CHAPTER 3

INVARIANT BIOMETRIC REPRESENTATION

3.1 NEED FOR INVARIANT BIOMETRIC REPRESENTATION

Representation of an object is the first step of pattern recognition. It has to be well defined, since it cannot be improved later in the process of training. Real imaging systems as well as imaging conditions are usually imperfect Jain et al (2000), Flusser (2005) and the observed image represents only a degraded version of the original image. A representation which gives good generalisation properties for a trained system is considered useful. This would prevent a learning system from having to discover all the various manifestations of a rigid object in its vision module. The representation must have sufficient descriptive power to allow discrimination between all dissimilar objects. Generally there are three major approaches to recognising objects that are deformed in various ways – brute force approach, image normalisation and invariant features Flusser (2005). Deformed objects are considered deformed versions of a template.

In the brute force approach, the parametric space of all possible degradations is searched, which means that the training set of each class should contain representatives of all the rotated and scaled versions. This requires a large storage space for each object, there is no extrapolation, no intelligent learning and it is necessary to learn about each object in each possible rendering. For example in face recognition, it requires, recognising a face under different possible sizes, positions, colors, illuminations, rotations,
gestures, makeup, beard, etc. Wallis and Rolls (1997). In the normalisation approach, the objects are transformed into a standard position before they enter the feature extraction stage. This approach is efficient; however it requires solving difficult inverse problems.

The basic idea of invariant features is to describe the object by a set of measurable quantities called invariant descriptors that are insensitive to deformations and provide enough discrimination power to distinguish objects belonging to different classes.

3.2 LITERATURE SURVEY

3.2.1 Fingerprint

Hrechak and McHugh (1990) use fingerprint minutiae for matching. The minutiae in each finger are represented as a vector. However the discrimination is reportedly low in this case. Since then, several approaches have been proposed to improve the reliability of the minutiae for automatic recognition. Jiang et al (2000) and Chikekerur et al (2006) suggest clustering of minutia points and matching between clusters. Ratha et al (2000) represent the arrangement of minutiae on the fingerprint as local graph like structures where each minutia is represented by a star. Star matching is then performed. Minutiae triangles whose vertices represent the minutiae and are popularly used for indexing, are also used for matching. Tan and Bhanu (2003) propose a method of counting the number of matching minutiae once the two fingerprints are aligned with respect to the similar triangles in both the prints. However, the non-linear distortion introduced in the fingerprint when the user moves his finger after placing it on the sensor for acquisition is a major problem. Techniques such as minutia clustering and warping Kwon et al (2006), multiple transformation based consolidation where sub-sets (clusters) of minutiae corresponding to different alignments of the fingerprint are
matched Bazen and Gerez (2003), normalisation, wherein the fingerprints are corrected to remove the distortion soon after acquiring Senior and Bolle (2001) are used to overcome this limitation. Further the number of minutiae identified in a print depends on the resolution of the image and also its size. In low cost consumer applications, the size and resolution of the fingerprint sensor are relatively smaller Maltoni et al (2003). Hence it becomes necessary to search for non-minutiae techniques.

Fingerprint texture, Level 3 (sweat pores) features and shape information are the common non-minutiae descriptors. Texture information generally constitutes the properties of the ridge lines such as its orientation and frequency. Coetzee and Botha (1993) perform analysis of the texture information obtained by transforming the image to its frequency domain. Wavelet transform Tico, Kuosmanan and Saarinen (2001) and Discrete Cosine Transform Tachaphetpiboon and Amornraksa (2005) are also used. Since frequency domain images are not robust against noise, others use this method along with the minutiae, in their hybrid model Jain et al (2001), Ross et al (2003). The most popular texture based approach is the fingercode Patil et al (2005). The fingerprint is tessellated with respect to the core point and texture information is extracted from each tessellation using a bank of eight Gabor filters. Later several variants of the fingercode approach were introduced. Generally, all fingercode approaches require that the core point of the image be identified accurately.

