CHAPTER - II
PRELIMINARIES

2.1. Definition

A graph \( G = (V, E) \) is a finite non-empty set \( V \) of objects called vertices together with a set \( E \) of unordered pairs of distinct vertices called edges.

2.2. Definition

The cardinality of the vertex set of graph \( G \) is called the \textbf{order} of \( G \) and is denoted by \( p \). The cardinality of its edge set is called the \textbf{size} of \( G \) and is denoted by \( q \). A graph with \( p \) vertices and \( q \) edges is called a \((p, q)\)-graph.

2.3. Definition

If \( e = (u, v) \) is an edge of \( G \), we write \( e = uv \) and we say that \( u \) and \( v \) are \textbf{adjacent} vertices in \( G \). If two vertices are adjacent, then they are said to be \textbf{neighbours}. Further, vertex \( u \) and edge \( e \) are said to be \textbf{incident} with each others, as are \( v \) and \( e \). If two distinct edges \( f \) and \( g \) are incident with a common vertex, then \( f \) and \( g \) are said to be \textbf{adjacent edges}.

2.4. Definition

An edge having the same vertex as both its end vertices is called a \textbf{self loop} or \textbf{loop}.

2.5. Definition

If there are more than one edge between the same given pair of vertices, these edges are called \textbf{parallel edges}. 
2.6. Definition

The number of edges incident on a vertex \( v \) with self-loops counted twice is called the **degree** of the vertex \( v \). It is denoted by \( d(v) \).

The **maximum degree** of a graph \( G \) is denoted by \( \Delta(G) \) and the **minimum** degree of \( G \) is denoted by \( \delta(G) \).

A vertex with degree 0 is called an **isolated vertex**.

A vertex with degree 1 is called a **pendant vertex**.

A graph is regular of degree \( r \) if every vertex of \( G \) has degree \( r \). Such graphs are called **\( r \)-regular graphs**.

2.7. Definition

A graph \( H \) is a **subgraph** of a graph \( G \), if all the vertices and all the edges of \( H \) are in \( G \), and each edge of \( H \) has the same end vertices as in \( G \).

2.8. Definition

A **spanning subgraph** of \( G \) is a subgraph \( H \) with \( V(H) = V(G) \). (i.e.) \( H \) and \( G \) having exactly the same vertex set.

2.9. Definition

A graph in which any two distinct vertices are adjacent is called a **complete graph** and denoted by \( K_p \).

2.10. Definition

Two graphs \( G \) and \( G' \) are said to be **isomorphic**, if there is a one to one correspondence between their vertices and between their edges, such that the incidence relationship is preserved.
2.11. Definition

A walk is a finite alternating sequence of vertices and edges, beginning and ending with vertices such that each edge is incident with the vertices proceeding and succeeding it [No edge appears more than once].

2.12. Definition

A walk which begins and ends with the same vertex is called a closed walk. A walk that is not closed is called an open walk.

2.13. Definition

If all the edges of a walk are distinct, then it is called a trial. An open walk in which no vertex appears more than once is called a path.

2.14. Definition

A walk in which no vertex (except the initial and the final vertex) appears more than once is called a cycle.

2.15. Definition

A graph $G$ is said to be connected if there is at least one path between every pair of vertices in $G$. Otherwise, $G$ is disconnected. A disconnected graph consists of two or more connected subgraphs. Each of these connected subgraph is called a component.

2.16. Definition

A graph $G$ is called bigraph (or) bipartite graph if $V(G)$ can be partitioned into two disjoint subsets $V_1, V_2$ such that every edge of $G$ joins a point of $V_1$ to a point of $V_2$. ($V_1, V_2$) is called a bipartition of $G$. 
2.17. Definition

A bipartite graph with bipartition \((V_1, V_2)\) is called a **complete bipartite graph** if every vertex of \(V_1\) is adjacent to every vertex of \(V_2\).

2.18. Definition

The **Cartesian product** \(G_1 \times G_2\) of two graphs \(G_1\) and \(G_2\) is the simple graph with \(V_1 \times V_2\) as its vertex set and two vertices \((u_1, v_1)\) and \((u_2, v_2)\) are adjacent in \(G_1 \times G_2\) iff either \(u_1 = u_2\) and \(v_1\) is adjacent to \(v_2\) in \(G_2\), or \(u_1\) is adjacent to \(u_2\) in \(G_1\) and \(v_1 = v_2\).

2.19. Definition

A graph with exactly one cycle is called a **unicyclic graph**.

2.20. Definition

Any cycle with a pendant edge attached at each vertex is called a **crown graph** and it is denoted by \(C_n^+(n \geq 3)\).

2.21. Definition

Armed crowns are obtained from cycle by attaching paths of equal lengths at each vertex of the cycle. We denote an armed crown by \(C_n \ominus P_m\) where \(P_m\) is a path of length \(m - 1\), where \(C_n\) is a \(n\)-cycle.

2.22. Definition

A **wheel** is a graph obtained from a cycle by adding a new vertex and edges joining it to all the vertices of the cycle, the new edges are called the spokes of the wheel. The wheel of \(n\) vertices is denoted by \(W_n\).
2.23. Definition

A graph is said to be $mC_n$-snake if it is a connected graph with $m$ blocks whose block-cutpoint graph is a path and each of the $m$ blocks is isomorphic to $C_n$.

2.24. Definition

Sparklers is a graph obtained by joining an end vertex of the path $P_m$, to the center of a star $K_{1,n}$ and is denoted by $P_m@K_{1,n}$.

2.25. Definition

The Bistar $B_{m,n}$ is the graph obtained from $K_2$ by joining $m$ pendant edges to one end of $K_2$ and $n$ pendant edges to the other end of $K_2$. The edge of $K_2$ is called the central edge of $B_{m,n}$.

2.26. Definition

A caterpillar is a graph derived from a path by hanging any number of leaves from the vertices of the path. The caterpillar can be considered as a sequence of stars $S_1 \cup S_2 \cup ... \cup S_r$, where each $S_i$ is a star with center $C_i$ and $n_i$ leaves, $i = 1, 2, ..., r$ and the leaves of $S_i$ include $C_{i-1}$ and $C_{i+1}$, $i = 1, 2, ..., r - 1$. It is denoted by $S_{n_1,n_2,...,n_r}$.

2.27. Definition

By a graph $mG$, we mean the disjoint union of $m$ copies of $G$.

2.28. Definition

If $G$ has order $n$, the corona of $G$ with $H$ denoted by $G \odot H$ is the graph obtained by taking one copy of $G$ and $n$ copies of $H$ and joining the $i^{th}$ vertex of $G$ with an edge to every vertex in the $i^{th}$ copy of $H$. 
2.29. Definition

Let $T$ be a tree and $u_0$ and $v_0$ be two adjacent vertices in $V(T)$. Let there be two pendant vertices $u$ and $v$ in $T$ such that the length of $u_0 - u$ path is equal to the length of $v_0 - v$ path. If the edge $u_0v_0$ is deleted from $T$ and $u$, $v$ are joined by an edge $uv$, then such a transformation of $T$ is called an elementary parallel transformation (or an EPT) and the edge $u_0v_0$ is called a transformable edge. If by a sequence of EPT’s $T$ can be reduced to a path, then $T$ is called a $T_p$-tree (transformed tree) and any such sequence regarded as a composition of mappings (EPT’s) denoted by $P$, is called a parallel transformation of $T$. The path, the image of $T$ under $P$ is denoted as $P(T)$. 