CHAPTER 3

DESIGNING OPTIMAL LQR CONTROLLERS FOR LFC OF TWO-AREA POWER SYSTEM NETWORK

3.1 CHAPTER OVERVIEW

The change in the power demand results in change in the frequency of the interconnected power system along with the power exchange between the different areas. The load frequency control can be achieved by proper regulation of the participating generating units. The area control error (ACE) has to be minimized for the frequency response enhancement and the integral square error (ISE) is generally taken as ACE. The objective of automatic load frequency controller (ALFC) is to reduce the integral square error to zero with the continuous change of demand of active power so that the total generated power of the system and the load requirement properly matches with each other [15, 18, 46]. The load on the power system consists of a variety of electrical drives. The equipment used for lighting purposes is basically resistive in nature and the rotating devices are basically a composite of the resistive and inductive components. The load power can be frequency dependant and non-frequency dependant, thus the speed-load characteristic of the composite load is given as:

\[ \Delta P_e = \Delta P_L + D \Delta \omega \]  \hspace{1cm} (3.1)

Where \( \Delta P_e \) is electrical power output change, \( \Delta P_L \) is the frequency independent load change and \( D \Delta \omega \) is the frequency sensitive load change. D is expressed as percent change in load to percent change in frequency.

The control of active power for LFC is possible by varying the parameters of various controllers and sources in the system. There are basically three types of possible connections of controllers in the power system network, namely, central controller, single agent and decentralized controller [47]. In case of central controller, only one control unit is used and it is connected to all the areas in the power system network to
control their parameters and maintain the generation and demand balance. Further, in single agent method the control action is performed on individual distributed generation system. In case of decentralized control each area is equipped with individual controller and local feedback control is used.

In this chapter, different controllers have been designed for LFC of two-area interconnected power system. The power system network comprises of two thermal power generation systems connected through tie-line. The first type of the controller is integral controller which has been fully optimized with particle swarm optimization (PSO) technique. The second type of controller is designed by using the dual properties of optimal control and classical control by using linear quadratic regulator (LQR) and integral controller for frequency regulation. The third controller is intelligent state feedback controller in which the optimum value of state feedback gain matrix (K) are obtained using PSO. Then, the performance comparison of the three controllers is carried to find out the best controller for the frequency regulation.

3.2 OBJECTIVES OF THE CHAPTER

The work carried out in this chapter addresses the challenge of designing the controller which incorporates the novel properties of optimal control, classical control and intelligent control to achieve the LFC of two-area network in an effective manner. The frequency response must satisfy the recent grid codes with minimum frequency deviation. The objectives of this chapter are as follows:

1. State space modeling of two-area thermal power system with eleven states.
2. Designing of PSO-based integral controller for the LFC of the system.
3. Frequency response enhancement with Linear Quadratic Regulator - Integral (LQR-I) controller.
4. Designing of PSO-based state feedback controller for automatic load frequency control of two-area network.
5. Performance comparison of PSO-based state feedback controller, LQR-I and integral only controller for load frequency control (LFC) of two-area network.
The remnant of this chapter is organized as follows: The concepts of optimal control, linear quadratic regulator, integral control and intelligent optimal control and PSO technique have been elaborated in Section 3.3 and 3.4. The designing of state space model with eleven states and transfer function modeling of two-area thermal network is done in Section 3.5. In the Section 3.6 the novel LFC approaches as described in Section 3.3 and 3.4 have been adapted for two-area network of Section 3.5. The Section 3.7 gives the results and comparison of various techniques to ascertain the superiority of the proposed controller and the summary of the chapter 3 is given in Section 3.8.

### 3.3 CLASSICAL CONTROL AND OPTIMAL CONTROL

The classical control is a heuristic approach based on Laplace transform for designing the linear time-invariant (LTI) controllers. The proportional (P), integral (I) and differential (D) and PID controllers are most widely used classical controllers in industry for various applications. The optimal control provides more effective control as it is based on cost function minimization [48, 49, 50]. For LTI systems, the linear quadratic regulator (LQR) is most commonly used control architecture in optimal control [51]. In the present work, the control benefits of both classical and optimal control have been adapted simultaneously. In order to design a simple, effective and easy-to-use controller which incorporates the benefits of both classical and optimal control, the LQR and integral controller have been used simultaneously for LFC of the two-area network. The effectiveness of optimal LQR control is enhanced further by optimizing the state feedback gain matrix with the help of particle swarm optimization technique.

#### 3.3.1 Classical Integral Control

In integral control, the control action is proportional to the integral of error signal. The \( K_i/s \) represents the integral controller transfer function where the \( K_i \) is the integral gain which should be optimized for an optimum response. When the integral controller is designed properly, the steady state error is zero and the system behavior is such that the response does not oscillate beyond the permissible limits in order to maintain the
stability of the system. The integral gain ($K_i$) should be chosen carefully. If the integral gain is too low, there can be asymptotic tracking and for too high values of $K_i$, the system can become unstable. The integral controller response is slow in nature but this controller has superiority in the form of low steady state error [52, 53]. The classical method for optimization was based on finding the optimum values of gains of integral controllers connected in different areas. The integral square error (ISE) is chosen as performance index of the system. The minimization of the performance index results in the optimum gains of all integral controllers for frequency control during particular loading condition. The main disadvantage of this method is that it does not give the optimal results in terms of peak overshoot and settling time.