Nanni and Lumini (2010) use the Local Binary Patterns introduced by Ojala et al (2002), where the image is subdivided into blocks and histograms are extracted from each block. However this method requires extraction of minutiae and alignment of the images based on their minutiae before the LBP is extracted. The Level 3 features which are the sweat pores along the fingerprint ridges, are highly discriminant, however detection of
pores require high resolution scanners. The ridge shape is used for fingerprint representation in several ways. Jiang et al (2001) use contour extraction to extract the ridge shape. But several methods that extract the ridge shape also include extraction of the Level 3 features. Rao and Black (1980) propose a syntactic approach to extract the fingerprint shape in the form of a string of primitives. However this method suffers from parsing complexities when the grammar tends to become more general. Moments are shape descriptors that are popularly used in image processing. Khotanzad and Hong (1990) use Zernike moments for invariant image recognition. Yang et al (2008) propose invariant moments based descriptors where seven moments are calculated from the image and they are claimed to give lower EER than any other features.

### 3.2.2 Palmprint

Early research on palmprint borrowed from fingerprint recognition methods. Shu and Zhang use the geometry features such as palm width and height, principal lines, wrinkles and minutiae which are similar to the features extracted from the fingerprint. These images require high resolution images for their extraction Zhang and Shu (1998), Zhang et al (2003). Texture based information is extracted by Wu et al (2004) who use Gabor filters and Zhang et al (2004). Li et al (2002) use Fourier Transform for representing the texture information. Zhang et al (2004) use 2D Gabor filters to extract the phase information which is called the palmcode from low resolution images. Kong, Zhang and Kamel (2006) suggest an improvement over the previous method by using multiple Gabor filters to obtain the so called Fusion code. Generally texture based methods are affected by lighting conditions.

Several citations are found in literature that uses Principal Component Analysis, Linear Discriminant Analysis and Independent Component Analysis for representing palmprints. Lu et al (2003), Wu et al
Similar to the fingerprints, shape descriptors are used for palmprint representation. Jin Soo Noh et al (2005) use Hu invariant moments, Yang et al (2010) use Zernike moments. The pseudo-Zernike moments which are derived from the Zernike moments, offer more features than the Zernike. However they have not been widely used for biometric recognition. Pang et al (2003) use Legendre, Pseudo-Zernike and Zernike moments, for palmprint matching. However, the performance is less because normalization of moments is required as they not inherently invariant to all types of deformations.

### 3.3 PROPOSED SHAPE DESCRIPTORS

The fingerprint and palmprint images are constituted from various line structures. It is also interesting to note that the human being’s visual system is thought to generate oriented line segments as a fundamental part of image processing http://www.intechopen.com. Outlines play a crucial role in how the human eye and mind interpret what is seen Ashbrook and Thacker (1998). Hence it is proposed to develop descriptors that quantify shapes in ways that agree with human intuition. In this context an attempt is made to find other applications which involve images with line structures. OCR applications are typical examples for this case. Script of languages such as the Chinese, Japanese, and Arabic and also the Indian languages such as Hindi, Tamil, Telugu, and Kannada are composed of lines of different orientation and symmetry. Especially, in Chinese character recognition, there are several complexities including the facts that Chinese characters are distinct and ideographic, the character size is very large, and a lot of structurally similar characters exist in the character set. Compared with the set of English letters, Chinese characters are more difficult to classify http://news.stanford.edu.

Moments are commonly used in statistics and computer science to specify a random distribution in a hierarchical way Dehghan and Faez (2007). Lower order moments describe the global characteristics of the distribution whereas the higher order moments are associated with details. Moment shape descriptors have been used as generic features for machine pattern recognition of letters and characters of all types of fonts including the handwritten ones Liu et al (2009). It is proposed that since the fingerprint and palmprint are also composed of line structures, the moment shape descriptors which are effectively used to classify letters and characters which appear very much similar (which in fact is the case among biometric images also) can also be used for fingerprint and palmprint recognition and classification.

A classification of moments is given in Figure 3.1.
The double arrows mean that each of these moments can be expressed within the other formulation while single arrows denote a sub-class relation. Among the Continuous Moments, the Legendre Moments (LM) and the Zernike Moments (ZM) are popularly used in literature. Among the Discrete Moments, the basic type is the Tchebichef Moments (TM) also called the Chebyshev moments (CM). These three moments are chosen as feature (shape) descriptors for fingerprint and palmprint images.

