \left( T-k \right) \left( 1-e^{\frac{kT-t_1}{1-\beta}} \right) - k^2 \theta \left( e^{T-k} - e^t \left( T-k \right) \right) + t_1 k(T-k) \left( e^{kT-\theta t + \theta t_1 - t_1 k} - e^{kT-t_1 k} \right) - k^2 (T-k) \left( e^{kT-\theta t + \theta t_1 - t_1 k} - e^{kT-t_1 k} \right) \right]
\]
\[
+ \frac{C_2}{N(\theta-k)(T-t_1)T} \left( -k - t_1 e^{\left( kT+\theta t_1 - t_1 k \right)} + ke \left( kT+\theta t_1 - t_1 k \right) - e^{kT} \left( T-1 + t_1 \left( T-k \right) \right) \right)
\]

Page 86 of 224
2.8 Computing Algorithm

We develop the following algorithm for computing the price of an item of the inventory for the model:

Step 1: Begin
Step 2: Input data
Step 3: Switch
Step 4: Case I (a)
Step 5: Compute selling points
Step 6: Case I (b)
Step 7: Compute selling points
Step 8: Case II
Step 9: Compute selling points
Step 10: Switch
Step 11: Case I (a)
Step 12: Compute inventory level
Step 13: Case I (b)
Step 14: Compute inventory level
Step 15: Case II
Step 16: Compute inventory level
Step 17: Switch
Step 18: Case I (a)
Step 19: HC ← first derivatives of HC w.r.t. Q
Step 20: TR ← first derivatives of TR w.r.t. Q
Step 21: PC ← first derivatives of PC w.r.t. Q
Step 22: Case I (b)
Step 23: HC ← first derivatives of HC w.r.t. Q
Step 24: TR ← first derivatives of TR w.r.t. Q
Step 25: PC ← first derivatives of PC w.r.t. Q
Step 26: Case II
Step 27: HC ← first derivatives of HC w.r.t. Q
Step 28: TR ← first derivatives of TR w.r.t. Q
Step 29: PC ← first derivatives of PC w.r.t. Q
Step 30: Switch
Step 31: Case I (a)
Step 32: Compute price
Step 33: Case I (b)
Step 34: Compute price
Step 35: Case II
Step 36: Compute price
Step 37: End

2.9 Sensitivity Analysis

In this section, we discuss the sensitivity analysis through numerical illustration of the model by developing the computer program in C++ and assuming following hypothetical data for program demonstration.

Case 1(a)

(i) Here, we compute inventory level and price of an item for the following data:
\( \alpha = 100, \beta = 0.6, N = 25, T = 50, t = 20, t_1 = 30, C_1 = 40, C_2 = 30; \)\\

<table>
<thead>
<tr>
<th>( S_0 )</th>
<th>( S )</th>
<th>( i )</th>
<th>( I )</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>1000</td>
<td>750</td>
<td>214.91</td>
<td>547.29</td>
</tr>
<tr>
<td>1000</td>
<td>10000</td>
<td>5000</td>
<td>2865.42</td>
<td>1542.40</td>
</tr>
<tr>
<td>10000</td>
<td>100000</td>
<td>90000</td>
<td>25788.79</td>
<td>4952.51</td>
</tr>
<tr>
<td>50000</td>
<td>100000</td>
<td>90000</td>
<td>25788.77</td>
<td>3714.44</td>
</tr>
<tr>
<td>100000</td>
<td>1000000</td>
<td>900000</td>
<td>257887.75</td>
<td>9330.25</td>
</tr>
</tbody>
</table>

(ii) We compute inventory level, deterioration rate, selling points and price of an item for the following data:
\( \alpha = 1000, \beta = 0.6, i = 10000, S_0 = 1000, S = 100000, T = 50, t = 20, t_1 = 30, k = 0.01, C_1 = 40, C_2 = 30; \)