3.3.2 Optimal Control

The problems in classical control can be overcome if the internal states of the system, which are variable in nature, are also considered while defining the performance index of the system. It gives rise to the use of optimal control theory for frequency regulation. The cost function in optimal control is the function of state variables and control variables. The optimal control is based on set of differential equations which define the path of control variables. In case of optimal control, the system has to be represented in state space form and the desired response in terms of cost function to be minimized.

The formulation of an optimal control problem requires the mathematical model of the system to be controlled in state space form, specification of the performance index, specification of all boundary conditions on states, information of all free variables and constraints to be satisfied by states and controls [54, 55].

The optimal control problem can be described by introducing the system dynamics as given below:

$$\dot{x} = F(x, u)$$  \hspace{1cm} (3.2)

$x$ and $u$ are state and control variables respectively. The initial state $x(0)$ is given by:

$$x(0) = x_0$$  \hspace{1cm} (3.3)
The objective function \( (J) \) consists of a function of the final state \( \psi[x(T)] \) and a cost function that is integrated over time.

\[
J = \psi[x(T)] + \int_0^T l(u, x)dt
\]  

(3.4)

A special case of linear quadratic (LQ) optimal control problem, which is of particular importance, arises when the objective function is a quadratic function of \( x \) and \( u \), and the system dynamics are described by the set of linear differential equations. The resulting feedback law which provides the state feedback control is known as the linear quadratic regulator (LQR). The theory of optimal control is concerned with operating a dynamic system at minimum cost.

Linear quadratic regulator (LQR) provides a way of stabilizing any linear system. It is worth pointing out that there are well established methods for solving the Ricatti equation (3.13) which has an important role in optimal control. This facilitates the design of linear quadratic regulators. A useful extension of the linear quadratic regulator ideas involves modifying the performance index to allow for a reference signal that the output of the system should track.

### 3.3.3 Linear Quadratic Regulator

The evolution of optimal control theory has led to the emergence of linear quadratic regulator which is an optimal multivariable feedback control approach that minimizes the excursion in state trajectories of a system while requiring minimum controller effort and improves stability performance. The optimal controller based on linear quadratic regulator aims to minimize a cost function which is most significant for the system. The optimal control with LQR is successfully used for multivariable control problems. The LQR design problem is primarily related to design a state feedback gain controller \( (K) \) such that the objective function \( (J) \) is minimized. In this technique, a feedback gain matrix is designed which minimizes the objective function in order to achieve some compromise between the use of control effort, the magnitude and the speed of response which gives a stable system. The LQR algorithm reduces the amount of work done by the control systems engineer to optimize the controller. However, the engineer still needs to specify the cost function parameters and compare the results with the specified design goals. Often the controller construction is an iterative process in which the
optimal controllers produced through the simulation are compared and then the parameters are adjusted to produce a controller more consistent with design goals.

One of the main advantages in the LQR theory is that the solution provides a feedback controller which minimizes the cost function (J) and the optimal results are attained in terms of all system parameters [56]. To ensure that the control problem is well defined, the state input matrix (A) and control input matrix (B) must be stabilizable. Stabilizability means that there exists a feedback gain (K) such that the closed loop system is stable. The state equation of a feedback system is given as:

\[ \dot{x} = (A - BK)x \]  \hspace{1cm} (3.5)

There must exist an acceptable solution with state feedback law:

\[ u = -Kx \]  \hspace{1cm} (3.6)

The solution of (3.5) is given by:

\[ x(t) = e^{(A-BK)t}x_0 \]  \hspace{1cm} (3.7)

The block diagram of LQR control is shown in Figure 3.1.

![Block diagram of LQR Control](image)

Figure 3.1: Block diagram of LQR Control

The state space (SS) representation of system is given by following state equation (3.8) and the output equation (3.9):

\[ \dot{X} = AX + BU \]  \hspace{1cm} (3.8)

\[ Y = CX \]  \hspace{1cm} (3.9)
Where the state input matrix A, input control matrix B and output matrix C are determined by system parameters. X is state vector, U is control input and Y is output of the control system. The cost function to be minimized in case of LQR is given as:

$$J_{LQR} = \frac{1}{2} \int_0^\infty (X^T Q X + U^T R U) dt$$

Q and R are the weight matrices. The state weight matrix Q is positive definite or positive semi-definite symmetrical matrix and control weight matrix R is positive definite symmetrical matrix. Q and R matrices as chosen as diagonal matrix. The value of the elements in Q and R is related to its contribution to the cost function $J_{LQR}$. The optimal control signal is given as:

$$U = -KX$$

(3.11)

$$K = R^{-1}B^TP$$

(3.12)

P can be calculated by matrix Riccatti equation given below:

$$A^TP + PA - PBR^{-1}B^TP + Q = 0$$

(3.13)

The weighting matrices Q and R are chosen carefully so that the performance index is minimum. The matrices A, B and C for the system analyzed in this work are given in the section 3.6.2.