3.4 MOMENTS

From a mathematical point of view, moments are projections of a function onto a polynomial basis similar to a Fourier Transform which is a projection onto a basis of harmonic functions. For any arbitrary function \( f(x, y) \), one may compute the moments as

\[
m_{pq} = \int \int x^p y^q f(x, y) dx dy
\]  

(3.3)

where \( p, q \) represent the moment order. For digitized images, the integral must be replaced with a summation over the domain covered by the image data. The digital approximation may be written as in Equation (3.4)
\[ m_{pq} = \sum_{i=1}^{M} \sum_{j=1}^{N} f(x_i, y_j) x_i^p y_j^q \]  
(3.4)

The geometric moments are basically projections of the image function onto the monomials, that is, \( \phi_{nm}(x, y) = x^n y^m \), the \((n+m)\)th order geometric moment, \( M_{nm} \), is defined as in Equation (3.5).

\[ M_{nm} = \int_{\mathbb{R}^2} x^n y^m f(x, y) dx dy \]  
(3.5)

The geometric moments are the most widely used in image analysis and pattern recognition tasks. This is essentially due to their simplicity, the invariance, the geometric meaning of the lower order values. The zeroth order moment \( M_{00} \) represents the total mass of the image, the two first order moments \( M_{10} \) and \( M_{01} \) provide the position of the center of the mass. The second order moments \( M_{20}, M_{11}, M_{02} \), can be used to determine several useful image features such as the principal axes, the image ellipse, and the radii of gyration, etc, Hu (1962).

The rotational moments of order ‘n’ with repetition ‘m’ has the kernel \( \phi_{nm}(r, \theta) = r^n e^{jm\theta} \). Then the 2D rotational moment defined in polar coordinates is given by Equation (3.6)

\[ D_{nm} = \int_0^{2\pi} \int_0^r r^n e^{jm\theta} f(r, \theta) r dr d\theta, \text{ for } m \leq n, n - m = \text{even} \]  
(3.6)

The rotational moments have the nice property of being invariant under image rotation. If the image is rotated by an angle \( \varphi \), the relation between the transformed and original moments is \( D'_{nm} = e^{jm\varphi} D_{nm} \). A rotation of \( \varphi \) is thus achieved by a phase change of rotational moments, so that the
magnitude remains invariant. This property makes the rotational moments, useful descriptors in pattern recognition.

The basic function of complex Moments is \( \phi_{nm}(x, y) = (x + iy)^n (x - iy)^m \), the 2D complex moment of order \((n+m)\) being defined as in Equation (3.7)

\[
C_{nm} = \int_{\mathbb{R}^2} (x + iy)^n (x - iy)^m f(x, y) dx dy
\]

(3.7)

The complex moments are related to rotational moments by \( C_{nm} = D_{n+m,n-m} \) and thus the rotation transformation of the image affects only the phase of the complex moments.

The geometric moment definition has the form of projection of \( f(x,y) \) onto the monomials \( x^n y^m \). Unfortunately the basis set \( \{x^n y^m\} \) is not orthogonal. Consequently these moments are not optimal with respect to information redundancy. To overcome the shortcomings associated with geometric moments, Teague suggested the use of orthogonal moments that are defined in terms of the continuous orthogonal polynomials such as Legendre and Zernike polynomials and later, the discrete orthogonal polynomials were also introduced.

