CHAPTER - IV
SOME ASSOCIATE CURVES IN A SUBSPACE OF A FINSLER SPACE

32. INTRODUCTION:

In this chapter using the definitions of the TA, PA and BA-vectors defined in previous chapter we have defined six new curves. The first three curves are such that the TA, PA and BA vectors lie respectively in the geodesic surface determined by the vector tangential to the curves of the congruence and its intrinsic derivative. In case of remaining three curves the vectors \( DT^i/\text{Ds} \), \( DP^i/\text{Ds} \) and \( DB^i/\text{Ds} \) lie in the same geodesic surface. We have obtained their differential equations and studied some of their properties including their relationship with other well known curves.
33. **$T_{\lambda} -$ CURVES**:  

We define a curve $C$ to be a $T_{\lambda}$ - curve when the unit vector $Y^i$ lies in the geodesic surface determined by $\lambda^*_{(\mu)}$ and $D\lambda^*_{(\mu)}/Ds$.

Thus we have

\[(33.1) \quad T^i = a_{(\mu)}\lambda^*_{(\mu)} + b_{(\mu)} D\lambda^*_{(\mu)}/Ds,\]

where $T^i$ is given by (22.5).

Equation (22.5) when multiplied by $g_{ij}(x,x')\lambda^*_{(\mu)}$ gives

\[(33.2) \quad g_{ij}(x,x')T^i\lambda^*_{(\mu)} = \pm \sin\theta_{(\mu)}.\]

Multiplying equation (33.1) by $g_{ij}(x,x')\lambda^*_{(\mu)}$ we get

\[(33.3) \quad a_{(\mu)} = \sin\theta_{(\mu)}.\]

From (33.1) and (33.3) we obtain

\[(33.4) \quad b_{(\mu)} = \cos\theta_{(\mu)}/K^*_{\lambda},\]

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where

\[(33.5) \quad K_\lambda^i = g_{ij}(x, x')(D\lambda_{(\mu)}^{ri}/Ds)(D\lambda_{(\mu)}^{sj}/Ds)\]

Multiplying equations (33.1) by \(g_{ij}(x, x')dx^j/ds\) and using equations (33.3) and (33.4) we obtain

\[(33.6) \quad \{K_\lambda^i \sin \theta_{(\mu)} + g_{\alpha\beta}(u, u')Q_{(\mu)}^{\alpha}\theta_{\beta}/ds\cos \theta_{(\mu)} = 0,\]

which yields:

**Theorem (33.1)**

When a curve is both a \(T_\lambda\)-curve and a \(N^*\)-curve it satisfies one of the following

(i) the magnitude of the vector \(D\lambda_{(\mu)}^{ri}/Ds\) is zero,

(ii) the vector \(\lambda_{(\mu)}^{ri}\) is orthogonal to the tangent to the curves \(C\).

(iii) the vector \(\lambda_{(\mu)}^{ri}\) is parallel to the tangent to the curve \(C\).
Multiplying equation (33.1) by $g_{ij}(x,x')q^i$ and using equation (33.3) and (33.4) we obtain

\begin{align}
(33.7) \quad \cos\theta_{(\mu)}[K^*_\lambda \cot\theta_{(\mu)} \{g_{\alpha\beta}(u,u')t^{*\alpha}_{(\mu)} p^\beta + \\
+ \sum_{\nu} C^*_\{(\mu\nu)\} R^{*-1}_{(\nu)} \} - \{g_{\alpha\beta}(u,u')Q^*_\{(\mu)\} p^\beta + \\
+ \sum_{\nu} r^{*}_{(\mu\nu)} R^{*-1}_{(\nu)} \} ] = 0.
\end{align}

Hence:

**Theorem (33.2)**

When a curve is both a $\mathfrak{T}_\lambda$-curve and a $R^*$-curve it satisfies one of the following

(i) the magnitude of the vector $D\lambda^*_{(\mu)}/Ds$ is zero.