### 3.3.4 LQR-Integral Control

LQR has low speed of response and steady-state error is also not zero always. It can generate the optimal controller for the particular cost function and model supplied. In order to improve the steady-state error, the regular LQR structure is modified to include feedback on the integral of the error [6, 57]. Integral feedback is a common technique in classical control. In a single-input-single-output (SISO) system, it can guarantee zero steady-state error. The combination of optimal control and classical control serves the purpose in a better way as the properties of both the kind of controllers are used in
control action. LQR-Integral controller performs better than integral only or LQR only controller for complex systems with large number of state variables.

3.4 INTELLIGENT OPTIMAL CONTROL

The control system for LFC of multi-area power system is complex due to large number of variables of the system which have to be optimized. In optimal control, as state space model is used and cost function is a function of state matrix (X), control vector (U) and weight matrices Q and R. The tuning of the variable parameters is essentially a time consuming task when optimal state feedback gain matrix (K) has to be obtained. Similarly, when integral controller is used the complexity of system arises due to number of controllers, frequency bias, droop and GRC. The optimal controller action can be further improved by the use of computationally intelligent technique for the tuning of state feedback gain matrix (K) of optimal controller. The large number of parameters can be optimized by using search based and evolutionary algorithms (EA). These algorithms help in achieving the fast and accurate response of the system. Particle swarm optimization is one of such computationally intelligent and powerful optimization algorithm which can be easily applied for various applications [22, 58]. In present work, PSO has been used for tuning of integral controller and LQR controller for obtaining the optimal state feedback gain matrix (K). The schematic block diagram of PSO-based state feedback controller is shown in Figure 3.2. The implementation of PSO-based state feedback controller for LFC of the two-area thermal network is explained in Section 3.6.

![Block diagram of proposed PSO-based state feedback controller](image)

Figure 3.2: Block diagram of proposed PSO-based state feedback controller
The computation of state feedback control gains is conventionally handled by LQR method via Riccati equation (3.13). The elements of Q and R matrices in the state feedback control design using LQR method have to be chosen carefully based on the understanding of the system. An intelligent method to resolve this problem of optimizing the state feedback gain matrix is proposed in this work by adopting PSO-based optimization. It computes the optimum state feedback gain (K) which is used in control law to compute the level of control signal. The result shows that the intelligent optimal control method reduces tuning time and improves the performance of the system.

3.4.1 Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a computational random optimization technique developed by Kennedy and Eberhart in 1995 [59]. It uses a simple mechanism that mimics swarm behavior in birds flocking and fish to guide the particles to search for global optimal solutions. PSO is proved to be a very efficient optimization algorithm by searching an entire high-dimensional problem space. PSO does not use the gradient of the ACE being optimized, so it does not require that the optimization problem be expressed as differential equation as required by classic optimization methods. Further, the solution is not trapped in the local optima as in case of classical optimization methods.

The particle swarm algorithm begins by creating the initial particles and assigning them initial velocities. It evaluates the objective function at each particle location and determines the best (lowest) function value and the best location. It updates velocities based on the current velocity, the particles individual best locations and the best locations of their neighbours. It then iteratively updates the particle locations. The new location as given in equation (3.14) is the old one plus the velocity modified as per the equation (3.15) to keep particles within bounds.

The iterations proceed until the algorithm reaches a stopping criterion. The main terminology used in PSO technique is that a particle is called bird, the solution is food, pbest is the the best fitness solution a particle has achieved so far, gbest is defined as the best position found by all the particles in the swarm. It will be updated whenever a particle reaches a position with better fitness value than the fitness value of the previous
global best. $V_{\text{max}}$ is the maximum range a particle can attain during one iteration and the learning factors $C_1$ and $C_2$ are chosen between 0 to 4.

The system is initialized by random solutions which get updated successively in order to get the global optima. The stopping criterion is determined by number of iterations and permissible error. The main processes include intensification which explores the previous solutions, finds the best solution of a given region. The diversification process searches new solutions and finds the regions with potentially the best solution.

The algorithmic steps of PSO technique are given below [55]:

1. Select the number of particles that constitute a swarm moving around in the search space looking for the best solution and initialize their position.
2. Each particle adjusts its travelling speed dynamically corresponding to the flying experiences of itself and its colleagues.
3. Each particle modifies its position according to its current position, its current velocity, the distance between its current position and $p_{\text{best}}$, the distance between its current position and $g_{\text{best}}$.
4. Move particles to their new positions. Go to step 2 until stopping criteria is satisfied.