<table>
<thead>
<tr>
<th>( N )</th>
<th>( I )</th>
<th>( \theta )</th>
<th>( N ) (t)</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>9394.19</td>
<td>0.00625</td>
<td>37</td>
<td>38.18</td>
</tr>
<tr>
<td>100</td>
<td>9394.19</td>
<td>0.00625</td>
<td>61</td>
<td>38.18</td>
</tr>
<tr>
<td>200</td>
<td>9394.19</td>
<td>0.00625</td>
<td>74</td>
<td>38.18</td>
</tr>
<tr>
<td>400</td>
<td>9394.19</td>
<td>0.00625</td>
<td>296</td>
<td>38.18</td>
</tr>
<tr>
<td>500</td>
<td>9394.19</td>
<td>0.00625</td>
<td>370</td>
<td>38.18</td>
</tr>
</tbody>
</table>

(iii) Here, we compute rate of deterioration, inventory level, selling points and price of an item for the following data:
\[ \alpha = 1000, \beta = 0.6, i = 50000, S_0 = 10000, S = 100000, N = 100, T = 50, t_1 = 40, k = 0.01, C_1 = 40, C_2 = 30; \]

**Table: 2.9. 3**

<table>
<thead>
<tr>
<th>T</th>
<th>( \theta )</th>
<th>I</th>
<th>( N(t) )</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.00625</td>
<td>41452.26</td>
<td>67</td>
<td>62</td>
</tr>
<tr>
<td>20</td>
<td>0.00625</td>
<td>44125.42</td>
<td>74</td>
<td>62</td>
</tr>
<tr>
<td>25</td>
<td>0.00625</td>
<td>45525.96</td>
<td>78</td>
<td>62</td>
</tr>
<tr>
<td>30</td>
<td>0.00625</td>
<td>46970.96</td>
<td>82</td>
<td>62</td>
</tr>
<tr>
<td>35</td>
<td>0.00625</td>
<td>48461.89</td>
<td>86</td>
<td>62</td>
</tr>
</tbody>
</table>

(iv) In this table, we compute rate of deterioration, inventory level, selling points and price of an item by changing the shape parameter \( \alpha \) and with following data:
\[ \beta = 0.6, i = 50000, S_0 = 10000, S = 100000, N = 100, T = 50, t = 20, t_1 = 30, C_1 = 40, C_2 = 30, k = 0.01. \]

**Table: 2.9.4**

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \theta )</th>
<th>I</th>
<th>( N(t) )</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.00625</td>
<td>46970.96</td>
<td>74</td>
<td>726.80</td>
</tr>
<tr>
<td>500</td>
<td>0.00625</td>
<td>46970.96</td>
<td>74</td>
<td>145.36</td>
</tr>
<tr>
<td>700</td>
<td>0.00625</td>
<td>46970.96</td>
<td>74</td>
<td>103.80</td>
</tr>
<tr>
<td>1000</td>
<td>0.00625</td>
<td>46970.96</td>
<td>74</td>
<td>72.68</td>
</tr>
<tr>
<td>2000</td>
<td>0.00625</td>
<td>46970.96</td>
<td>74</td>
<td>36.34</td>
</tr>
</tbody>
</table>
(v) Here, we compute deterioration rate, inventory level, price of an item for different values of end-level inventory from following data: 
\( \alpha =1000, \beta =0.6, S_0 =100000, S =1000000, N =100, T=50, t=20, 
\ t_1=30, C_1=40, C_2=30, k = 0. \)

**Table: 2.9.5**

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( i )</th>
<th>( I )</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00625</td>
<td>20000</td>
<td>1878.84</td>
<td>20.06</td>
</tr>
<tr>
<td>0.00625</td>
<td>40000</td>
<td>37576.77</td>
<td>66.47</td>
</tr>
<tr>
<td>0.00625</td>
<td>50000</td>
<td>46970.96</td>
<td>72.67</td>
</tr>
<tr>
<td>0.00625</td>
<td>60000</td>
<td>56365.15</td>
<td>78.18</td>
</tr>
<tr>
<td>0.00625</td>
<td>80000</td>
<td>75153.53</td>
<td>87.71</td>
</tr>
</tbody>
</table>

(vi) Here, we compute deterioration rate, inventory level, price of an item for different values constant stock, initial stock and with following data: 
\( \alpha=1000, \beta=0.6, \ i=1000, \ T=50, \ t=20, \ t_1=30, \ C_1=40, \ C_2=30, 
\ N=100, k=0.01; \)