3.4.1 Orthogonal Moments

Orthogonal moments are moments to an orthogonal or weighted orthogonal-polynomial basis. Orthogonal (OG) polynomials can be evaluated by recurrent relations that can be efficiently implemented by means of multiplication with special matrices. They avoid high dynamic range of moment values that may lead to loss of precision due to overflow or underflow. This is possible because, unlike standard powers, the values of OG polynomials lie inside a narrow interval such as \([-1, 1]\). Orthogonal moments
are also robust to noise. 2D orthogonal polynomials and moments are divided into two basic groups: the polynomials orthogonal on a rectangle and the polynomials orthogonal on a disk. The polynomials orthogonal on a rectangle originate from 1D OG polynomials whose 2D versions are created as products of 1D polynomials in x and y. The main advantage of the moments orthogonal on a rectangle is that they preserve the orthogonality even on sampled or discretised images. They can be made translation and scale invariant, but making them rotation invariant is more complicated. The polynomials orthogonal on a disk are intrinsically 2D functions. They are constructed as products of a radial factor which is usually a 1D OG polynomial and an angular factor which is usually a harmonic function. When implementing these moments, an image must be mapped into a disk of orthogonality which creates resampling problems. However they are inherently rotation invariant.

Using orthogonal moments, an attempt is made to derive feature descriptors that are Translation, Rotation and Scaling (TRS) invariant. TRS is the simplest transformation of spatial coordinates. It is sometimes called a similarity transform and can be described as in Equation (3.8)

\[ x' = sR x + t \]  \hspace{1cm} (3.8)

where ‘t’ is a translation vector, ‘s’ is a positive scaling factor and ‘R’ is a rotation matrix

\[ R = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \]

where ‘\( \alpha \)’ is the angle of rotation. Invariance with respect to TRS is required in almost all applications because the object should be correctly recognised, regardless of its position and orientation in the scene and of the object-camera distance.
The continuous orthogonal moments namely the Legendre and Pseudo-Zernike moments; and a discrete orthogonal moment namely the Chebyshev moment are chosen for fingerprint and palmprint representation. The Legendre moments are not inherently invariant to TSR. Translation invariance is achieved by subtracting the centroid from the Legendre polynomials and then extracting the moments. The moments are then expressed in terms of the geometric moments to achieve scaling and rotation invariance. The pseudo-Zernike moments are inherently invariant to rotation. They are made scale and translation invariant in the same manner as the Legendre moments. The Chebyshev moments are made translation, scale and rotation invariant in the same manner as the Legendre moments. The methods carried out for achieving invariance in the moments are explained in the subsequent sections.

### 3.4.2 Legendre Moments

The Legendre moment of order \( (m + n) \) is defined as in Equation (3.9)

\[
\lambda_{mn} = \frac{(2m+1)(2n+1)}{4} \int_{-1}^{1} \int_{-1}^{1} P_m(x) P_n(y) f(x, y) dx dy
\]

(3.9)

where \( m, n = 0,1,2, \ldots, \infty \) and \( P_m \) and \( P_n \) are the Legendre polynomials Shu et al (2007). The \( n \)th order Legendre polynomial is given by

\[
P_n(x) = \sum_{j=0}^{n} a_{nj} x^j \]

where \( a_{nj} \) are the Legendre coefficients given by Equation (3.10)

\[
a_{nj} = (-1)^{(n-j)/2} \frac{1}{2^n} \frac{(n+j)!}{\left(\frac{n-j}{2}\right)!\left(\frac{n+j}{2}\right)!} \quad \text{where } (n-j) = \text{even}
\]

(3.10)
The Legendre polynomials which are orthogonal over the interval [-1, 1] can be taken as a product of \( P_m(x) \) and \( P_n(y) \) and the result is an orthogonal set of polynomials over a square. For orthogonality to exist in the moments, the image function \( f(x,y) \) is also defined over the same interval as the basis set. So for a discrete image Hosny (2007) with the current pixel \( P_{xy} \), the Equation is now (3.11).

\[
\lambda_{mn} = \frac{(2m+1)(2n+1)}{4} \sum_x \sum_y P_m(x)P_n(y)P_{xy}
\]  

(3.11)

where \( x,y \) are defined in the interval [-1, 1], that is over a square area.