(ii) the vector $\lambda^*_{(\mu)}$ is orthogonal to the tangent to the curve $C$,

(iii) the given curve is a hyperasymptotic curve,

(Mishra and Sinha, 1965).
Multiplying equation (33.1) by $g_\eta(x, x')\eta^{(r)}_\eta$ we obtain on simplification the following equation:

\[(33.8) \quad \cos\theta_{(\mu)} [\cot\theta_{(\mu)}K^*_{(\mu)}] - g_\eta^\alpha (u, u')t^{\alpha\beta} + \sum_{(\mu)} C^*_{(\mu)} \cos\zeta_{(\mu)} - \{g_\eta^\alpha (u, u')Q^{\alpha\beta}_{(\mu)} + \sum_{(\mu)} t^*_{(\mu)} \cos\zeta_{(\mu)}\} = 0,\]

which implies

**Theorem (33.3)**

When curve is both a $T_\lambda$-curve and an $O^*$-curve it satisfies one of the following:

(i) the magnitude of the vector $D\lambda^{(r)}_{(\mu)}/Ds$ is zero,

(ii) the vector $\lambda^{(r)}_{(\mu)}$ is orthogonal to the tangent to the curve $C$,

(iii) the given curve is a union curve.
34. $P_\lambda$ - CURVES:

We define a curve $C$ to be a $P_\lambda$ - curve when the vector $P^i$ lies in the geodesic surface determined by $\lambda^*_i(\mu)$ and $D\lambda^*_i(\mu)/Ds$.

Thus we have

\[(34.1) \quad P^i = C_{(\mu)} \lambda^*_i(\mu) + d_{(\mu)} D\lambda^*_i(\mu)/Ds,\]

where $P^i$ is given by (22.9)

Multiplying equation (34.1) by $g_{ij}(x, x')\lambda^*_i$ we get

\[(34.2) \quad C_{(\mu)} = \sin \theta_{2(\mu)}.\]

From equation (34.1) and (34.2), we can easily obtain

\[(34.3) \quad d_{(\mu)} = \cos \theta_{2(\mu)}/K^*_\lambda.\]

Multiplying equation (34.1) by $g_{ij}(x, x')dx_j/ds$, we obtain on simplification

\[(34.4) \quad \cos \theta_{2(\mu)} [K^*_\lambda \cot \theta_{2(\mu)} \left\{ g_{\alpha\beta}(u, u') t^*_\alpha(t_{(\mu)}) du^\beta/ds \right\} -

\left\{ g_{\alpha\beta}(u, u') Q^*_\alpha(t_{(\mu)}) du^\beta/ds \right\} ] = 0 ,\]
Hence we have:

**Theorem (34.1)**

When a given curve is both a $P_\alpha$-curve and a $N^*$-curve, it satisfies one of the following

(i) the magnitude of the vector $D\lambda^*_\alpha / Ds$ is zero,

(ii) the first curvature vector to the curve $C$ is orthogonal to the congruence of curves,

(iii) the given curve is a hypernormal curve, (Rastogi and Trivedi, 1972).

Multiplying equation (34.1) by $g_{ij}(x, x')q^j$ we obtain

\[
\cos \theta_{2(\mu)} [K^*_\alpha K^*_h \sin \theta_{2(\mu)} + \{g_{\alpha\beta}(u, u')Q^\alpha_{(\mu)} p^\beta +
\]

\[
+ \sum_{(\mu, \nu)} r^*_\alpha R^*_\alpha_{(\nu)} ] = 0.
\]

Thus:

**Theorem (34.2)**

When a curve is both a $P_\alpha$-curve and a $R^*$-curve it
satisfies one of the following:

(i) the magnitude of the vector $\frac{D\lambda^{*i}_{(\mu)}}{Ds}$ is zero,

(ii) the given curve is a geodesic in $F_n$,

(iii) the first curvature vector to the curve $C$ is orthogonal to the congruences of curves,

(iv) the vector $q^i$ is parallel to $\lambda^{*i}_{(\mu)}$.