The PSO Pseudocode is mentioned below:

$[x^*] = \text{PSO}( )$

$P = \text{Particle\_Initialization}();$

For $i=1$ to $\text{it\_max}$

For each particle $p$ in $P$ do

$fp = f(p);$
If \( fp \) is better than \( f(p\text{Best}) \)

\[ \text{pBest} = p; \]

end

end

\( g\text{Best} = \text{best p in P}; \)

For each particle \( p \) in \( P \) do

\[ v = v + c1 \ast \text{rand} \ast (p\text{Best} - p) + c2 \ast \text{rand} \ast (g\text{Best} - p); \]

\[ p = p + v; \]

end

end

Hence Particle update rule is given as:

\[ p = p + v \]  \hspace{1cm} (3.14) \\

with

\[ v = v + c1 \ast \text{rand} \ast (p\text{Best} - p) + c2 \ast \text{rand} \ast (g\text{Best} - p) \]  \hspace{1cm} (3.15) \\

where \( p \) is particle’s position, \( v \) is path direction, \( c1 \) is weight of local information, \( c2 \) is weight of global information, \( p\text{Best} \) is best position of the particle, \( g\text{Best} \) is best position of the swarm, \( \text{rand} \) is random variable. The flow chart of PSO technique is shown in Figure 3.3.

The advantages of PSO are that it is insensitive to scaling of design variables, simple implementation, easily parallelized for concurrent processing, derivative free, very few algorithm parameters and it is a very efficient global search algorithm. The disadvantages are the tendency to a fast and premature convergence in mid optimum points and slow local search ability.
3.5 MODELING OF POWER SYSTEM NETWORK

The performance of Integral, LQR-Integral and PSO-based state feedback controller is analyzed for the two-area network with interconnected thermal power plants. Two power system areas are connected to each other by a tie-line and the reheat turbine is used in both the power plants. Each thermal plant taken for performance comparison of different load frequency controllers is of generation capacity 2000 MW. The system
model developed is made linear by making certain assumptions and approximations as in the mathematical model [18]. The model of steam turbine is made linear by assuming single equivalent time constant. The two-area system mainly comprised of two balancing areas (BA) with each one having governor system, turbine, reheat stage of turbine, generator and frequency controller.

The power flowing from BA1 to BA2 may be expressed as:

\[
P_{12} = \frac{V_1 V_2}{X} \sin(\theta_1 - \theta_2) \tag{3.16}
\]

\[
X = X_1 + X_2 + X_{\text{tie}} \tag{3.17}
\]

Where X is the total reactance of the transmission system (Ω), \( V_1 \) and \( V_2 \) are the voltage levels at both the areas (V), \( \theta_1 \) and \( \theta_2 \) are the angles of power at area-1 and area-2 respectively.

The transfer function of the governor is:

\[
G_g(s) = \frac{1}{T_g s + 1} \tag{3.18}
\]

Transfer function of the steam turbine is:

\[
G_t(s) = \frac{1}{T_t s + 1} \tag{3.19}
\]

The transfer function for reheat stage is given by:

\[
\Delta P_r = \frac{1 + sK_r T_r}{1 + sT_r S} \Delta P_g \tag{3.20}
\]

Transfer function of the generator is:

\[
G_p(s) = \frac{K_p}{T_p s + 1} \tag{3.21}
\]

\[K_p=1/D\text{ and } T_p=2H/f\]

Where \( T_g \) is governor time constant (second), \( T_t \) turbine time constant which is generally in the range of 0.2-2 seconds, \( T_p \) is power system time constant, H is inertia constant of area-i (s), \( K_p \) is power system gain (Hz/puMW), D is equivalent system
damping coefficient of control area (MW/Hz), \( K_r \) is steam turbine reheat constant and \( f \) is nominal system frequency (Hz). The parameters of the system are given in Table 3.1.

Table 3.1: Parameters of Two-area Thermal Power System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency ( f )</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Governor time constant ( T_g )</td>
<td>0.08 s</td>
</tr>
<tr>
<td>Maximum Tie-Line power</td>
<td></td>
</tr>
<tr>
<td>( P_{t_{\text{iemax}}} )</td>
<td>200 MW</td>
</tr>
<tr>
<td>Reheat time constant ( T_r )</td>
<td>10 s</td>
</tr>
<tr>
<td>Tie-line time constant ( T_{12} )</td>
<td>0.544 s</td>
</tr>
<tr>
<td>Tie-line gain ( a_{12} )</td>
<td>0.5</td>
</tr>
<tr>
<td>Steam turbine reheat constant ( K_r )</td>
<td>0.5</td>
</tr>
<tr>
<td>Steam turbine time constant ( T_t )</td>
<td>0.3</td>
</tr>
<tr>
<td>Power system gain ( K_p )</td>
<td>120 Hz/pu MW</td>
</tr>
<tr>
<td>Power system time constant ( T_p )</td>
<td>20 s</td>
</tr>
</tbody>
</table>

The model of the two-area thermal power system network with each plant having generating capacity of 2000 MW is designed on the basis on transfer function approach in MATLAB/SIMULINK environment. The state space modeling of the system has also been done in this section to analyze the performance of the LQR-I, PSO-based state feedback controllers. The power system transfer function model has been developed by combining the transfer function for different parts of thermal power network such as generator, governor, turbine, reheat and tie line power. All the important parameters like
frequency bias ($B_i$), power-frequency droop ($R_i$) and generation rate constraint (GRC) have been taken into consideration while modeling the system [60-64]. The block diagram of two-area thermal system network with eleven state variables for automatic frequency regulation of a linear network is shown Figure 3.4.

