Preface
The present thesis is an outcome of my research work carried out during the period 2010-2014 in the Department of Post-graduate studies and Research in Mathematics at K. S. Saket (P.G.) College, Ayodhya, Faizabad (U.P.) under the supervision of Dr. P. C. Yadav, (Retd.) Associate Professor and Head of the Department.

The whole thesis has been divided into six chapters and each chapter has further been subdivided into sections. In numbering the equations the decimal notation has been employed. References to the equations are of the form (CSE), where C, S and E stand for the corresponding chapter, section and equation respectively. If C coincides with the chapter in hand, it is omitted; the numbers in the square brackets refer to the references given at the end of the chapter. The notation \((-k/h)\) means the subtraction from the former term obtained after the interchange of the indices \(k\) and \(h\), for instance \(\Omega_{kh}(-k/h) = \Omega_{kh} - \Omega_{hk}\), the symbols \(\partial_k, \partial_x, ( ), v, V\) and \(L_v\) respectively denote the partial differentiation with respect to \(x^k\), the partial differentiation with respect to \(\dot{x}^k\), the Berwald's covariant differentiation, the Cartan's first and second covariant differentiation, the projective covariant differentiation and Lie-differentiation with respect to the given infinitesimal point transformation.

The first chapter of the thesis is introductory, in which an account of the nature and contents of the preliminary details has been given. Some results and definitions useful for the later work have also been mentioned theorem. The Finsler space \(F^n_s\) equipped with non-symmetric connection \(\Gamma_{jk}^i (x, \dot{x})\) that is based on a non-symmetric fundamental tensor \(g_{ij} (x, \dot{x}) \neq (g_{ji} (x, \dot{x}))\) has been defined in this chapter. We have introduced the concepts of \(\Theta\)-
covariant differentiation and $\Theta$ covariant differentiations of a tensor quantity with respect to $x^k$ and have obtained the curvature tensor $R_{ijk}^h(x, \dot{x})$ and $\dot{R}_{ijk}^h(x, \dot{x})$ respectively because of the duality in the nature of the two covariant derivatives.