Now, considering the normalization process of Legendre moments in order to make them invariant, the images are enhanced, binarised and the region of interest is extracted before the moments are extracted. In binary images, the centroid Hosny (2007) is given by Equation (3.12).

\[
x = \frac{m_{10}}{m_{00}}, \quad y = \frac{m_{01}}{m_{00}}
\]

(3.12)

The Legendre moment can be made location invariant by subtracting the centroid Mendez et al (2000) so that the polynomials \( P_m(x - x_0) \) and \( P_n(y - y_0) \) are substituted in the equation (3.11) instead of \( P_m(x) \) and \( P_n(y) \). Similarly it is now necessary to make the moments invariant to scaling of the object. Geometric moments are known as the simplest type among all moments. The two dimensional geometric moment of order \( p + q \) is expressed as in Equation (3.12)

\[
m_{pq} = \int \int x^p y^q f(x,y) dx dy
\]

(3.12)
where \( p, q = 0, 1, 2, \ldots, n \) and \( I \) is the region of the pixel space in which the image intensity function \( f(x, y) \) is defined. The discretisation Chong, Raveendran and Mukundan (2004) of Equation (3.12) yields

\[
m_{pq} = \sum_{p} \sum_{q} x^{p} y^{q} f(x, y)
\]

(3.13)

The first order moments \( m_{10} \) and \( m_{01} \) provide the moments intensity according to the ‘x’ and ‘y’ axis of the image respectively as shown in Equation (3.14):

\[
m_{10} = \sum_{p} \sum_{q} x f(x, y),
\]

(3.14a)

\[
m_{01} = \sum_{p} \sum_{q} y f(x, y)
\]

(3.14b)

Therefore, when an object or part of the object occupies a higher position in the \( x \) – axis, the geometric moment will produce large numbers. Therefore image coordinate values are normalized to values less than one so that the moment values after normalization tend to zero instead of infinity as the moment order increases. However these will have to be represented using floating point values, which again increases the computational complexity. As an alternative, the geometric moments can be developed to be central moments which have the property of invariance to translation. The definition of central moments is given by subtracting each pixel of the geometric moments with the centroid Pang et al (2003) as given below:

\[
\mu_{pq} = \sum_{p} \sum_{q} (x - \overline{x})^{p} (y - \overline{y})^{q} f(x, y)
\]

(3.15)
This is because, the image centroid moves with the image and the central moments are defined with respect to this point as the origin. When scaling normalization is applied to the moments, they change as

\[ \eta_{pq} \frac{\mu_{pq}}{\mu_{00}} \gamma = [(p + q)/2] + 1 \]  
(3.16)

Geometric moments are however invariant to rotation. Therefore, if the Legendre moments could be expressed in terms of the geometric moments Pang et al (2003), then Legendre moments which are invariant to rotation, translations and scaling would be obtained. This is possible since Legendre polynomials are functions of basic monomials \( x^p \) and hence can be expressed as in Equation (3.17)

\[ L_{pq} = \frac{(2p+1)(2q+1)}{4} \sum_{k=0}^{p} \sum_{j=0}^{q} a_{pk}a_{qj} m_{kj} \]  
(3.17)

where \( a_{pk} \) and \( a_{qj} \) are the coefficients. It can be noted that Legendre moments of order ‘r’ depend only on the geometric moments of the same order or less. For instance, for Legendre moments upto order 2, the moment values are given by Equation (3.18)

\[ L_{00} = m_{00} \]  
(3.18a)

\[ L_{10} = \frac{3}{4} m_{00} \]  
(3.18b)

\[ L_{20} = \frac{5}{8}(3m_{20} - m_{00}) \]  
(3.18c)

\[ L_{11} = \frac{9}{4} m_{11} \]  
(3.18d)
While these Legendre moments are defined over Cartesian coordinates inside a square, the Pseudo-zernike moments are defined over the polar coordinates inside a circle.

### 3.4.3 Pseudo-Zernike Moments

The two dimensional Zernike moments Liao and Pawlak (1998) of order $p$ with repetition $q$ of an image intensity function $f(r, \theta)$ are defined as:

$$Z_{pq} = \frac{p + 1}{\pi} \int \int_{0}^{1} V_{pq}(r, \theta) f(r, \theta) r dr d\theta; \quad |r| \leq 1$$