Figure 3.4: Two-area thermal power system network with eleven states
For designing the state space model of the system, the power system state variables have been determined to obtain the set of equations of the interconnected thermal power network in state space [65]. The model has been developed taking reheat into consideration for both the thermal power systems. Total eleven state variables for the system shown in Figure 3.4 are taken as $\Delta f_1$ as $x_1$, $\Delta P_{t1}$ as $x_2$, $\Delta P_{r1}$ as $x_3$, $\Delta P_{g1}$ as $x_4$, $\Delta f_2$ as $x_5$, $\Delta P_{t2}$ as $x_6$, $\Delta P_{r2}$ as $x_7$, $\Delta P_{g2}$ as $x_8$, $\Delta P_{tie(1,2)}$ as $x_9$, $\int (ACE_1) dt$ as $x_{10}$, $\int (ACE_2) dt$ as $x_{11}$. The $\Delta f_i$ is frequency deviation in area-i $(i=1,2)$, $\Delta P_{t}$ is turbine output power, $\Delta P_{r}$ is reheat power, $\Delta P_{g}$ is generator power, $\Delta P_{tie(1,2)}$ is tie line power, $ACE_i$ is area control error. State equations for the different blocks of the system are given as:

For block 1:

$$x_1 + T_{P1} \frac{dx_1}{dt} = K_{P1} (x_2 - x_9)$$  \hspace{1cm} (3.22)$$

For block 2:

$$x_2 + T_{t1} \frac{dx_2}{dt} = x_3$$  \hspace{1cm} (3.23)$$

For block 3:

$$x_3 + T_{r1} \frac{dx_3}{dt} = x_4 + K_{r1} T_{r1} \left( \frac{dx_4}{dt} \right)$$  \hspace{1cm} (3.24)$$

For block 4:

$$x_4 + T_{g1} \frac{dx_4}{dt} = -x_1/R_1 + u_1$$  \hspace{1cm} (3.25)$$

For block 5:

$$x_5 + T_{r2} \frac{dx_5}{dt} = K_{r2} (x_6 + x_9)$$  \hspace{1cm} (3.26)$$

For block 6:

$$x_6 + T_{r2} \frac{dx_6}{dt} = x_7$$  \hspace{1cm} (3.27)$$

For block 7:

$$x_7 + T_{r2} \frac{dx_7}{dt} = x_8 + K_{r2} T_{r2} \left( \frac{dx_7}{dt} \right)$$  \hspace{1cm} (3.28)$$

For block 8:

$$x_8 + T_{g2} \frac{dx_8}{dt} = -x_5/R_2 + u_2$$  \hspace{1cm} (3.29)$$

For block 9:

$$\frac{dx_9}{dt} = 2\pi T_0 x_1 - 2\pi T_0 x_5$$  \hspace{1cm} (3.30)$$

For block 10:

$$x_{10} = B_1 x_1 + x_9$$  \hspace{1cm} (3.31)$$

For block 11:

$$x_{11} = B_2 x_5 - x_9$$  \hspace{1cm} (3.32)$$

Where $T_g$ is governor time constant, $T_t$ turbine time constant, $T_p$ is power system time constant, $T_r$ is reheat time constant, $B$ is frequency bias, $R$ is frequency-droop, $T_0$ is tie-line time constant. The control inputs of the system are $u_1$ and $u_2$. The MATLAB/SIMULINK model of two-area thermal network with eleven states is shown in Figure 3.5. The state vector and control vector for the above system are given as :

State Vector $(X) = [X_1 X_2 X_3 X_4 X_5 X_6 X_7 X_8 X_9 X_{10} X_{11}]$ \hspace{1cm} (3.33)$

Control Vector $(u) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ \hspace{1cm} (3.34)$
Figure 3.5: MATLAB/SIMULINK model for LFC of two-area network using LQR- Integral controller
The state vector $X$ comprises of all eleven states and control vector has two control inputs $u_1$ and $u_2$. For the analysis of the performance of LQR-Integral controller and PSO-based state feedback controller, the state space model is used. The integral controller performance is analyzed with transfer function model.

### 3.6 DESIGNING OF CONTROLLERS FOR LFC OF TWO-AREA POWER SYSTEM NETWORK

In this section, the novel LFC approaches using integral controller, LQR-Integral controller and PSO-based state feedback gain (K) controller as described in Section 3.3 and 3.4 have been adapted for LFC of two-area network. The mathematical modeling in the form of transfer function and state space representation of the two-area thermal system is already described in Section 3.5.

#### 3.6.1 PSO-based Integral Controller

The first controller used for frequency response enhancement of the system given in Section 3.5 is an integral controller. The integral gain ($K_i$) is tuned by PSO technique to achieve the robust performance. As there are certain structural limits for the parameters which have to be considered for designing of a controller, particle swarm optimization (PSO) technique is used to find the control parameter of the integral load frequency controller within the structural constraints [59]. The performance index can be given as integral square of error (ISE) given by equation as below:

$$\text{ISE} = \frac{1}{2} \int (\Delta f_1^2 + \Delta f_2^2) \, dt \quad (3.35)$$

Where $\Delta f_1$ is the frequency variation in area-1 and $\Delta f_2$ is the frequency variation in area-2. The major advantage of integral controllers is that they have the unique ability to return the controlled variable back to the exact set point following a disturbance. Disadvantages of the integral control mode are that it responds relatively slowly to an error signal and that it can initially allow a large deviation at the instant the error is produced. This can lead to system instability and cyclic operation. For this reason, the
integral control mode is not normally used alone, but is combined with another control mode. The LQR-Integral control gives the better performance for load frequency control where the integral gain act as a state of the system.