Multiplying equation (34.1) by $g_{ij}(x,x')\eta_{(r)}^i$ we get

$$
(34.6) \quad \cos\theta_{2(\mu)}[K^*_{\lambda} \cot\theta_{2(\mu)} \{g_{ab}(u,u')t^{*a}_{(\mu)}\xi^b_{(r)} + \\
+ \sum_u C^*_{(\mu)} \cos\xi_{(ur)} + \{g_{ab}(u,u')Q^{*a}_{(r)} + \]

$$

$$
+ \sum_u r^{*}_{(\mu)} \cos\xi_{(ur)} = 0,
$$

which yields the following:

**Theorem (34.3)**

When a curve is both a $P_\lambda$-curve and an $O^*$-curve it satisfies one of the following:

(i) the magnitude of the vector $\frac{D\lambda^{*i}_{(\mu)}}{Ds}$ is zero,
(ii) the vector $\lambda_{(\mu)}^{i}$ is orthogonal to $q^i$,

(iii) the given curve is a union curve (Behari and Prakash, 1960).

35. \underline{B_{k} - CURVE}:

We define a curve C to be a $B_{k} -$ curve when the vector $B^i$ lies in the geodesic surface determined by $\lambda_{(\mu)}^{i}$ and $D\lambda_{(\mu)}^{i} / Ds$

Thus for a $B_{k} -$ curve we have

\begin{equation}
B^i = e_{(\mu)}^{i} \lambda_{(\mu)}^{i} + f_{(\mu)} \frac{D\lambda_{(\mu)}^{i}}{Ds},
\end{equation}

where $B^i$ is given by (22.14)

Multiplying equation (35.1) by $g_{y}(x,x')\lambda_{(\mu)}^{i}$ we obtain

\begin{equation}
e_{(\mu)} = \sin \theta_{r(\mu)}.
\end{equation}

From equation (35.1) and (35.2) we obtain

\begin{equation}
f_{(\mu)} = \cos \theta_{r(\mu)} / K_{k}^{i}.
\end{equation}
Multiplying equation (35.1) by $g_{ij}(x,x')dx^j/ds$ we obtain

\begin{equation}
(35.4) \quad \cos \theta_{r(\mu)}[K^*_\lambda \cot \theta_{r(\mu)} \{g_{\alpha\beta}(u,u')t^a_{(\mu)}du^\beta/ds\} -
-\{g_{\alpha\beta}(u,u')Q^{*\alpha}_{(\mu)}du^\beta/ds\}] = 0.
\end{equation}

Hence:

**Theorem (35.1)**

When a given curve is both a $B_\lambda$-curve and an $N^*$curve, it satisfies one of the following:

(i) the magnitude of the vector $D\lambda^{*i}_{(\mu)}/Ds$ is zero,

(ii) the vector $\lambda^{*i}_{(\mu)}$ is orthogonal to $\eta^i_{(r)}$

(iii) the given curve is a hypernormal curve.

Multiplying equation (35.1) by $g_{ij}(x,x')q^j$, we obtain

\begin{equation}
(35.5) \quad \cos \theta_{r(\mu)}[K^*_\lambda \cot \theta_{r(\mu)} \{g_{\alpha\beta}(u,u')t^a_{(\mu)}p^\beta + \sum_{\mu} C^{*}_{(\mu)}R^{*\mu}_{(\nu)}\} -
-\{g_{\alpha\beta}(u,u')Q^{*\alpha}_{(\mu)}p^\beta + \sum_{\mu} r^*_{(\mu)}R^{*\mu}_{(\nu)}\}] = 0.
\end{equation}

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which yields the following:

**Theorem (35.2)**

When a given curve is both a $B_\lambda$-curve and a $R^*$-curve it satisfies one of the following:

(i) the magnitude of the vector $D\lambda^*_i/DS$ is zero,

(ii) the vector $\lambda^*_i$ is orthogonal to $\eta^i_{(r)}$

(iii) the given curve is a hyperasymptotic curve.