**Table: 2.9.6**

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( S_0 )</th>
<th>( S )</th>
<th>( I )</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00625</td>
<td>5000</td>
<td>10000</td>
<td>9394.19</td>
<td>38.17</td>
</tr>
<tr>
<td>0.00625</td>
<td>6000</td>
<td>100000</td>
<td>9394.19</td>
<td>38.17</td>
</tr>
<tr>
<td>0.00625</td>
<td>7000</td>
<td>1000000</td>
<td>9394.19</td>
<td>38.17</td>
</tr>
<tr>
<td>0.00625</td>
<td>8000</td>
<td>1000000</td>
<td>9394.19</td>
<td>38.17</td>
</tr>
<tr>
<td>0.00625</td>
<td>9000</td>
<td>1000000</td>
<td>9394.19</td>
<td>38.17</td>
</tr>
</tbody>
</table>
(vii) In this table, we compute deterioration rate, inventory level, price of an item for changing the size parameter and with following data:
\[ i = 1000, S_0 = 1000, S = 10000, T = 50, t = 35, t_1 = 20, C_1 = 40, C_2 = 30, N = 100, k = 0.01; \]

**Table: 2.9.7**

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \theta )</th>
<th>( I )</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.00625</td>
<td>7515.35</td>
<td>34.91</td>
</tr>
<tr>
<td>0.01</td>
<td>0.00990</td>
<td>7245.94</td>
<td>361325..00</td>
</tr>
<tr>
<td>0.06</td>
<td>0.009430</td>
<td>7239.49</td>
<td>3887452.91</td>
</tr>
<tr>
<td>0.001</td>
<td>0.009901</td>
<td>7243.09</td>
<td>622147.50</td>
</tr>
<tr>
<td>0.006</td>
<td>0.00994</td>
<td>90538.65</td>
<td>7659878.64</td>
</tr>
</tbody>
</table>

**Case 1(b)**

(i) We compute inventory level and price of a unit inventory item for different end-level inventory.

**Table: 2.9.8**

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( i )</th>
<th>( I )</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>500</td>
<td>500.077</td>
<td>32.56</td>
</tr>
<tr>
<td>5</td>
<td>700</td>
<td>700.15</td>
<td>29.01</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>1000.03</td>
<td>25.58</td>
</tr>
<tr>
<td>5</td>
<td>1500</td>
<td>1500.21</td>
<td>22.17</td>
</tr>
<tr>
<td>5</td>
<td>1700</td>
<td>1700.23</td>
<td>21.22</td>
</tr>
</tbody>
</table>
(ii) In this table, we compute inventory level and price of an item by changing shape parameter and end-level inventory with considering following data:
\[ S_0=100, S=10000, \beta=0.000006, T=15, t=5, t_1=10, C_1=10, C_2=30, k=0.01; \]

**Table: 2.9.9**

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( i )</th>
<th>( N )</th>
<th>( I )</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1000</td>
<td>20</td>
<td>1000.03</td>
<td>23.00</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>20</td>
<td>1000.15</td>
<td>8.35</td>
</tr>
<tr>
<td>10</td>
<td>1000</td>
<td>20</td>
<td>1000.23</td>
<td>4.21</td>
</tr>
<tr>
<td>12</td>
<td>1000</td>
<td>20</td>
<td>1000.29</td>
<td>2.15</td>
</tr>
<tr>
<td>14</td>
<td>1000</td>
<td>20</td>
<td>1000.41</td>
<td>0.81</td>
</tr>
</tbody>
</table>

(ii) Here, we compute price of an item and number of selling points by changing the size parameter and with following data:
\[ \alpha=50, S=100000, S_0 = 100, T = 15, t = 5, t_1=10, C_1= 10, C_2=30, k=0.1; \]

**Table: 2.9.10**

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( i )</th>
<th>( N )</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1000</td>
<td>20</td>
<td>51.66</td>
</tr>
<tr>
<td>0.2</td>
<td>1000</td>
<td>20</td>
<td>27.03</td>
</tr>
<tr>
<td>0.3</td>
<td>1000</td>
<td>20</td>
<td>15.72</td>
</tr>
<tr>
<td>0.4</td>
<td>1000</td>
<td>20</td>
<td>10.73</td>
</tr>
<tr>
<td>0.5</td>
<td>1000</td>
<td>20</td>
<td>9.01</td>
</tr>
</tbody>
</table>
(iii) In this table, we compute price of an item by different values of selling points and with following data:
\[ \alpha = 50, \beta = 0.000006, S = 100000, S_0 = 100, i = 1000, t_1 = 10, C_1 = 10, C_2 = 30, k = 0.01. \]