The second chapter of the thesis deals with 1-recurrent Finsler spaces. This chapter has been divided into five sections of which the first section is introductory. The second section deals with special projective recurrent tensor fields. In this section we have defined special projective tensor fields as a linear combination of the projective entity $Q^i_j(x, \dot{x})$ and Berwald’s deviation tensor field $H^i_j(x, \dot{x})$ in the form $M^i_j(x, \dot{x}) = F(x, \dot{x})H^i_j + P(x, \dot{x})Q^i_j$, where $F(x, \dot{x})$ is the fundamental function and $P(x, \dot{x})$ is a scalar function of degree one in its directional arguments. Further, the special projective tensor fields $M^{i}_{hi}(x, \dot{x})$ and $M^{i}_{bhi}(x, \dot{x})$ have also been defined as $M^{i}_{hi}(x, \dot{x}) = 2/3 \dot{\partial}_{[h}M^{i}_{j]}$ and $M^{i}_{bhi}(x, \dot{x}) = \dot{\partial}_tM^{i}_{hi} = 2/3 \dot{\partial}_t\dot{\partial}_{[h}M^{i}_{j]}$, in this continuation we have also stated that an $n$-dimensional Finsler space $F^*_n$ is said to be $M-\Theta$ recurrent if the special projective curvature tensor field $M^{i}_{bhi}(x, \dot{x})$ satisfies $M^{i}_{bhi} + _{k} = \lambda_k M^{i}_{bhi}$ where $\lambda_k = \lambda_k(x, \dot{x})$ is a non-null recurrence vector. After giving these basic definitions we have derived result in the form of theorems telling as to what relationship recurrence vector $\lambda_k(x, \dot{x})$ satisfies in a $M-\Theta$-recurrent $F^*_n$ of first order along with the necessary and sufficient conditions telling as to when in a $M-\Theta$-recurrent $F^*_n$ and also in an affinely connected $M-\Theta$-recurrent $F^*_n$ of first order $M^{i}_{hi}-\Theta$-recurrent $F^*_n$ will be $M^{i}_{bhi}-\Theta$-recurrent. We have also derived the
relationship which hold in between the fundamental function $F(x,\dot{x})$ and the scalar function $P(x,\dot{x})$ in an affinely connected $M-\Theta$-recurrent $F_n^*$ of order me and in the last have derived the necessary and sufficient condition under which an $M_{kh}^i - \Theta$-recurrent $F_n^*$ will be $M-\Theta$-recurrent, few more results in the form of corollaries have also been derived in this section. In the third section studies have been carried out with special reference to special recurrent normal projective curvature tensor of order one. In this section we have stated that in a Finsler space, if the normal projective curvature tensor is characterized by $N_{jkh}^i(m) = \mu_m N_{jkh}^i$ and the normal projective curvature tensor is decomposable in the form $N_{jkh}^i = y_j^i \Psi_{kh}$ then such a space is said to be Normal projective recurrent space or NPR-space. With such as the background we have derived results in the form of theorems telling as to what relationship the normal projective curvature tensor $N_{jkh}^i$ satisfies in a normal projective recurrent Finsler space, in this continuation we have also derived that the decomposed tensor fields $y_j^i$ and $\Psi_{kh}$ are recurrent of order one if in a normal projective recurrent Finsler space the normal projective curvature tensor be decomposable in the form as has been stated in the foregoing lines and in the last have derived that a Finsler space is normal projective recurrent if the recurrent normal projective curvature tensor in the space under consideration be decomposable in the form $N_{jkh}^i = y_j^i \Psi_{jkh}$. In the fourth section of the chapter studies of recurrent Berwald’s curvature tensor of order one have been carried out. In this section firstly we have derived the connection in between Berwald’s connection and Berwald’s deviation tensor field in a
recurrent Finsler space of first order and thereafter have derived that a Berwald’s recurrent Finsler space of first order characterized by
\[ H^i_{jk}(m) = \lambda_m H^i_{jkh}, \quad H^i_{jkh} \neq 0 \]
always satisfies either of the two conditions (a) the space under consideration is a Landsberg space or (b) \( H^i_j = 0 \) and in the sequel have also derived that if a Berwald recurrent Finsler space of first order be assumed to be \( P^* \) then the space is Riemannian or \( H^i_j = 0 \) and also the conditions under which a recurrent Berwald space is necessarily a Landsberg space and in the last have derived the necessary and sufficient condition under which the associate curvature tensor \( H_{ijkh} \) is recurrent in a Finsler space with recurrent Berwald curvature tensor and have also observed that the associates of the Berwalds deviation tensor \( H^i_{kh} \) and \( H^i_h \) are recurrent in a Finsler space with recurrent Berwald curvature tensor.
The fifth and the last section of the chapter deals with \( U^h \)-recurrent Finsler spaces, in this section we have stated that a Finsler space will be called \( U^h \)-recurrent Finsler space if the tensor \( U^i_{jk} \) is recurrent in the sense of Cartan, i.e.
\[ U^i_{jk} = \lambda_m U^i_{jkm}, \quad U^i_{jk} \neq 0. \] After giving this definition we have derived that in a \( U^h \)-recurrent Finsler space the tensor \( G^i_{jkh} - \ell^i \ell_r G^r_{jkh} \) is \( h \)-recurrent, we have also derived the necessary and sufficient condition which should hold in order that in a \( U^h \)-recurrent Finsler space the tensor \( G^i_{jkh} \) is \( h \)-recurrent and in the last have derived the requirements which should be fulfilled under which in a \( U^h \)-recurrent Finsler space, the directional derivative of the tensor \( G^i_{jkh} \) in the direction of the contra vector \( v^j(x^\ell) \) is proportional to \( G^i_{jkh} \).
The third chapter of the thesis with recurrences in special Finsler spaces. This chapter has been divided into four sections of which the first section is introductory, in this section we have stated that a Finsler space \( F_n(n > 2) \) with the non-zero length \( C \) of the torsion tensor \( C^i \) is called semi \( C \)-reducible if \( C_{ijk} = \frac{p}{n + 1} \)

\[
(h_j C_k + h_{jk} C_i + h_{ki} C_j) + \frac{q}{C} C_i C_j C_k,
\]

where \( p \) and \( q = (1 - p) \) do not vanish and \( h_{ij} \) is the angular metric tensor. A semi \( C \)-reducible space is of the first kind or of the second kind according as \( p \neq \frac{n + 1}{2} \) or \( p = \frac{n + 1}{2} \) and in the continuation have also stated that a non-