Along with integral gain ($K_i$), the speed regulation parameter ($R_{i}$) and frequency bias ($B_{i}$) parameters of both the systems are also optimized using PSO for the LFC of two-area network. The six optimized variable and gains are found as: $B_1 = B_2 = 2$, $K_{i1} = 0.02$, $K_{i2} = 0.015$, $1/R_1 = 0.3$, $1/R_2 = 0.85$.

### 3.6.2 LQR-Integral Control

In the second method for the LFC of two-area thermal power network, the linear quadratic regulator is used to optimize the integral controller gain as well as the states of system. First algebraic Riccati equation has been used to solve for the matrix $P$ with equation (3.13). The performance index in this method is given in equation (3.43). For designing of weight matrices $Q$ and $R$ the proper understanding of the system is essential. $Q$ and $R$ are mainly the diagonal matrices. Both the matrices related to the weights such as $Q$ and $R$ are chosen judiciously for the stability of the system and for proper designing of LQ regulator. The optimal regulators are designed by changing the diagonal elements of $Q$ and the elements of matrix $R$ are not changed [60, 66]. The weight matrices are also selected according to the predefined specifications of the system and desired response of the system. For PSO-based state feedback controller the initial values of $Q$ and $R$ are selected as unity for both the diagonal matrices as given in equations (3.36) and (3.37) in order to reduce the complexity of the system.

\[
Q = \begin{bmatrix}
Q_{11} & 0 & 0 & 0 \\
0 & Q_{22} & 0 & 0 \\
0 & 0 & Q_{33} & 0 \\
0 & 0 & 0 & Q_{44}
\end{bmatrix}
\]  \hspace{1cm} (3.36)

\[
R = \begin{bmatrix}
R_{11} & 0 & 0 & 0 \\
0 & R_{22} & 0 & 0 \\
0 & 0 & R_{33} & 0 \\
0 & 0 & 0 & R_{44}
\end{bmatrix}
\]  \hspace{1cm} (3.37)
The matrices $A$ and $B$ for the system are written by observing the state differential equations (3.22)-(3.32) and specifications of the system. The state input matrix $A$ is $11\times11$ matrix and input control matrix $B$ is $11\times2$ matrix.

$$A = \begin{bmatrix}
-\frac{1}{T_{p1}} & \frac{K_{p1}}{T_{p1}} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{K_{p1}}{T_{p1}} & 0 & 0 \\
0 & -\frac{1}{T_{r1}} & \frac{1}{T_{r1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{-K_{r1}}{R_{1}T_{g1}} & 0 & -\frac{1}{T_{r1}} & \left(\frac{1}{T_{r1}} - \frac{K_{r1}}{T_{g1}}\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{-1}{R_{1}T_{g1}} & 0 & 0 & -\frac{1}{T_{g1}} & \frac{K_{p2}}{T_{p2}} & 0 & 0 & \frac{K_{p2}}{T_{p2}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{-1}{T_{p2}} & \frac{K_{p2}}{T_{p2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{T_{r2}} & \frac{1}{T_{r2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{-K_{r2}}{R_{2}T_{g2}} & 0 & -\frac{-1}{T_{r2}} & \left(\frac{1}{T_{r2}} - \frac{K_{r2}}{T_{g2}}\right) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-1}{R_{2}T_{g2}} & 0 & 0 & -\frac{-1}{T_{g2}} & 0 & 0 & 0 \\
2\pi T^{0} & 0 & 0 & 0 & -2\pi T^{0} & 0 & 0 & 0 & 0 & 0 & 0 \\
B_{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & B_{2} & 0 & 0 & 0 & -1 & 0 & 0
\end{bmatrix} \quad (3.38)$$

$$B = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
K_{r1} & 0 \\
\frac{1}{T_{g1}} & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\frac{K_{r2}}{T_{g2}} & 0 \\
0 & \frac{1}{T_{g2}} \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix} \quad (3.39)$$
The system state matrix $A$, input control matrix $B$ and output matrix $C$, while considering the parameters of the system (Two-area thermal systems 2000 MW each) can be given as:

$$A = \begin{bmatrix}
-0.05 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 \\
0 & -3.33 & 3.33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2.08 & 0 & -0.1 & -6.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-4.16 & 0 & 0 & -12.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.05 & 6 & 0 & 0 & 6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -3.33 & 3.33 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -2.08 & 0 & -0.1 & -6.1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -4.16 & 0 & 0 & -12.5 & 0 & 0 & 0 \\
3.42 & 0 & 0 & 0 & -3.42 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -10 & 0 \\
\end{bmatrix} \quad (3.40)$$

$$B = \begin{bmatrix}
0 \\
0 \\
6.25 \\
12.5 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} \quad (3.41)$$