(3.19)

where Zernike polynomials $V_{pq}(r, \theta)$ are defined as:

$$V_{pq}(r, \theta) = R_{pq}(r)e^{-jq\theta}; \quad j = \sqrt{-1}$$

(3.20)

and the real-valued radial polynomial, $R_{pq}(r)$, is defined as

$$R_{pq}(r) = \sum_{k=0}^{\frac{|q|}{2}} (-1)^{k} \frac{(p-k)!}{k!(p+|q| - k)!} \left(\frac{p+|q|}{2} - k\right)! r^{p-2k}$$

(3.21)

where $0 \leq |q| \leq p$ and $p - |q|$ is even. For an $N \times N$ image with intensity function $f(i, j)$, the discrete approximation Liao and Pawlak (1998) is defined as Equation (3.22)

$$Z_{pq} = \frac{2(p + 1)}{(N + 1)^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} R_{pq}(r_{ij}) e^{-jq\theta} f(i, j)$$

(3.22)

To map the $N \times N$ image inside the unit circle, the image coordinate transformation is calculated through the formula in Equation (3.23).

$$x_i = \frac{\sqrt{2}}{N-1} i - \frac{1}{\sqrt{2}}, \quad y_j = \frac{\sqrt{2}}{N-1} j - \frac{1}{\sqrt{2}}$$

(3.23)
The Zernike moments in (3.19) become pseudo-zernike moments if the radial polynomials, $R_{pq}$, defined as in Equation (3.20) with its condition $p-|q|=$ even is eliminated Teague (1980). Therefore, pseudo-zernike moments offer more feature vectors than Zernike moments since pseudo-zernike polynomial contains $(p+1)^2$ linearly independent polynomials of order $\leq p$, whereas Zernike polynomial contains only $\frac{1}{2} (p+1)(p+2)$ linearly independent polynomials due to condition of $p-|q|=$ even.

The Pseudo-zernike moments can be made scale and translation invariant in a similar manner as is done for the Legendre moments. Due to their inherent properties, these moments are already rotation invariant. Since the pseudo-zernike moments are more robust to noise than the Legendre moments Teague (1980) and are also inherently invariant to rotation, they are widely preferred in pattern recognition applications.

### 3.4.4 Chebyshev Moments

Mukundan introduced the Chebyshev moments to image processing Flusser et al (1983). The discrete nature of Chebyshev moments decreases the discretization errors and makes it superior to other statistical moments. This also results in reduction in computational time and increase in system accuracy. The $n$th order discrete Tchebichef polynomials are defined as in Equation (3.24)

$$T_n(x)=(1-N)_n\, _3F_2(-n, -x, 1+n;1, 1-N; 1),$$

$n, x = 0, 1, 2, \ldots, N-1$ \hspace{1cm} (3.24)

they can also be written as in Equation (3.25)
\begin{align}
T_n(x) = n! \sum_{k=0}^{n} (-1)^{n-k} \binom{n-k}{n} \binom{n+k}{x} (3.25) 
\end{align}

which satisfies the relation of orthogonality in Equation (3.26)

\begin{align}
\sum_{x=0}^{N-1} T_n(x)T_m(x) = \sigma(n,N)\delta_{mn} (3.26) 
\end{align}

with the normalizing factor in Equation (3.27)

\begin{align}
\sigma(n,N) &= \frac{N(N^2 - 1^2)(N^2 - 2^2)...(N^2 - n^2)}{2n+1} (3.27a) \\
= (2n)\binom{N+n}{2n+1} &\text{ n=0, 1, 2, … N } (3.27b) 
\end{align}

\begin{align}
\rho(m,N) &= \frac{N(1 - \frac{1}{N^2})(1 - \frac{2^2}{N^2})...(1 - \frac{n^2}{N^2})}{2n+1} \text{ n=0,1,2,…,N-1 } (3.28) 
\end{align}

The recurrence formula is as in Equation (3.29)

\begin{align}
(n+1)T_{n+1}(x) = (2n+1)(2x-N+1)T_n(x) - n(N^2 - n^2)T_{n-1}(x) \text{, n=1,2,… } (3.29) 
\end{align}

with the initial values \( T_0(x)=1 \) and \( T_1(x)=2x+1-N \)