Riemannian Finsler space \( F^n \) is called \( C^h \)-recurrent, if the \((h)hv\)-torsion tensor \( C_{ijk} \) satisfies \( C_{ijk|\ell} = k_{i} C_{ijk} \) where \( k_{\ell} \) is a covariant vector field. We have also stated that a Finsler space in which the Berwald’s connection \( G^i_{jk} \) is independent of directional arguments is called a Berwald space and such a space is characterized by \( C_{ijk|\ell} = 0 \), from this statement it is obvious that every Berwald space is a Landsberg space. The second section of the chapter has been devoted to the study of properties of recurrence. In this section we have said that a tensor field \( T_{ij} \) called \( h \)-recurrent if \( T_{ij|k} = \lambda_k T_{ij} \), where \( \lambda_k \) is a covariant vector field whereas a vector field \( T_i \) is called \( h \)-recurrent if for a covariant vector field \( k_h \), we have \( T_{i|h} = k_h T_i \). With such as our basic assumptions we have derived results stating that in a \( C^h \)-recurrent Finsler space the covariant vector field \( C_i \) is \( h \)-recurrent, we have also derived the necessary and sufficient condition in order that a
C-reducible Finsler space is $C^h$-recurrent and in this continuation we have also derived the condition under which a C2-like Finsler space is $C^h$-recurrent. In the sequel we have derived the necessary and sufficient condition in order that a semi C2-like Finsler space of second kind is $C^h$-recurrent. In the later portions of this section we have stated that a non-Riemannian Finsler space is called $S^h$ or $T^h$-recurrent according the $v$-curvature tensor $S_{hijk}$ or the $T$-tensor $T_{hijk}$ satisfies $S_{hijk|\ell} = \Psi_{\ell} S_{hijk}$ and $T_{hijk|\ell} = \lambda_{\ell} T_{hijk}$ respectively, where $\Psi_{\ell}$ is covariant vector field and thereafter have derived that a $C^h$-recurrent Finsler space is $S^h$-recurrent, a non flat S3-like Finsler space is $S^h$-recurrent where an S3-like Finsler space is characterized by $S_{i j k |\ell} = \frac{S}{(n-1)(n-2)} (h_{ik} h_{j \ell} - h_{i \ell} h_{jk})$ and in the last have derived the necessary and sufficient conditions under which C-reducible Finsler space is $S^h$-recurrent and an S4-like Finsler space is $S^h$-recurrent. The third section of the chapter has been devoted to the study of $T^h$-recurrent Finsler spaces. In this section we have started our studies with the statement that a Finsler space is said to satisfy the $T$-condition if the $T$-tensor of the space vanishes where the $T$-tensor $T_{hijk}$ can be expressed in the following two forms

$$T_{hijk} = \Phi \left( h_{hi} h_{jk} + h_{ij} h_{ki} + h_{hk} h_{ij} \right)$$

and

$$T_{hijk} = h_{hi} A_{jk} + h_{ij} A_{hk} + h_{hk} A_{ij} + h_{ij} A_{hk} + h_{jk} A_{hi}$$

where $\Phi$ is a scalar and $A_{ij}$ is a symmetric indicatory tensor. With such as the basics of this section we have derived results talking as to when a C-reducible Finsler space is $T^h$-recurrent and a two dimensional Finsler space is $T^h$-recurrent and in the last have derived
the necessary and sufficient condition under which the $T$-tensor in
the form stated in the foregoing lives is $T^h$-recurrent. The fourth and
the last section of the chapter has been devoted to the study of $P^h$-
recurrent Finsler spaces. In this section we have derived the necessary
and sufficient conditions under which a $P^r$-Finsler space with scalar
coefficient $\lambda$ is a Landsberg space, a $C^h$-recurrent Finsler space with
recurrence vector field $k_t$ is a Landsberg space, a non-Landsberg $P^r$-
Finsler space is $P^h$-recurrent, a $P$-reducible Finsler space is $P^h$-
recurrent and a C-reducible Finsler space is $C^h$-recurrent.