<table>
<thead>
<tr>
<th>N</th>
<th>T</th>
<th>t</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>15</td>
<td>5</td>
<td>71.23</td>
</tr>
<tr>
<td>50</td>
<td>20</td>
<td>5</td>
<td>63.73</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>7</td>
<td>56.26</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
<td>8</td>
<td>58.23</td>
</tr>
<tr>
<td>50</td>
<td>15</td>
<td>8</td>
<td>58.26</td>
</tr>
</tbody>
</table>

**Case II**

We compute price of an item and inventory-level for different values of shape parameter with following data:
\[ N = 50, T = 3, t = 2, t_1 = 1, C_1 = 10, C_2 = 30 \text{ and } \theta = 0.006. \]

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( S_0 )</th>
<th>( S )</th>
<th>( i )</th>
<th>( I )</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.6</td>
<td>100</td>
<td>1000</td>
<td>800</td>
<td>4558.70</td>
<td>60.70</td>
</tr>
<tr>
<td>50</td>
<td>0.6</td>
<td>100</td>
<td>1000</td>
<td>800</td>
<td>2676.96</td>
<td>60.70</td>
</tr>
<tr>
<td>40</td>
<td>0.6</td>
<td>100</td>
<td>1000</td>
<td>800</td>
<td>2300.61</td>
<td>60.70</td>
</tr>
<tr>
<td>20</td>
<td>0.6</td>
<td>100</td>
<td>1000</td>
<td>800</td>
<td>1547.91</td>
<td>60.70</td>
</tr>
<tr>
<td>10</td>
<td>0.6</td>
<td>100</td>
<td>1000</td>
<td>800</td>
<td>1171.56</td>
<td>60.70</td>
</tr>
</tbody>
</table>
(ii) In this table, we compute the price of an item and inventory level for different values of deterioration rate and different values of selling points with following data:
\[ \alpha = 50, \beta = 0.6, t = 2, T = 4, t_1 = 1, C_1 = 10, C_2 = 30, S_0 = 100, S = 1000, i = 800, k = 0.6: \]

<table>
<thead>
<tr>
<th>N</th>
<th>( \theta )</th>
<th>I</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.06</td>
<td>32113.57</td>
<td>35086.29</td>
</tr>
<tr>
<td>25</td>
<td>0.08</td>
<td>33308.94</td>
<td>36437.58</td>
</tr>
<tr>
<td>50</td>
<td>0.006</td>
<td>24544.48</td>
<td>26582.33</td>
</tr>
<tr>
<td>50</td>
<td>0.0006</td>
<td>24333.97</td>
<td>26342.88</td>
</tr>
<tr>
<td>100</td>
<td>0.00006</td>
<td>18434.83</td>
<td>19739.39</td>
</tr>
</tbody>
</table>

(iii) Here, we compute the price, inventory-level, and number of selling points at that instant of time, by changing size-parameter and with following data:
\[ a = 50, S_0 = 100, S = 1000, I = 800, N = 25, T = 4, t = 2, N = 25, t_1 = 1, \\
C_1 = 10, C_2 = 30, \theta = 0.0000006; \]

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>I</th>
<th>N (t)</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>2664.01</td>
<td>8</td>
<td>31579.89</td>
</tr>
<tr>
<td>0.08</td>
<td>3373.05</td>
<td>8</td>
<td>31579.89</td>
</tr>
<tr>
<td>0.10</td>
<td>3621.30</td>
<td>8</td>
<td>31579.89</td>
</tr>
</tbody>
</table>
(iv) In this table, we compute the price of an item, inventory level and number of selling points at that instant by different constant stock and different end-level inventory with following data: \( \alpha=50, \beta=0.6, S_0=100, S=1000, N=25, T=4, t=2, t_1=1, C_1=10, C_2=30, \theta=0.0000006; \)