Multiplying equation (35.1) by $g_{ij}(x,x')\eta^i_{(r)}$, we obtain

\[
(35.6) \quad \cos \theta_{r(\mu)}[\sin \theta_{r(\mu)}K^*_\lambda + \{g_{uv}(u,u')Q^{*a}_{(\mu)}\xi^b_{(r)} + \\
+ \sum_{\nu} r^*_{(\mu\nu)} \cos \xi_{(3\nu)}\} = 0.
\]

which yields:

**Theorem (35.3)**

When a curve is both a $B_\lambda$-curve and an $O^*$-curve it satisfies one of the following:

(i) the magnitude of the vector $D\lambda^*_i/DS$ is zero,
(ii) the vector $\lambda^i_{(\mu)}$ is parallel to $\eta^i_{(r)}$

(iii) the given curve is a union curve.

36. **IT CURVE**:

We define a curve $C$ to be an IT Curve when the vector $DT^i/\text{Ds}$ lies in the geodesic surface determined by $\lambda^i_{(\mu)}$ and $D\lambda^i_{(\mu)}/\text{Ds}$.

Thus we have

$$DT^i/\text{Ds} = A_{(\mu)}\lambda^i_{(\mu)} + B_{(\mu)}D\lambda^i_{(\mu)}/\text{Ds},$$

where $DT^i/\text{Ds}$ is given by (22.6).

Multiplying equation (36.1) by $g_{ij}(x,x')\lambda^j_{(\mu)}$, we obtain

$$A_{(\mu)} = K_T \cos \phi_{(\mu)},$$

where $\phi_{(\mu)}$ is the angle between $DT^i/\text{Ds}$ and $\lambda^i_{(\mu)}$ and $K_T$ is the magnitude of the vector $DT^i/\text{Ds}$.

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From equation (36.1) and (36.2) we obtain

\[ B_{(\mu)} = K_\tau \sin \phi_{(\mu)} / K^*_{\lambda} \]  

Multiplying equation (36.1) by \( g_{ij}(x,x')dx^j/ds \) and using equations (36.2) and (36.3) we obtain

\[ K^*_{\lambda}[g_{\alpha\beta}(u,u')L^\alpha_{(\mu)} du^\beta / ds - K_\tau \cos \phi_{(\mu)} \cos \theta_{(\mu)}] = \]

\[ = K_\tau \sin \phi_{(\mu)} g_{\alpha\beta}(u,u')Q^*_{\alpha\beta} du^\beta / ds, \]

which yields the following:

**Theorem (36.1)**

When an IT-curve is both a N*-curve and a hypernormal curve (Rastogi and Trivedi, 1972) it satisfies either of the following:

(i) the magnitude of the vector \( D\kappa^*_{(\mu)}/Ds \) is zero,

(ii) the vector tangential to the curves, is orthogonal to the intrinsic derivative of the TA-vector.
Multiplying equation (36.1) by $g_{ij}(x,x')q^j$, we obtain

\begin{equation}
(36.5) \quad K_x^a\left[\{g_{\alpha\beta}(u,u')L_{(\mu)}^\alpha p^\beta + \sum_{\nu} M_{(\mu\nu)} R^{*\nu\nu}_{(\nu)}\} - \right.
\end{equation}

\begin{equation}
- K_T \cos \phi_{(\mu)} \{g_{\alpha\beta}(u,u')t_{(\mu)}^\alpha p^\beta + \sum_{\nu} C_{(\mu\nu)} R^{*\nu\nu}_{(\nu)}\}\right]
\end{equation}

\begin{equation}
= K_T \sin \phi_{(\mu)} \{g_{\alpha\beta}(u,u')Q_{(\mu)}^\alpha p^\beta + \sum_{\nu} r_{(\mu\nu)} R^{*\nu\nu}_{(\nu)}\}.
\end{equation}

Hence we have:

**Theorem (36.2)**

When an IT-curve is both a hyperasymptotic curve and a R*-curve (Rastogi and Trivedi, 1972) it satisfies either of the following:

(i) the magnitude of the vector $D\lambda^*_i/(D\nu)$ is zero,

(ii) the first curvature vector to the curve $C$ is orthogonal to the intrinsic derivative of the TA-vector.