The fourth chapter of the thesis has been devoted to the study of
curvature and torsion tensor of $h$-recurrent Finsler connection and
projective transformation in affinely connected recurrent Finsler
space. This chapter has mainly been divided into five sections of
which the first section is introductory, in this section a brief account
of the preliminary details has been given which shall be of use in the
later sections. The studies in the second section has been carried out
under the assumption that the $h$-covariant derivative of the $(h)hv$-
torsion tensor $C_{ijk}$ with respect to the Finsler connection vanishes
identically. Consequent upon this assumption we have derived results
telling that the coefficient $F_{jk}^i$ are function of position only if (i) $F_n$
be assumed to be a Berwald space and (ii) the $h$-covariant derivative of
the $(h)hv$-torsion tensor $C_{ijk}$ with respect to the Finsler connection
vanishes identically and also that $F_n$ is a Riemannian space if the
tensor field $B^i_j = D^i_j + \frac{1}{2}(a^i_j \delta^i_j + a^i_j \dot{x}^i + a^i_j \dot{x}_j)$ where $D^i_j$ is the
deflection tensor given by $D^i_j = \dot{x}^k F_{kj}^i - N^i_j$. In the third section of the
chapter we have considered such $h$-recurrent Finsler connection.
whose \((v)h\)-torsion tensor \(P^i_{jk}\) is given by \(P^i_{jk} = -\hat{\partial}_k B^i_j\) where
\[
B^i_j = -A^i_{j;\ell} + D^i_j + \frac{1}{2}(a_o \delta^i_j + a_j \dot{x}^i + a^i \dot{x}_k)
\]
and thereafter have derived results in the form of a theorem. In the fourth section we have discussed projective transformation in affinely connected recurrent Finsler spaces while in the fifth and the last section of the chapter we have discussed projective transformations in recurrent and Ricci recurrent Finsler spaces and have derived results which hold in Ricci recurrent and in an affinely connected Ricci recurrent Finsler space admitting infinitesimal projective transformation such results have also been derived in an affinely connected recurrent Finsler space admitting infinitesimal projective transformation.