It is advantageous to work with orthonormal polynomials whose norm equals one as shown in (3.30)

\begin{align}
\sum_{x=0}^{N-1} (\tilde{T}_n(x))^2 = 1 (3.30) 
\end{align}

Chebyshev polynomials and their associated moments do not show much change in their values of dynamic range of values, nor any numerical
instability for large values of \( N \). This suggests that normalize the polynomials to the magnitude by \( N^n \) and then to normalize them by \( q(n,N) \) to obtain the orthonormal polynomials. It results in the modified definition of the discrete Chebyshev polynomials as shown in Equation (3.31)

\[
\hat{T}_0(x) = \frac{1}{\sqrt{N}} \tag{3.31a}
\]

\[
\hat{T}_1(x) = (2x + 1 - N(\frac{3}{\sqrt{N(N^2 - 1)}}) \tag{3.31b}
\]

\[
\hat{T}_n(x) = (\alpha_1x + \alpha_2)\hat{T}_{n-1}(x) - \alpha_3\hat{T}_{n-2}(x) \tag{3.31c}
\]

The values \( \alpha_1, \alpha_2, \alpha_3 \) are computed using Equation (3.32)

\[
\alpha_1 = \frac{2}{n} \sqrt{\frac{4n^2 - 1}{N^2 - n^2}} \tag{3.32a}
\]

\[
\alpha_2 = \frac{(1 - N)}{n} \sqrt{\frac{4n^2 - 1}{N^2 - n^2}} \tag{3.32b}
\]

\[
\alpha_3 = \frac{n - 1}{n} \sqrt{\frac{2n + 1}{2N - 3} \sqrt{\frac{N^2 - (n - 1)^2}{N^2 - n^2}}} \tag{3.32c}
\]

Then the discrete Chebyshev moments for digital images are given by Equation (3.33)

\[
\tau_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \hat{T}_m(x)\hat{T}_n(y)f(x,y)dxdy \tag{3.33}
\]

The 2-D Chebyshev moments are less complex in nature but the greatest disadvantage is variance to rotation, translation and scaling of the image. Since the geometric moments are invariant to rotation, translation and
scaling Chebyshev moments are expressed in terms of geometric moments to overcome this problem.

Geometric moments of an image \( f(x, y) \) are expressed using the discrete sum approximation as

\[
g_{mn} = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} x^m y^n f(x, y) \tag{3.34}
\]

Chebyshev moments of the same image may be expressed in terms of geometric moments as

\[
T_{mn} = A_m A_n \sum_{k=0}^{N} C_k(m, N) \sum_{l=0}^{N} C_k(n, N) \times \sum_{i=0}^{k} \sum_{j=0}^{l} s^{(i)}_k s^{(j)}_l g_{ij} \tag{3.35}
\]

Where \( s^{(i)}_k \) are the Stirling numbers and \( A_m \) is given in Equation (3.36)

\[
A_m = \frac{1}{\sigma(m, N) \rho(m, N)} \tag{3.36}
\]

3.5 PRE-PROCESSING OF FINGERPRINT AND PALMPRINT IMAGES

The fingerprint images are taken from the FVC 2002 databases http://bias.csr.unibo.it/fvc2002/. The palmprint images available in the publically available PolyU Palmprint database http://www.comp.polyu.edu.hk/~biometrics/ are used.
3.5.1 Fingerprint

A conventional Gaussian filter is used for noise removal and smoothening. A fingerprint’s uniqueness is characterised by the ridges which are nothing but sharp discontinuities in the image. These discontinuities are variations in gray levels or the intensities. They are therefore affected when the illumination is not uniform or sufficient. Hence a fingerprint enhancement method adopted should be able to improve the contrast so as to make all details in the image prominent and also for an edge sharpening to make the edge segmentation more efficient. The wavelet-based normalization method Du and Ward (2005) was found to be very much suitable for the above requirements. In this method the contrast and edges of the fingerprint images are enhanced simultaneously using the wavelet transform. Wavelet-based image analysis decomposes an image into approximate coefficients and detail coefficients. Contrast enhancement can be done by histogram equalization of the approximation coefficients and meanwhile edge enhancement can be achieved by multiplying the detail coefficients with a scalar (>1). A normalized image is obtained from the modified coefficients by inverse wavelet transform. Using trial and error, db10 1st level wavelet decomposition and 1.5 as the scalar value for multiplying the detail coefficient were fixed.
Binarisation of the image is done using an adaptive threshold based binarization method Raghavendra et al (2009). This method involves transformation of the pixel values to 1 in case if the value of the pixel is larger than the mean intensity value of the current block in which the pixel is present, as seen in equation (3.1).