Multiplying equation (36.1) by $g_{ij}(x,x')\eta_{(r)}^i$
we obtain

\[
(36.6) \quad K^*_1 \left[ \{ g_{\alpha \beta}(u, u') L^\alpha_{(\mu)} \zeta^\beta_{(r)} \} + \sum M_{(\mu \nu)} \cos \zeta_{(ur)} \right] - \\
- K_1 \cos \phi \{ g_{\alpha \beta}(u, u') t^{*}_{(\mu \nu)} \zeta^\beta_{(r)} + \sum C^*_{(\mu \nu)} \cos \zeta_{(ur)} \} = \\
= K_1 \sin \phi \{ g_{\alpha \beta}(u, u') Q^{* \alpha}_{(\mu \nu)} \zeta^\beta_{(r)} + \sum r^{*}_{(\mu \nu)} \cos \zeta_{(ur)} \}.
\]

which yields:

**Theorem (36.3)**

When an IT-curve is both a union curve and an O*-curve (Rastogi and Trivedi, 1972) it satisfies either of the following:

(i) the magnitude of the vector \( D\lambda^{*}_{(\mu)}/Ds \) is zero,

(ii) the binormals to the curve \( C \) are orthogonal to the intrinsic derivative of the TA-vector.

Multiplying equation (36.1) by \( g_{ij}(x, x') T^i \), we obtain

\[
(36.7) \quad K^*_1 \left[ K^*_1 \cos \phi_{(\mu)} \sin \theta_{(\mu)} \right] - \\
- \sin \phi_{(\mu)} \cot \theta_{(\mu)} \{ g_{\alpha \beta}(u, u') Q^{\alpha}_{(\mu)} \, du^\beta/\, ds \} = 0
\]
Thus we have:

**Theorem (37.4)**

When an IT-curve satisfies one of the following:

(a) the congruences of curves are in the direction of the intrinsic derivative of the TA-vector,

(b) the given curve is a N*-curve,

(c) the congruence of curves is orthogonal to the tangent to the curve C,

then it satisfies one of the following:

(i) the magnitude of the vector $D\lambda^i_{(\mu)}/Ds$ is zero

(ii) the congruence is orthogonal to the intrinsic derivative of the TA-vector,

(iii) the congruence is parallel to the tangent to the curve C, provided $K_T=0$.

**Remark**

Condition (a) and (ii); (c) and (iii) do not hold simultaneously.
37. **IP Curve**: 

We define a curve \( C \) to be an IP-curve when the vector \( DP^i/\text{Ds} \) lies in the geodesic surface determined by \( \lambda^*_{(\mu)} \) and \( D\lambda^*_{(\mu)}/\text{Ds} \).

Thus for an IP-curve, we have

\[(37.1) \quad DP^i/\text{Ds} = \overline{C}_{(\mu)}\lambda^*_{(\mu) i} + \overline{D}_{(\mu)} D\lambda^*_{(\mu) i}/\text{Ds},\]

where \( \overline{C}_{(\mu)} \) and \( \overline{D}_{(\mu)} \) are arbitrary constants to be determined and \( DP^i/\text{Ds} \) is given by (22.11).

Multiplying equation (37.1) by \( g_{ij}(x,x')\lambda^*_{(\mu) j} \), we get

\[(37.2) \quad \overline{C}_{(\mu)} = K_p \cos \phi_{2(\mu)},\]

where \( \phi_{2(\mu)} \) is the angle between \( DP^i/\text{Ds} \) and \( \lambda^*_{(\mu) j} \) and \( K_p^2 = g_{ij}(x,x')(DP^i/\text{Ds})(DP^j/\text{Ds}) \).