The fifth chapter of the thesis deals with semi-parallel and semi-concircular vector fields in a Finsler space. The aim of this chapter is to generalize the concept of a normalized semi-parallel \(h\)-vector field in some special Finsler spaces and to discuss semi-C-concircular vector fields and a semi P-concircular vector field in a Finsler space. The chapter in hand has been divided into three section of which the first sections is introductory, in this section it has been stated that a non-zero vector field \(v_i\) in a Finsler space is said to be semi-parallel if there exists a scalar \(\lambda(x^i)\) and \(v(x^i)\) satisfying the following conditions: (a) \(v_i\) is a function of positional coordinates only (b) \(C^i_{jk} v_i = 0\) and (c) \(v_{ij} = \lambda(g_{ij} + v_i v_j)\) where \(v_{ij}\) denotes the \(h\)-covariant derivative of \(v_i\) with respect to \(x^j\). Singh U.P. and Bindu Kumari denoted the normalized vector field by \(X_i\). Thus, \(X_i = \frac{v_i}{v}\) where \(v\) denotes the length of the vector field \(v_i\), i.e. \(v^2 = v_i v^i\). They showed that the
condition (c) mentioned above is equivalent to
\[ X_{\hat{i} j} = \frac{\lambda}{v} \left( g_{ij} - X_i X_j \right) \]
and in view of these observations they defined a normalized semi-
parallel vector field \( X_i \) by the conditions (a) \( X_i \) is a function of
positional coordinates only, (b) \( C^i_{jk} X_i = 0 \) and (c)
\[ X_{\hat{i} j} = \rho \left( g_{ij} - X_i X_j \right) \]
where \( \rho \) is a scalar point function. The section
two of this chapter deals with generalised normalized semi-parallel \( h \)-
vector field in special Finsler spaces. It has been stated in this section
that a normalized vector field \( X_i \) in a Finsler space \( F_n \) is said to be
normalized semi-parallel \( h \)-vector field if (a) \( X_i \big|_j = 0 \), (b)
\[ FC^i_{jk} X_i = \sigma h_{jk} \]
and (c) \( X_{\hat{i} j} = \rho \left( g_{ij} - X_i X_j \right) \) where \( \sigma \) and \( \rho \) are non-
zero scalar function of \( x^i \) only and it is also assumed that \( X_i \) is
positively homogeneous function of degree zero in their directional
arguments and in this continuation if has also been stated that a
normalized vector field \( X_i \) in a Finsler space is said to be a generalized
normalized semi-parallel \( h \)-vector field if (a) \( X_i \big|_j = 0 \), (b) \( P^i_{jk} X_i = \mu h_{jk} \)
and (c) \( X_{\hat{i} j} = \rho \left( g_{ij} - X_i X_j \right) \) where \( \mu \) and \( \rho \) are non-zero scalar
functions, \( X_i \) is positively homogeneous of degree in its directional
argument. Consequent upon these basic assumptions we have derived
results in the form of theorems telling (i) The angular metric tensor
\( h_{ij} \) can never be written as the product of two covariant vectors (2) In
a Landsberg space there exists no generalized semi-parallel \( h \)-vector
field (3) These exists no generalised normalized semi-parallel \( h \)-vector
field in a Berwald space (4) A P-reducible Finsler space does not admit
a generalized normalized semi-parallel \( h \)-vector field (5) A C-reducible
Finsler space does not admit a generalized normalized admit a
generalized normalized semi-parallel $h$-vector field. (6) A Randers' or Kropina space does not admit a generalized normalized semi-parallel $h$-vector field. The third and the last section of the chapter deals with semi concircular vector field in a Finsler space. In this section we have stated that a conformal transformation is said to be a concircular transformation a C-concircular transformation and a P-concircular transformation if the function $\sigma_h$ is a solution of the following equations (a) $\sigma_{j(k)} = \sigma_j \sigma_k + \Phi g_{jk}$, (b) $\sigma_{j(k)} = \sigma_j \sigma_k + \Phi g_{jk}$, $C^i_{jk} \sigma_i = 0$ and (c) $\sigma_{j(k)} = \sigma_j \sigma_k + \Phi g_{jk}$, $P^i_{jk} \sigma_i = 0$ respectively, where $\sigma_h$ and $\Phi$ contain $x^i$ only. The vector field $\sigma_h(x^i)$ is called a concircular vector field, a C-concircular vector field and a P-concircular vector field according as it satisfies (a), (b) and (c) respectively, in this continuation we have also stated that a conformal transformation is said to be a semi-concircular transformation and a semi P-concircular transformation if the function $\sigma_h$ is a solution of the equations (a') $\sigma_{j(k)} = \sigma_j \sigma_k + \Phi g_{jk}$, $FC^i_{jk} \sigma_i = \eta h_{jk}$ and (b') $\sigma_{j(k)} = \sigma_j \sigma_k + \Phi g_{jk}$, $P^i_{jk} \sigma_i = \mu h_{jk}$ respectively. Functions $\eta$ and $\mu$ are homogenous of degree zero in its directional arguments. The vector field $\sigma_h(x^i)$ is called a semi-concircular vector field and semi P-concircular vector field according as it satisfies (a') and (b') respectively. The condition $FC^i_{jk} \sigma_i = \eta h_{jk}$ and $P^i_{jk} \sigma_i = \mu h_{jk}$ are called semi C-condition and semi P-condition. Consequent upon these basic assumptions we have derived results in the form of theorems telling as to what conditions will be satisfied when a semi C-concircular transformation becomes semi P-concircular, few more results have also been derived in this continuation.
In the sixth and the last chapter of the thesis studies have been carried out with special reference to Cartan’s connection $\Gamma^s_{jk}$. This chapter has been divided into seven sections of which the first section is introductory, in sections two and three have given a conformal change to the connection coefficient $\Gamma^s_{jk}$ in order that the infinitesimal point transformation $\bar{x}^i = x^i + v^i(x)dt$ be an infinitesimal conformal transformation and thereafter have derived results in the form of theorems, while in the subsequent sections we have given a projective change to the same connection coefficient in order that the infinitesimal point transformation $\bar{x}^i = x^i + v^i(x)$ be an infinitesimal projective transformation and subsequently have derived results in a symmetric Finsler space. In the last two sections studies have been carried out with special reference to infinitesimal normal projective transformation.

A selected bibliography consisting of the references of the existing literature on the subject is given in the end.

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