\[
I_{new}(n_1,n_2) = \begin{cases} 1 & \text{if } I_{old}(n_1,n_2) \geq \text{Local Mean} \\ 0 & \text{otherwise} \end{cases}
\] (3.37)

A two step method is followed to segment the region of interest from the binarized image. The first step involves estimation of the block direction along with direction variety check. The second step involves two morphological operations called ‘OPEN’ and ‘CLOSE’. The operator ‘OPEN’ expands the image, eliminates the peaks due to background noise and the latter shrinks the images and removes minute cavities.

3.5.2 Palmprint

![Figure 3.3 Palmprint Processing](image)

Gaussian smoothing is applied to the input image for noise removal. The filtered image is then binarised using an appropriate threshold ‘t’.

\[
B(x,y) = 1, \text{ if } O(x,y) \geq t
\] (3.38a)
B(x, y) = 0, if O(x, y) < t \hspace{1cm} (3.38b)

where, B(x, y) and O(x, y) are the binary image and the original image, respectively. The center of the palm is extracted since it contains the principal lines and wrinkles. Wavelet based normalization is used to enhance the extracted region of the palm Du and Ward (2005).

### 3.6 INARIANT FINGERPRINT AND PALMPRINT REPRESENTATION USING OG MOMENTS

#### 3.6.1 Fingerprint

The pseudo-Zernike, Legendre and Chebyshev moments are extracted from the fingerprint RoIs. Then the images are translated, scaled and rotated at an angle as shown in Figure 3.5. The Legendre, pseudo-Zernike and Chebyshev descriptors are again extracted for the translated, scaled and rotated images separately. The absolute difference between the two is calculated and plotted. The absolute errors for the translation, scaling and rotation are plotted in Figure 3.6. It is noted that the absolute error of the translation, scaling and rotational invariance are minimal and there is a slight increase in error when compared to the classical translation invariant, scale invariant and non-rotational descriptor.

![Figure 3.4 Deformed Fingerprint images](image)

(a) Original Fingerprint Image (b) Translated Image (c) Scaled Image (d) Rotated Image
Figure 3.5 TSR invariance in fingerprints (a) Translated Image (b) Scaled Image (c) Rotated Image
3.6.2 Palmprint

The pseudo-Zernike, Legendre and Chebyshev descriptors are extracted from the palmprint RoIs. Then the images are translated, scaled and rotated and again the moments are extracted for each case separately. The absolute errors for the translation, scaling and rotation are plotted in Figure 3.6. It is noted that the absolute error of the translation, scaling and rotational invariance proposed for Legendre, pseudo-Zernike and Chebyshev descriptors are minimal and there is a slight increase in error when compared to the classical translation invariant, scale invariant and non-rotational descriptor.

![Figure 3.6 Deformed palmprint images](image)

(a) Original Fingerprint Image  (b) Translated Image  (c) Scaled Image  (d) Rotated Image

![Translation Invariance Graph](image)
Figure 3.7  TSR invariance in palmprints (a) Translated Image  (b) Scaled Image (c) Rotated Image
For conducting the experiments, the fingerprints are taken from the FVC 2002 database. There are four different databases in FVC 2002 and the DB1 set has been used. It is composed of 800 fingerprint images derived from 100 individuals with eight images per finger. The palmprints are taken from the PolyU Palmprint database. The database contains 7752 gray scale images corresponding to 386 users.

3.7 CONCLUSION

In this chapter an overview of moments, particularly the orthogonal moments is provided. The primary motivation for using them is their stable numerical implementation and their high correlations since their standard powers are nearly independent. However, these orthogonal moments do have a disability that the image must always be mapped into the area of orthogonality since their numerical behaviour is worse outside the area of orthogonality.