From equations (37.1) and (37.2) we obtain

\[(37.3) \quad D_{(\mu)} = K_p \sin \phi_{2(\mu)}/K_p^*,\]

Multiplying equation (37.1) by \( g_{ij}(x,x')ds^i/\text{ds}, g_{ij}(x,x')q^i \) and \( g_{ij}(x,x')\eta^i_{(r)} \) and using (37.2) and (37.3) we
obtain the following equations respectively:

\[(37.4)\]

\[K_1^* \left[ \epsilon_{\alpha\beta}(u, u') R_{(\mu)}^\alpha \frac{du^\beta}{ds} - K_p \cos \phi_{2(\mu)} \cos \theta_{2(\mu)} \right] = \]

\[= K_p \sin \phi_{2(\mu)} \epsilon_{\alpha\beta}(u, u') Q_{(\mu)}^\alpha \frac{du^\beta}{ds},\]

\[K_2^* \left[ \epsilon_{\alpha\beta}(u, u') R_{(\mu)}^{*\beta} + \sum_{\nu} O_{(\mu\nu)}^{*\beta} R_{(\nu)}^{*\beta-1} \right] - \]

\[= K_p \cos \phi_{2(\mu)} \epsilon_{\alpha\beta}(u, u') t_{(\mu)}^{*\beta} + \sum_{\nu} C_{(\mu\nu)}^{*\beta} R_{(\nu)}^{*\beta-1} \] =

\[= K_p \sin \phi_{2(\mu)} \epsilon_{\alpha\beta}(u, u') Q_{(\mu)}^{*\beta} + \sum_{\nu} r_{(\mu\nu)}^{*\beta} R_{(\nu)}^{*\beta-1} \]

and

\[k_1^* \left[ \epsilon_{\alpha\beta}(u, u') R_{(\mu)}^\alpha \xi_{(r)}^\beta + \sum_{\nu} O_{(\mu\nu)}^\alpha \cos \zeta_{(\nu)} \right] - \]

\[= K_p \cos \phi_{2(\mu)} \epsilon_{\alpha\beta}(u, u') t_{(\mu)}^\alpha \xi_{(r)}^\beta + \sum_{\nu} C_{(\mu\nu)}^\alpha \cos \zeta_{(\nu)} \] =

\[= K_p \sin \phi_{2(\mu)} \epsilon_{\alpha\beta}(u, u') Q_{(\mu)}^{\alpha\beta} \xi_{(r)}^\beta + \sum_{\nu} r_{(\mu\nu)}^\alpha \cos \zeta_{(\nu)} \] =

From equations (37.4), (37.5) and (37.6), we obtain the
following theorems:

**Theorem (37.1)**

When an IP-curve is both a hypernormal curve and a N*-curve, it satisfies either of the following:

(i) the magnitude of the vector $D\lambda_i^{*i}/Ds$ is zero,

(ii) the tangent to the curve $C$ is orthogonal to the intrinsic derivative of the PA-vector.

**Theorem (37.2)**

When an IP-curve is both a hyperasymptotic curve and a R*-curve, it satisfies either of the following:

(i) the magnitude of the vector $D\lambda_i^{*i}/Ds$ is zero,

(ii) the first curvature vector to the curve $C$ is orthogonal to the intrinsic derivative of the PA-vector.

**Remark**

A result similar to theorem (36.4) can also be obtained for IP-curves.
38. **IB-CURVES** :

We define a curve $C$ to be an IB-curve when the vector $DB^i/DS$ lies in the geodesic surface determined by $\lambda^{*i}_{(\mu)}$ and $D\lambda^{*i}_{(\mu)}/DS$.

Thus for an IB-curve, we have

\[(38.1) \quad DB^i/DS = E_{(\mu)}\lambda^{*i}_{(\mu)} + F_{(\mu)} D\lambda^{*i}_{(\mu)}/DS,\]

where $DB^i/DS$ is given by (22.16).