**Table: 2.9.15**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>28413.0</td>
<td>8</td>
<td>31579.88</td>
</tr>
<tr>
<td>100</td>
<td>400</td>
<td>28613.0</td>
<td>8</td>
<td>31579.88</td>
</tr>
<tr>
<td>100</td>
<td>500</td>
<td>28713.0</td>
<td>8</td>
<td>31579.88</td>
</tr>
<tr>
<td>200</td>
<td>700</td>
<td>43462.92</td>
<td>8</td>
<td>31579.88</td>
</tr>
<tr>
<td>700</td>
<td>900</td>
<td>91579.31</td>
<td>8</td>
<td>31579.88</td>
</tr>
</tbody>
</table>

**2.10 Important observations drawn from the model**

The following observations have been drawn from the model under consideration.

From the table (2.9.1), it is clear that when initial stock, constant stock and end level of inventory are changed then price is also affected. It is also evident that price is more affected by end level stock.
Table (2.9.2) shows that price of an item does not depend on the number of selling points in the market. This observation, in turn, proves that model has been investigated under the condition of perfect competition. Further, it affects inventory level at instant of time.

Table (2.9.3) reveals that if we consider different time instants, price of an item does not change i.e. price of an item remains constant in different time instants under perfect competition in one cycle.

It is obvious from the table (2.9.4) that increasing shape parameter $\alpha$, inventory level remains unaffected but corresponding price of an inventory item decreases.

From table (2.9.5), it is evident that when end-level inventory increases then inventory level increases and corresponding price of an item also increases.

Table (2.9.6) exhibits that initial and constant stocks do not affect the price of an item.

By table (2.9.7), we conclude that by decreasing the value of size parameter $\beta$, price will increase and inventory level will also increase.

Table (2.9.8) makes it clear that if the value of end-level inventory increases, the corresponding price of inventory of an item decreases.

Table (2.9.9) shows that by increasing shape-parameter, price of an item decreases.

From table (2.9.10), it is obvious that by increasing size-parameter, price of an inventory item decreases.
Table (2.9.11) shows that for large value of cycle time, price of unit item decreases & when number of selling points at end of cycle time decreases then price of an item slightly increases.

It is evidenced by the table (2.9.12) that by increasing shape parameter $\alpha$, inventory level also rises and price does not depend on inventory level. It means that price of an item does not change on the basis of stock at any selling point.

Table (2.9.13) shows that when deteriorating rate decreases, the price of an item also decreases. It is also to mention that when number of selling points increases then the price of an item will decrease.

It is shown from the table (2.9.14) that when size parameter of inventory increases then inventory level also increases but price of an item remains unaltered.

From table (2.9.15), it is obvious that change in constant stock and end-level inventory amounts to change in the inventory level but price of an item remains unaltered in perfect competition.

2.11 Conclusions and Future Research

Finally, we can conclude with the remark that this EOQ model considers the method of marginal revenue and marginal cost as an important economic analysis of the model consisting some interesting behaviour, which lies in the fact that if marginal revenue is greater than marginal cost, marginal profit is positive, and if marginal revenue is less than marginal cost, marginal profit is negative. When marginal revenue equals marginal cost, marginal profit tapers off to zero. Since total profit
increases when marginal profit is positive and total profit decreases when marginal profit is negative, it must reach a maximum where marginal profit is zero or where marginal cost equals marginal revenue, which is a quintessential behaviour for the perfect condition. It has been empirically observed that calculation of the marginal cost may come out more accurate if the formation of regression lines is possible in the context of above problem. This is a useful finding for the market structure of the economy, which needs its critical analysis from theoretical as well as applications point of view.

Various observations laid down (tables 2.9.1-2.9.15) for the model, can intuitively help simulate various problems in this field of the research. This also paves the way for the further study of the model under other market structures such as imperfect, monopolistic, oligopolistic, monopsonic competitions and elastic demands etc. which are essential characteristics of any dynamic market economy. This chapter can also be useful for some important case studies involving deteriorating items under perfect conditions.
REFERENCES


40. Joan Robinson, What is perfect competition, Quarterly Journal of Economics 104 (1934) 331.