The output matrix $C$ is 1x11 matrix as given below:

$$C = [1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \quad (3.42)$$
The system performance index while using LQR-Integral controller is as follows:

\[
\text{Performance Index} = 0.5 \int (x'Qx + u'Ru) \, dt
\]  
(3.43)

In the MATLAB/Simulink environment the state space model is given by the equation below:

\[
sys = \text{ss}(A,B,C,D);
\]  
(3.44)

The state feedback gain matrix \((K)\) can be calculated with LQR method as follows [67, 68]:

\[
[K] = \text{lqr}(\text{sys}, Q, R);
\]  
(3.45)

Further, the value of matrix \(K\) obtained with LQR-Integral controller for the system under study is given as:

\[
K = \begin{bmatrix}
0.489 & 1.087 & 2.399 & 0.717 & 0.026 & 0.003 & 0.162 & 0.053 & 0.823 & 1 & 0 \\
0.003 & 0.003 & 0.162 & 0.053 & 0.489 & 1.087 & 2.399 & 0.717 & 0.823 & 0 & 1
\end{bmatrix}
\]  
(3.46)

The frequency bias parameters with LQR-I controllers are \(B_1 = 1.57\) and \(B_2 = 2.05\), integral gains \(K_{i1} = 0.017\), \(K_{i2} = 0.018\), the speed regulation parameters are \(R_1 = 0.146\) and \(R_2 = 0.146\).

### 3.6.3 PSO-based State Feedback Controller

The performance index for PSO-based state feedback controller can be given as integral square of error (ISE) as given below:

\[
\text{ISE} = 1/2 \int (\Delta f_1^2 + \Delta f_2^2) \, dt
\]  
(3.47)

In the PSO-based state feedback method the elements of state feedback gain matrix \((K)\) act as the particles. The pseudo code for using the PSO for the calculation of optimized value of state feedback gain matrix is given as:
For every particle:

Initialize

Do

Solve the equation (3.45) for obtaining the initial values of matrix K

Calculate the particle velocity using the equation given below:

\[ v_i (t) = v_i (t-1) + c_1 \varphi_1 (p \text{ (pbest} - x_i (t-1)) + c_2 \varphi_2 (p \text{ (gbest} - x_i (t-1)) \]  \hspace{1cm} (3.48)

Update particle position according to equations (3.48) and (3.49):

\[ x_i (t) = x_i (t-1) + v_i (t) \]  \hspace{1cm} (3.49)

Till the stopping criterion is met.

End

The present value of ACE is compared to best value of ACE in previous iterations (pbest). Then this new pbest is considered as the current value for the pbest. The particle at which the least ACE is obtained becomes gbest. The optimum values of elements of matrix K of dimension 2X11 are obtained with PSO technique as given in equation (3.50). The system model on which the PSO technique has been applied is described in Section 3.5. This is a fast and efficient tuning approach. The population size taken in PSO method is 100, search range is taken as [-3, 3], acceleration constants are chosen as \( c_1=1.5, \ c_2=1.5 \) and iteration number is taken as 100. As the main advantage of PSO is that the initial value of the parameter to be optimized can be taken as zero and the suitable range for optimum value is given for fast convergence. So initially each element of K matrix has been taken as zero.

The optimum control vector is obtained as per the equation (3.11) using the optimized state feedback gain matrix (K). As per the system and its loading conditions the final optimized state feedback gain matrix (K) after applying PSO method is:

\[
K = \begin{bmatrix}
0.012 & 0.071 & 2.856 & 0.741 & 0.729 & 0.639 & 0.469 & 0.182 & 1.348 & 1 & 0
\end{bmatrix} \hspace{1cm} (3.50)
\]
3.7 SIMULATION RESULTS AND DISCUSSION

The performance of the three load frequency controllers such as integral controller, LQR-I controller and PSO-based state feedback controller has been compared in this section. The two-area interconnected power system network model is explained in section 3.5. Three types of controllers have been used for frequency response enhancement of the system considering one controller at a time. The various parameters of different controllers have been optimized as described in the earlier sections 3.6.1, 3.6.2 and 3.6.3. The LFC response of the system using these optimized controllers has been discussed and then performance comparison is done for integral controller, LQR-Integral controller and PSO-based state feedback gain controller.

Two different operating conditions are incorporated to demonstrate the effectiveness of proposed PSO-based state feedback controller (PSO-LQR-I) and to compare the performance with LQR-I and Integral controller. In first case the effect of load variation of 1% in area-1 is analysed on the frequency response of both the areas. In our case since the aim is to control the frequency, the second case presents the effect of load variation of 1% in area-2 and frequency response enhancement with different controllers.

The load has been increased by 1% at a time in one thermal power area for the simulation study done in MATLAB/SIMULINK environment. It has been observed that the load change in area-1 results in the frequency variation in the area-1 as well as in area-2. Then the performance of controllers is analyzed by increasing the load by same amount in area-2. The optimally designed integral controller, LQR-Integral controller and PSO-based state feedback gain (K) matrix controller helps in keeping all the state variables in proper range along with the permissible frequency deviation during load change.