From equation (38.1) we easily obtain

\[(38.2) \quad E_{(\mu)} = \cos \phi_{r(\mu)} K_B\]

and

\[(38.3) \quad F_{(\mu)} = K_B \sin \phi_{r(\mu)}/K^*_\lambda,\]

where $\phi_{r(\mu)}$ is the angle between $DB^i/DS$ and $\lambda^{*i}_{(\mu)}$ and $K_B$ is the magnitude of the vector $DB^i/DS$.

Multiplying equation (38.1) by $g_{y(i)(i)} dx^i/DS$, $g_{y(i)}(x,x') q^j$ and $g_{y(i)}(x,x') \eta^j_{(r)}$ and using (38.2) and (38.3), we
obtain the following equations respectively:

\[ (38.4) \quad K^*_\lambda \{ \varepsilon_{\alpha \beta} (u, u') S^\alpha_{(\mu)} \} \frac{du^\beta}{ds} - K_B \cos \phi_{(\mu)} \cos \theta_{(\mu)} \]

\[ = K_B \sin \phi_{(\mu)} \varepsilon_{\alpha \beta} (u, u') \Omega^\alpha_{(\mu)} \frac{du^\beta}{ds}, \]

\[ (38.5) \quad k^*_\lambda \{ \varepsilon_{\alpha \beta} (u, u') S^\alpha_{(\mu)} \} P^\beta + \sum_{\nu} U_{(\mu\nu)} R^*_{(\nu)} - \]

\[ - K_B \cos \phi_{(\mu)} \{ \varepsilon_{\alpha \beta} (u, u') \Omega^\alpha_{(\mu)} \} P^\beta + \sum_{\nu} C^*_{(\mu\nu)} R^*_{(\nu)} \}

\[ = K_B \sin \phi_{(\mu)} \{ \varepsilon_{\alpha \beta} (u, u') \Omega^\alpha_{(\mu)} \} P^\beta + \sum_{\nu} r^*_{(\mu\nu)} R^*_{(\nu)}, \]

and

\[ (38.6) \quad K^*_\lambda \{ \varepsilon_{\alpha \beta} (u, u') S^\alpha_{(\mu)} \} \varepsilon^\beta_{(r)} + \sum_{\nu} U_{(\mu\nu)} \cos \zeta_{(ur)} \}

\[ - K_B \cos \phi_{(\mu)} \{ \varepsilon_{\alpha \beta} (u, u') \Omega^\alpha_{(\mu)} \} \varepsilon^\beta_{(r)} + \sum_{\nu} C^*_{(\mu\nu)} \cos \zeta_{(ur)} \}

\[ = K_B \sin \phi_{(\mu)} \{ \varepsilon_{\alpha \beta} (u, u') \Omega^\alpha_{(\mu)} \} \varepsilon^\beta_{(r)} + \sum_{\nu} r^*_{(\mu\nu)} \cos \zeta_{(ur)} \}, \]

which yield following theorems:

**Theorem (38.1)**

When an IB-curve is both a hypernormal curve and a

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N*-curve, it satisfies either of the following:

(i) the magnitude of the vector $D\lambda_{(\mu)}^*/Ds$ is zero,

(ii) the tangent to the curve $C$ is orthogonal to the intrinsic derivative of the BA-vector,

**Theorem (38.2)**

When an IB-curve is both a hyperasymptotic curve and a R*-curve it satisfies either of the following:

(i) the magnitude of the vector $D\lambda_{(\mu)}^*/Ds$ is zero,

(ii) the first curvature vector to the curve $C$ is orthogonal to the intrinsic derivative of the BA-vector,

**Theorem (38.3)**

When an IB-curve is both a union curve and an O*-curve, it satisfies either of the following:

(i) the magnitude of the vector $D\lambda_{(\mu)}^*/Ds$ is zero.

(ii) the binormals to the curve $C$ are orthogonal to the intrinsic derivative of the BA-vector.
REFERENCES