The variation in frequency for both the areas after loading the area-1 has been shown in Figure 3.6 and Figure 3.7 for integral controller and LQR-integral controller. The loading effect in area-2 and thus frequency responses with integral and LQR-integral controllers have been given in Figure 3.8 and Figure 3.9.
The nomenclature used for various frequency responses is as follows:

delf11: Frequency response of power system Area-1 for 1% load deviation in Area-1
delf21: Frequency response of power system Area-2 for 1% load deviation in Area-1
delf12: Frequency response of power system Area-1 for 1% load deviation in Area-2
delf22: Frequency response of power system Area-2 for 1% load deviation in Area-2

Figure 3.6: Frequency response of area -1 if 1% load variation occurs in area-1

Figure 3.7: Frequency response of area -2 if 1% load variation occurs in area-1
Figure 3.8: Frequency response of area -1 if 1% load variation occurs in area-2

Figure 3.9: Frequency response of area -2 if 1% load variation occurs in area-2
The frequency response of area-1 and area-2 for 1% load variation in area-1 and using PSO-based state feedback controller is shown in Figure 3.10 and Figure 3.11.

Figure 3.10: Frequency response for area-1 if 1% load variation occurs in area-1 using
PSO-LQR-I controller

Figure 3.11: Frequency response of area-2 if 1% load variation occurs in area-1 using
PSO-LQR-I controller
Based on the above results, the variation in frequency for 1% load change in area-1 and area-2 while changing the load in one area at a time in each case is shown in Table 3.2.

Table 3.2: Performance Comparison of Integral, LQR-Integral and PSO-based State Feedback Controller

<table>
<thead>
<tr>
<th>Frequency Response (peak undershoot) of power system Area-1 and power system Area-2 for 1% load deviation in Area-1</th>
<th>Integral controller</th>
<th>LQR-Integral controller</th>
<th>PSO-based state feedback controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>delf11(Hz)</td>
<td>-0.07</td>
<td>-0.03</td>
<td>-0.015</td>
</tr>
<tr>
<td>delf21(Hz)</td>
<td>-0.07</td>
<td>-0.04</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

Peak undershoot of Area-1 and Area-2 for 1% load change in Area-2

<table>
<thead>
<tr>
<th>Peak undershoot of Area-1 and Area-2 for 1% load change in Area-2</th>
<th>Integral controller</th>
<th>LQR-Integral controller</th>
<th>PSO-based state feedback controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>delf12(Hz)</td>
<td>-0.12</td>
<td>-0.04</td>
<td>-0.03</td>
</tr>
<tr>
<td>delf22(Hz)</td>
<td>-0.12</td>
<td>-0.03</td>
<td>-0.015</td>
</tr>
</tbody>
</table>

Figure 3.12: Comparative analysis of various controllers for 1% load change in area-1
It is concluded from the Figure 3.6 – Figure 3.11 that parameters integral gain ($K_i$), frequency bias ($B_i$) and speed regulation parameter ($R_i$) are best optimized with proposed PSO based state feedback controller (PSO-LQR-I) as the response is converged to global optima, whereas the existing integral controller and LQR-I controller have not fully optimized the parameters and the response is converged to local optima [29,65]. PSO-LQR-I technique is showing best frequency performance in all time domain specifications such as rise time, peak over shoot, settling time and peak undershoot.

The table 3.2 shows frequency response comparison in terms of peak undershoot and peak overshoot of power system area-1 and power system area-2 for 1% load deviation in area-1 and area-2 at a time, it can be seen that the parameters like peak undershoot and peak overshoot minimum with proposed PSO based state feedback controller.

The comparison of responses of three controllers in Figure 3.12 and Figure 3.13 in terms of undershoot and overshoot for load variations shows that for same load variation
in different areas, the best performance is with PSO-based state feedback controller (PSO-LQR-I), then the LQR-I controller followed by integral controller performance.

3.8 SUMMARY

In this work the performance comparison of three controllers such as PSO-based state feedback controller, LQR-Integral controller and Integral controller performance has been done. The system investigated for frequency response enhancement comprises of two thermal systems with reheat turbines. The two systems are interconnected with tie line. For the comparison of the performance of three types of controllers, the transfer function model and state space model with eleven states are developed for the system. Transfer function model has been used for finding the response of integral controller and PSO-based state feedback controller. Further, the state space based model is designed to observe the LQR-Integral controller performance. The load variation in each area at one time is 1% and responses are obtained for each controller. The frequency variation is observed in both the areas although the load changes in one area only at one time. The frequency deviation is minimum when PSO-based state feedback controller is used followed by the LQR-Integral controller. The maximum variation in frequency is in case of integral controller, which is although fully optimized with powerful computational technique PSO. This comparison of frequency controller is essentially useful in identifying the most suitable frequency controller which controls the frequency upon load changes for the interconnected power network. The present comparison gives the most efficient PSO-based state feedback controller which gives the minimum frequency deviation upon load variation in any area.